Ibn Sīnā's assumptions on structure of propositions (Highly provisional)

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1 Introduction

Throughout his logical writings, Ibn Sīnā constantly uses analyses of sentences. These analyses are clearly based on some background assumptions about the nature of language and the relationship between syntax and meaning. But Ibn Sīnā never — as far as I know — sets out his linguistic assumptions systematically, and never tells us where he took them from. This paper is a very preliminary attempt to formulate a framework that makes sense of Ibn Sīnā's assumptions about sentence structure. We have to introduce some theoretical notions that don't translate anything in Ibn Sīnā; the test of the result is whether our framework allows us to give in modern terms a convincing account of what Ibn Sīnā did in fact say.

We will work with a kind of dependency grammar (see for example Matthews [3] Chapter 4). The basic notion of such a grammar is a binary relation between linguistic units A and B, expressed by saying that B *depends on* A, in symbols

$$B \longrightarrow A.$$

The arrow can be written at any angle, as long as the items at its foot and its head are the thing that depends and the thing it depends on. We can also describe the item at the head of the arrow as the *head*; more on this below.

The main criterion for saying that B depends on A is that Ibn Sīnā describes B as being 'attached to' (*yalḥaq 'ilā*, *yaqtarin bi* etc.) A. He is not entirely consistent about directions of attachment, but my impression is

that the inconsistencies are at the level of about 5% and are probably just stylistic aberrations.

A second but important criterion is that when Ibn Sīnā thinks of B as being attached to A, he tends to speak of the two together, A and B, as being the same thing as A, so that for example if B is replaced by B', he speaks of the result as the same item but with a different property. Compare the way we say 'My car has just had new tyres fitted', but we wouldn't — except as a joke — say 'My tyres have just had a new car fitted'. This way of speaking, treating the dependent item as an accidental property of the head, is thoroughly unfortunate; Ibn Sīnā himself makes a penetrating attack on it (see section 5 below).

In many places Ibn Sīnā says that some compound phrase plays a role 'as a whole' (*jumla*, *majmū^c*), or even that the internal contents of some phrase are unreachable for certain purposes. To represent this I draw a box around the phrase, and I refer to the process of putting an item in a box as 'grouping' (with Adobe Illustrator in mind, perhaps). Strictly this notion is an addition to the pure dependency framework.

The notion of grouping connects with a useful idea introduced by Leonard Bloomfield in his *Language*:

> ... the resultant phrase may belong to the same form-class as one (or more) of the constituents. For instance, *poor John* is a proper-noun expression, and so is the constituent *John*; the forms *John* and *poor John* have, on the whole, the same functions. Accordingly, we say that the English character-substance construction (as in *poor John*, *fresh milk*, and the like) is an *endocentric* construction. (Bloomfield [2] p. 194)

In an endocentric construction the compound phrase $A \longleftarrow B$ plays the same linguistic role (in some appropriate sense of linguistic role) as its head A. So if A in turn can be made dependent on some third unit C, then so can the grouping of $A \longleftarrow B$.

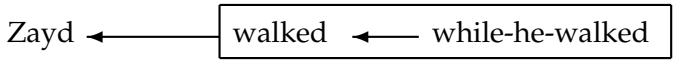
We see a concrete instance of this in section 10 below. Leaving out some details not relevant here, Ibn Sīnā notes first that 'walked' can be attached to 'Zayd', forming the sentence

$$\text{Zayd} \longleftarrow \text{walked}$$

But also 'while-he-walked' can be attached to 'walked', giving an endocentric compound

$$\text{walked} \longleftarrow \text{while-he-walked}.$$

This allows us to group the compound and attach it to 'Zayd' in the same way that we attached 'walked':



But many constructions are not endocentric. One that Ibn Sīnā discusses in several places is the construction which adds a quantifier to a universal:

$$\text{every} \longrightarrow \text{human}$$

The construction is exocentric (i.e. not endocentric) because 'every human' is not — at least prima facie — a universal; you can't sensibly classify things into those which are every human and those which aren't every human. And in fact Ibn Sīnā definitely wouldn't want to group this compound, because the Ammonius-Russell semantics for quantifiers, which Ibn Sīnā endorses, requires us to be able to separate out the quantifier from the universal that it is attached to. (The Ammonius-Russell semantics says, for example, that 'Every A is a B' is true if and only if every thing that satisfies A also satisfies B.)

When he introduced the notion of endocentric constructions, Bloomfield made a connection with heads:

> In subordinative endocentric constructions, the resultant phrase belongs to the same form-class as one of the constituents, which we call the *head*: thus, *poor John* belongs to the same form-class as *John*, which we accordingly call the head ... (Bloomfield [2] p. 195)

Note that this is not our notion of head, because we count 'human' as the head of 'every human' although the construction is not endocentric. In fact this is one of the places where our second criterion for headness is most solid; Ibn Sīnā constantly regards the quantifier as an adjunct of the universal, and in his symbolisations he often uses the same symbol indifferently for a universal or a quantified universal.

While writing these notes I came across the project 'Arabic language computing applied to the Quran', Eric Atwell and Kais Dukes, Department of Computing, Leeds University. They are annotating the text of the Qur'an for computer analysis of its semantics. The sentence analyses that they are using are in terms of dependencies, and they are extremely close to those I am attributing to Ibn Sīnā. This is a complete coincidence — I never came across this group before, and their analyses reminded me of

nothing I know in medieval Arabic linguistics. I contacted them at once, and they told me that they take the head of a phrase to be what the traditional Arabic grammarians described as *ʿāmil*, i.e. the head controls the inflections of the dependent item. (For example in 'beautiful Nadia', 'Nadia' would be the head because it forces the adjective 'beautiful' to take a feminine inflection.) This criterion for headship is utterly different from the ones I used. I doubt that it plays any direct role in Ibn Sīnā's own thinking, because he was very much aware that there is little correlation between the inflections of different languages, so that they are not a good guide to the underlying semantics. I want to discuss this further with the Leeds group, for two main reasons. First, the coincidence between their analyses and my interpretations of Ibn Sīnā's analyses suggests some underlying reason that needs to be uncovered. And second, it raises the question whether Ibn Sīnā's analyses were influenced by the Arabic linguists at least as much as by the Greek-based logical tradition. Discussions of Kees Versteegh and others, about how far Greek antecedents influenced the early Qur'anic linguists, are almost certainly relevant here.

2 Meaning versus syntax

It should be stressed at once that Ibn Sīnā always regarded himself as *describing* languages, not as *inventing* them. He used standard technical terms and invented a few more; but this is a trivial countexample, since these terms were always used within Arabic (or Persian) and not as ingredients of an alternative artificial language.

So Allan Bäck is unfortunately wide of the mark when he says

> Ibn Sīnā claims that it is fruitless to hunt for rules of inference in ordinary language, which is so context-dependent. Instead, a technical language, based on philosophical analysis, is to be worked out, and logical rules to be devised for it. (Bäck [1] p. 95.)

Bäck bases this conclusion on mistranslations of *ḳāṣṣī* and *ḳawāṣṣ* on p. 108 of *ʿIbāra* as 'specialist(s)'. The first word should be translated as 'narrow (meaning)', which is one of its standard senses in Ibn Sīnā. Probably the second should be translated as 'careful speakers'. Neither here nor anywhere else that I know of in Ibn Sīnā's logic is there any suggestion that sentences are to be translated or analysed into a 'technical language, based on philosophical analysis'.

In fact Ibn Sīnā condemns the use of artificial languages:

> If we [do this with the expression], we are using it according to an artificial stipulation, as we explained. The effect [of the convention] is that when we say a sentence ϕ we don't at all mean by ϕ what ϕ ought to mean. Instead of that, when we say ϕ, we say it and it means something that the artificial convention says it should. And you learn that 'this is a stone' and other drudgery. (*Qiyās* 45.6–10)

He often appeals to the normal meaning (*mafhūm*) of expressions, and to their normal usage (*musta^{c}mal*).

Sentences of a language are obviously structured objects. Ibn Sīnā believes that the structure of a sentence reflects that of its meaning, so sentence meanings are structured objects too. But he calls attention to examples where sentences in one language translate into differently-structured sentences of another language. Any such example will show that some feature of sentence structure is purely at the level of syntax and not at the level of meaning. Logicians do their work at the level of meaning.

For example a complex meaning can be expressed by a complex phrase in one language and a single word in another:

> One should say firstly in all this that logic is not concerned with what happens in one language as opposed to another. ... It often happens that one language assigns an atomic expression to a complex meaning, ... while another language expresses this compound meaning only by a complex expression. (*^{c}Ibāra* 19.16–20. 3)

> The form of a construction sometimes varies from one language to another. For example the element added to the other element can come first in one language and second in another. There is nothing in nature to make subject and predicate come in one particular order in a sentence. (*^{c}Ibāra* 31.2–4)

Here Ibn Sīnā says explicitly that the order of words in a sentence is not an invariant of the meaning. (This was one of the main reasons why I chose the dependency format; dependencies can be written in any order.) This puts Ibn Sīnā sharply at odds with some of the leading western Scholastics, who used Latin word order to express things like scope, and often wrote as if Latin word order is the 'natural' order.

There are some cases where Ibn Sīnā says that elements A and B should be 'adjacent'. For example

> The quantifier should be adjacent to (*yujāwara*) the subject, and the copula should be adjacent to the predicate. (*ᶜIbāra* 112.115-113.1)

I am not sure how to take remarks like this. He may simply be saying that certain dependencies hold; but he could mean that some dependencies are so important that languages always put the two relevant words next to each other. (This is certainly false, but I don't know whether Ibn Sīnā would have known any languages that illustrate it. Today linguists tend to cite Sanskrit and Australian bushman languages.)

The analyses in this paper will all be of *sentences* and other expressions in a language, usually English. But the intention is to capture the structure of the underlying meanings, as Ibn Sīnā sees it. Ibn Sīnā has no special notation for meanings; he discusses them through expressions that have those meanings. Where I do want to refer to a meaning rather than a word that carries the meaning, I follow Jackendoff's notation. Thus [HORSE] is the meaning of 'horse', and [EVERY HORSE] is the meaning of 'every horse'.

3 Are the diagrams adequate for meaning?

In practice there are two requirements on a representation of the structures of meanings of propositions.

(a) *Adequacy for meaning* Given a dependency diagram of a proposition, we have to be able to read off from it what the meaning of the proposition is.

(b) *Reachability from the syntax* We have to be able to read off, from the surface form of the sentence, what dependency diagrams are possible for it.

Ibn Sīnā never discusses such things systematically, but piecemeal he says a good deal about them. Note that (b) has to depend on the language, but (a) is purely at the level of meaning and should be independent of the language. In this section we review (a).

If the same two items of meaning can be combined in two different ways to produce different results, then the formalism needs to be able to distinguish the kinds of combination.

We already distinguish the two directions of an arrow. But more than this is needed. For example we can combine [ZAYD] and [MUSICAL] to say

Zayd is musical.

But equally we can combine them to form the proposition

Zayd is not musical.

or the description

musical Zayd.

So there can be several different flavours of arrow going from 'Zayd' to 'musical'. In the formalism we can distinguish them with labels. In principle this probably as much as we need in order to guarantee adequacy for meaning.

In some examples below I put labels '+c' or '-c' on copula arrows to indicate that the copula is affirmative or negative. The notation '[+c]' means that there is no explicit item in the syntax to represent the affirmative copula. (Ibn Sīnā talks about this sort of thing a lot.)

One important problem for recovering meaning from diagrams is where an element has two other elements dependent on it, and we need to relate these other elements in order to make sense of the compound. For example we can attach 'white' to the universal 'person', and we can also attach 'every' to 'person':

$$\text{white} \longrightarrow \text{person} \longleftarrow \text{every}$$

How to interpret the phrase as a whole? In this particular case one's instinct is that 'white' and 'person' should first be grouped together to give the compound universal 'white person', and that the quantifier is understood as if it was attached to this group. But Ibn Sīnā goes nowhere near explaining this. The one contribution that he does make is to point out that 'every person' is not a universal; so maybe this blocks attaching 'white' to it, and hence 'white' has to be attached first. But note that this is a significant argument, because it rests on the meanings of the words and not on

any surface features of the syntax. We will come back to the question with some more testing examples below.

Ibn Sīnā explains in several places that attaching 'every' or 'all' to a universal produces a compound that can't be read as a universal, at least in any straightforward way. A universal is a structured object which includes among other things a 'nature' (*ṭabīʿa*) or 'whatness' (*māhiyya*), which we can regard as a criterion for dividing things into two categories: those which 'satisfy' the universal and those which don't. Ibn Sīnā's vocabulary in this area is quite subtle and I gloss over some details not relevant to sentence construction.

Ibn Sīnā observes that [EVERY LAUGHER] is not a universal that is satisfied by laughers; and since there is no other sensible candidate for it to be satisfied by, that establishes for practical purposes that it is not a universal at all. Ibn Sīnā does allow the 'twisted' (*munḥarif*) interpretation that [EVERY LAUGHER] is a universal that is true of all and only those things that are every laugher; but he is disparaging about the usefulness of this approach.

Thus:

> The nature [HUMAN] with no universal or existential quantifier added is just that. Of course it is; [ALL LAUGHERS] is not a description that each individual laugher satisfies. (*ʿIbāra* 57.9–11; cf. also 64.15–17)

Compare a famous passage from *Madḵal*:

> [ANIMAL] is a meaning, regardless of whether it is satisfied by things in the real world or in the mind. In itself it is neither universally quantified nor existentially quantified. If it was universally quantified, i.e if [ANIMAL] was universally quantified, then no individual could be an animal. What is universally quantified is [EVERY ANIMAL]. (*Madḵal* 65.11–14)

The point is that if [ANIMAL] was the same meaning as [EVERY ANIMAL], then to satisfy it, an individual would have to be every animal, which is absurd. For nearly a thousand years metaphysicians have been mistranslating this innocent linguistic observation in hopes of finding in it a profound piece of ontology; the mistranslation is said to have deeply influenced Duns Scotus.

4 Are the diagrams reachable from the syntax?

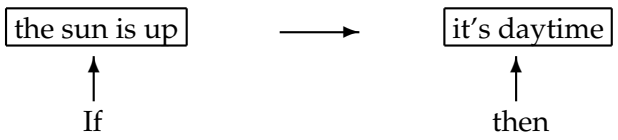
In principle there is no reason why each meaningful sentence of a language should correspond to just one possible semantic construction. Languages differ in how much they leave to the context of utterance and the intelligence of the speaker and hearer. Ibn Sīnā takes it for granted that there are bound to be structurally ambiguous sentences. He discusses a number of examples in *Safsaṭa*, criticising Plato for having ignored this kind of ambiguity.

Nevertheless, we need to have ways of expressing ourselves clearly. Discussing subject-predicate compounds and propositional compounds, Ibn Sīnā notes that a language should have a device for expressing the direction of the dependency. But in Arabic at any rate, these two constructions use different devices. In the case of propositional compounds, Arabic adds particles at the beginning of one or both of the clauses: 'If', 'then'. This is essentially the same device as adding inflections to nouns to indicate their relationship. Particles added for this kind of structural reason are regularly referred to by the Arabic linguists as 'addition' (*ziyāda*), one of Ibn Sīnā's favourite words.

In more detail,

> As to the propositional compound, it splits into distinct propositions between which there is a copula (*ribāṭ*) consisting of particles or expressions linked to one or both of the sentences. (*ʿIbāra* 40.17–41.1)

So a typical example might be



The two sentences are grouped into boxes, because Ibn Sīnā says the proposition combines two sentences. The arrow is from left to right because Ibn Sīnā says the lefhand proposition expresses a condition on the righthand one.

In passing, I remark that for a logician it is not always clear which clause expresses a condition on the other. In the case 'If ϕ then ψ' we would all reckon that the first clause expresses the condition. But in the case 'ϕ only if ψ', which means much the same, which clause makes the condition? Ibn Sīnā discusses exactly this case at *Qiyās* 252.3f. His conclusion is that the

muqaddam expresses a condition on the *tālin*. Unfortunately this leaves us none the wiser, because these two words can mean 'first clause' and 'second clause', but equally well they can mean 'antecedent' and 'consequent'!

We turn to the copula construction which forms sentences by combining a subject term with a predicate term. Which is the head? Since Ibn Sīnā regularly says that the predicate is predicated 'onto' or 'to' (*ᶜalā* or *li*) the subject, and I think never uses any similar expression in the other direction, I will take the subject as the head. This construction is about as far from endocentric as one could get, since the inputs are generally nouns and end result is a sentence. So there is no hope of calling on our second criterion of headship, that Ibn Sīnā speaks of the sentence as really being the subject but with a predicate attached.

Arabic has several different devices for showing which is subject and which is predicate, the main one being word order. Ibn Sīnā claims that in this construction it's essential to have at least an implicit syntactic element that stands for the copula itself.

> So if one wants the inner aspect [of the proposition] to be imitated by the expression, the expression will need to contain three meaningful elements: something signifying the meaning of the subject, a second thing signifying the meaning of the predicate, and a third thing signifying the attachment and link between the former two. If the two meanings [HUMAN] and [ANIMAL] come together under the gaze of the mind, there is nothing in the two meanings themselves to determine that either one is the predicate or the subject, or more generally that either one of them is related to something else. (*ᶜIbāra* 38.4–8)

This is an argument from reachability, but it is wholly unconvincing when he fails to apply it to any other constructions. We can learn a bit more when we see what functions he gives to this syntactic element.

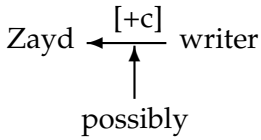
There is a class of verbs that are often used to supply copulas. Modern linguists know them as 'copular verbs', but Ibn Sīnā's name for them is *wujūdī*, a name of purely historical interest that is presumably a calque on the Greek *huparktikos*. These verbs play the expletive function of supplying a verb when there is no verb in the predicate. (Ibn Sīnā's word for 'expletive' seems to be *naqāwiṣu l-dalāla*, cf. *ᶜIbāra* 28.14.) They also supply a tense when there is no tense in the predicate, for example when the predicate is a noun. That could provide an essential part of the meaning, in which case the copular verb should probably be seen as forming part of the predicate

rather than a separate item. But Ibn Sīnā himself notes that often the tense is expletive too, because the intended meaning is tenseless, as for example in

> Every set of three things is (*takūnu*) odd-numbered. (*ʿIbāra* 39.14.)

A quite different reason for having a syntactic element to represent the arrow is that Ibn Sīnā sometimes wants to attach other elements to this one. He presumably intends that this corresponds to something at the level of meaning: he must think that it's possible to attach meanings not to other meanings, but to a dependency between meanings. This is a very odd idea, and strictly within the dependency framework it's impossible. Some examples that Ibn Sīnā gives are removable. For example he adds a negation particle to an affirmative copula; we can regard this as a device at the level of syntax for representing a negative copula.

But other examples are much less removable, for example where Ibn Sīnā attaches a modailty to a copula. We will go along with it to the extent of allowing diagrams like



But the idea is still odd. It pushes us back to the adequacy question: how are we supposed to get a meaning out of a diagram like this?

Another role of syntax is to constrain the possible combinations of elements. This is important for reachability: a highly constrained syntax tends to limit severely the number of diagrams that we need to consider, and hence cuts down on structural ambiguity. Ibn Sīnā regards it as one of the main roles of noun inflections and prepositions to enable or disable various combinations. For example

> In Arabic [the effect of] this is it becomes incorrect to connect to [a noun] each of the things that are naturally connected to nouns. One doesn't attach 'in' to 'Zayd' in the nominative. In the accusative, 'Zayd' won't have attached to it 'hit' or 'was' or 'is an animal'; and the same holds for the genitive 'Zayd's'. (*ʿIbāra* 14.10–13)

5 *taḥrīf*

The following remarks appear in *Qiyās*.

> Rather, the condition [expressed by the whole sentence] transforms the parts from being propositions. Thus when you say
>
> If p
>
> then what you say is neither true nor false. And when you say
>
> then q.
>
> what you say is neither true nor false, since the 'then' gets its justification from signifying the [relation of] following. Unless of course we are speaking a language which has no way of marking the second clause as being the second clause, apart from the fact that it follows [the first clause]. In this case the second clause on its own could be true or false, because one could read it in a way that leaves out part of its intended meaning. But if it is read in a way that gives it all of its real meaning [in context], then it is as it would be if 'then' was attached to it. (*Qiyās* 236.3–9)

The main point here is that even though 'The sun is up' says something either true or false, the phrase 'If the sun is up' doesn't. This seems uncontroversial. Perhaps the most interesting feature of the passage is the suggestion that there might be languages with no particles for 'if' and 'then'. There are in fact a few such languages in the north-west Caucasus, for example Abkhaz and East and West Circassian. These languages can't have close equivalents of 'if' and 'then' because they have no subordinate clauses. They are near enough to Persia that Ibn Sīnā might have known people who spoke the languages, or he might have read descriptions of these languages by Persian linguists. It's a pity he says no more about them.

But in *ᶜIbāra* he phrases himself differently:

> If you say 'If the sun is up' and then say no more, then you remove your statement (*qawluka*) 'The sun is up' from any connection with assent, and so this statement by itself (*hāḏā l-qawl waḥdahu*) is not true and not false. (*ᶜIbāra* 41 3f)

Here Ibn Sīnā seems to be claiming that an expression with a certain meaning has its meaning changed by being put into a context. His name for

this process is *taḥrīf*, 'deflection'. The word doesn't appear at this point in *ᶜIbāra*, but Ibn Sīnā uses it with essentially the same example at *Qiyās* 68.6–8 (twice) and at *Mašriqiyyūn* 61.11.

Ibn Sīnā's account is puzzling because it is so obviously a misdescription. The expressions 'The sun is up' and 'If the sun is up' are not the same expression. The first is true or false, the second isn't. Adding 'If' at the beginning of the sentence doesn't change any properties of the sentence, it creates a new expression which is not a sentence.

The reason for Ibn Sīnā's confusion seems to be his language for talking about heads, which we mentioned in the first section above. He seems to think of a sentence with 'If' added at the beginning as like a car with a potato in the exhaust — it's the same car, but now it doesn't work properly. Curiously Ibn Sīnā himself calls attention to the dangers of this loose language:

> There is a common kind of error about things that are joined together. It occurs through not recognising that a thing which is combined with another thing is not the whole arising from the [first] and the thing taken with it; just as one added to six, when we consider one together with six, is not the sum of one and six, which is seven. (*ᶜIbāra* 15.9–12)

He says precisely that in the construction

$$\text{one} \longleftarrow \text{six}$$

the head term 'one' has a different value from the constructed whole, so one should avoid identifying the two.

6 Atomic meanings

Compound meanings are built up from atomic or simple meanings. The paradigm examples of atomic meanings are the meanings of common nouns. These paradigm meanings have 'natures', just as universals do (and in fact many universals just are the meanings of common nouns). The nature of a meaning serves to determine what things satisfy the meaning.

A meaning (not necessarily atomic) is said to be 'particular' if it carries a feature indicating that it can be properly used only if there is exactly one thing that satisfies it. If it carries no such feature it is said to be a 'universal'. A meaning is said to be 'declarative' (*jāzim*) if it is properly used only to classify situations, in such a way that we can speak of a situation making

it true or false. A situation satisfies it by making it true; hence the use of 'satisfied' (*mawjūd*) to mean 'true'.

We can think of a meaning has having an 'argument', which is a slot where other meanings can be fitted in; for example [WHITE] has a slot where we can fit in the meaning [ZAYD], and the result of fitting [ZAYD] into the slot is a piece of information, either that Zayd satisfies [WHITE] (i.e. is white) or that he doesn't. This is mostly metaphor, but not entirely. Ibn Sīnā points to the personal inflections of verbs and regards them as 'indeterminate subjects'. Thus *yamšī* means 'he walks', and the element *ya* can be thought of as a personal pronoun which is indefinite — unless we use the verb in a way which makes it determinate who 'he' is. Nouns don't have indeterminate subjects in this way, at least according to Ibn Sīnā. Probably he missed a trick here; today many linguists follow Frege and regard common nouns as having an argument.

In Ibn Sīnā's account of meaning a very important point is that meanings have many implicit arguments that are held indeterminate unless one needs to make them determinate. Mostly they are optional arguments (i.e. there is no need to recognise them when using the meaning), but one argument Ibn Sīnā singles out as obligatory even though it is often not explicit. This is the time argument. In a world where things keep changing, every statement about the world should be regarded as referring to some point or interval of time.

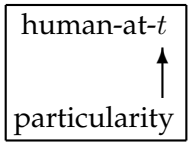
In an early section of *Qiyās*, section 1.3, Ibn Sīnā introduces a number of sentences which can't be understood unless we realise that the predicates, and in most cases the subjects too, carry a time argument, and that the normal meaning of the sentence in scientific discourse implies something about the quantification of these time arguments. Following Oscar Mitchell, I call such sentences 'two-dimensional': they have one dimension for the subject individuals and one dimension for times. It will become clear later that for Ibn Sīnā the relationship between the subject quantifiers and the time quantifiers in two-dimensional sentences is both crucial and difficult.

Time is certainly not the only kind of argument. At *ʿIbāra* 43.12–44.4 Ibn Sīnā mentions some optional arguments: the instrument, the part of the thing (Ethiopians are black outside but not necessarily inside) and the x in 'half of x'. Elsewhere he has longer lists. What is supposed to happen to these arguments when they are ignored?

I think his intended semantics is that when an argument is not explicitly mentioned, the default assumption (other things being equal) is that the argument is existentially quantified. But he adds that this doesn't mean

that in practice you can express an existential quantification just by not specifying one of the arguments. For example you can't express 'Someone in the world is walking' by saying *yamšī* (*ʿIbāra* 21.11). Presumably also he would have agreed that for some particular kinds of expression there are particular default assumptions; to say that an Ethiopian is black is to say that his skin is black, not that some part of him is black.

The net effect is as follows; I am not sure how far this goes beyond Ibn Sīnā's explicit intentions. Suppose we have a meaning [HUMAN]. There is an implicit time argument, which we can express with a variable: [HUMAN-AT-t]. We count a thing as human if and only if it is human at some time; though it's a fact about humanity that if you are human at some time then you are also human at any time while you exist, as Ibn Sīnā constantly repeats. So when we use the meaning [HUMAN] without calling any attention to the time argument, it's as if we were using [HUMAN] with t taken 'with particularity', in other words, existentially quantified. Thus in most contexts the word 'human' carries the implicit structure

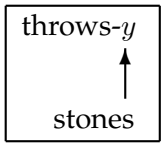


(I put the arrow pointing to t to show that this is the quantified variable.)

But there are places where the implicit argument plays a different role. I wish he said more about two examples that he introduces very early in *ʿIbāra*:

> 'herder of sheep' and 'thrower of stones' (*ʿIbāra* 12.14)

The semantics of these phrases requires us to notice that 'herd' and 'throw' are transitive verbs, so they have an object argument; the words 'sheep' and 'stones' qualify this argument. So the required diagram is along the lines



Ibn Sīnā's treatment of relations seems to contain many sound insights, but almost never explained properly.

We will see in section 8 below how this treatment of implicit arguments will lead Ibn Sīnā into serious trouble with negation, particularly since he has no usable notion of scope.

7 Quantifiers

We have already covered several points about quantification. A quantifier is attached to a universal and captures one of its arguments. In what we said about the secondary arguments, a further point was implicit: if a quantifier is attached at one of the optional arguments, the other arguments (including the main argument) remain available to be satisfied or not satisfied.

For example the verb 'herds y' has two arguments, the subject argument and the object argument. We can ask whether a particular ordered pair satisfies this verb. Ibn Sīnā is perfectly happy with this kind of logical technology, for example at *Qiyās* 476.2ff where he discusses the conditions under which the ordered pair of Zayd and a time satisfies the condition [RICH AT TIME t]. As we saw above, we can capture the object argument y with a kind of sortal quantifier, so that we get the compound phrase 'herds sheep'. This now has just its main argument, available for testing whether or not Zayd or ᶜUmar herds sheep. Ibn Sīnā doesn't have slick explanations of all this, but it seems highly likely that he would have accepted a Tarski-style explanation as in tune with his own thinking, at least for this kind of example.

More can be said. When we apply the universal [HERDS SHEEP] to [ZAYD], the sheep quantifier is hidden; as Ibn Sīnā would say, it is 'a part of the predicate'. Hence it is not available to play any syntactic or semantic role in the predication, except through the meaning of the predicate as a whole. Ibn Sīnā makes exactly this point about the predication

$$\text{the human} \longleftarrow \text{receptive to every skill.}$$

(ᶜ*Ibāra* 64.9–12.)

We turn to the quantifiers that play an explicit role in recognising the truth or falsehood of the sentence as a whole. (Modern logicians might talk of the 'quantifier prefix' here, but remember that for Ibn Sīnā the word order is not in principle relevant to the meaning.)

In [4] Zia Movahed rightly quotes *Qiyās*:

> In the sentence 'Whenever ϕ then ψ', the meaning of the expression 'whenever' doesn't just consist of universal quantification over the intention (??), as if one said 'Every occasion on which ϕ holds is one on which ψ holds'; but what is universally quantified in it is every circumstance connected with the expression, so that there is no set of circumstances in which ϕ is true and ψ is not also true. (*Qiyās* 265.1–5)

These remarks of Ibn Sīnā need some interpretation. Two natural questions are as follows.

(a) Is Ibn Sīnā saying that the time argument is really a circumstance argument, so that 'at all times' should be understood as 'in all circumstances'? Or is he saying that 'at all times' is really shorthand for 'at all times and for all parts of the objects in question and for all values of the indefinite arguments y, z, ...'? In other words, is the 'every' in 'every circumstance connected with the expression' a quantification over situations or over arguments of the expression?

The first interpretation is less radical, and it almost makes Ibn Sīnā quote Boole, who quantified over 'times and conjunctions of circumstances'. One argument in its favour is that Ibn Sīnā several times glosses 'time' (*waqt* or *zamān*) with the word *ḥāl*, which does indeed often mean 'circumstance'. (As when we say *kayfa ḥālak*? How are your circumstances?) Examples are at *Qiyās* 26.5, 65.4, 89.2. But another reading is that Ibn Sīnā uses *waqt* and *zamān* to mean intervals of time (as at *Qiyās* 128.5 and *Burhān* 231.9 Badawi), while *ḥāl* means a point of time.

The second interpretation is much more radical, and I hope it is the truth. It would put Ibn Sīnā's quantifiers at roughly the same level as Frege's 'latin letters', and certainly make Ibn Sīnā's logic adequate for all of first-order logic, if only Ibn Sīnā's standards of precision were improved. I won't pursue this further here. Maybe it's only a dream.

(b) Is Ibn Sīnā talking about what is in effect a quantification over possible situations? In other words, when we quantify over the circumstances or the implicit arguments, is the range of values limited to actual ones or does it extend to possible values?

This question has important consequences for the interpretation of quantifier modality below. Here I would say only that Ibn Sīnā says in a number of places that quantification is over actual instances of the subject term. I

know no places where he makes any concessions to the notion of a possible individual. In at least one case he is quite rude about this notion:

> ... unless of course they mean the quantifier which we can now form in our minds, and which some people have elaborated in shameless detail, that some potentially ashamed things are potentially people. In one place I did lay down some restrictions on this notion. If anything like this were legitimate, ... (*Qiyiās* 209.3–5)

(Where was this place?)

8 Scope of negation

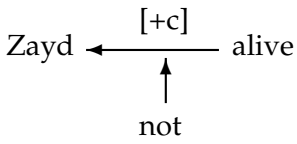
Ibn Sīnā allows negation to occur in two places. First, it can be attached to a universal so as to form another universal of a sort:

> For example 'non-human' is a compound of a noun and the particle of negation. (*ʿIbāra* 12.15f)

Second, it can be attached to predicate-plus-copula.

> ['is not alive'] is a compound of ['is alive'] and the particle of negation, as in the sentence 'Zayd is not alive'. The phrase 'is alive' is what, if it weren't for the particle of negation, would be affirmed about Zayd. (*ʿIbāra* 35.6–8)

This is a little puzzling. Ibn Sīnā has not previously suggested that the copula and the predicate form a unit together. Most likely this is one of the places where he thinks of the negation particle as attached to the copula:



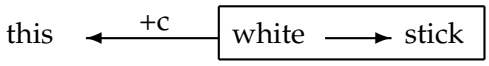
As far as I know, these are the only two places for negation that Ibn Sīnā officially recognises. But no Arabic speaker can avoid using the verb *laysa*, which stands at the beginning of a sentence and expresses 'It is not the case that'. So the language forces every Arabic logician to have a sentence negation, unlike Boethius who thought in Latin and (Chris Martin tells us) never encountered the idea of sentence negation. Nevertheless Ibn Sīnā

shows a strong tendency to push negations inwards by De Morgan-type rules whenever he can. Typical is this:

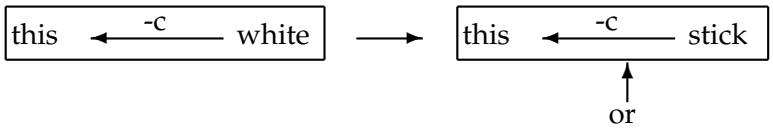
> Turning to contingent propositions, when we say
>
> > It's contingent that every B is an A.
>
> its contradictory negation is
>
> > It's not (*laysa*) contingent that every B is an A.
>
> or rather
>
> > It's either necessary or impossible that [simply every B is an A].
>
> given how contingency works. (*Qiyās* 49.16–50.1)

There is a question what operation Ibn Sīnā thought he was performing by moving the negation inwards in this example. But we can't tackle that until we see (below) how he handled modalities.

Ibn Sīnā can move negations inwards without difficulty. But it's far from clear what rules he is using. For example at *Qiyās* 492.17f he remarks that the negation of 'white stick' is not 'stick that is non-white'; otherwise the moon would be a non-white stick. One would expect him to explain what the negation of 'white stick' really is. But presumably his problem is that he doesn't recognise disjunctive terms; so he would need to move up to sentence level to carry out the negation. Thus the negation of



would have to be something along the lines



How would you shake this down into simple general rules?

A fortiori, how would you describe the scopes of the various negations? I can save you the trouble of looking up how Ibn Sīnā answers this question. He simply has no notion of the scope of negations. All his manipulations of negation may well be ad hoc paraphrases of particular sentence forms.

For example, suppose we have a simple subject-predicate statement with a family of existentially quantified parameters in the predicate. A modern logician would say: negating the statement negates the predicate, and when we move the negation onto the predicate term, the quantifiers will move out of the scope of the negation, so they have to switch between universal and existential. For example the negation of

> Zayd sometimes writes.

will be

> Zayd never writes.

Ibn Sīnā would get there by common sense, but there is no evidence that he would have any clue of the general rules. Asked for general rules, his first instinct is to remove the quantifiers altogether and require that the parameters be given fixed values:

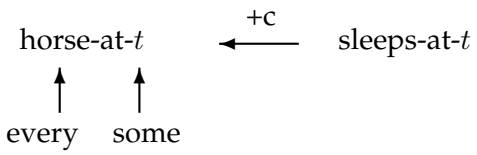
> This is the contradictory — I mean that the affirmative and the negative have opposite truth values. This opposition is determinate if the meaning of the affirmative proposition is determined from every aspect, and the negative proposition includes all the same determinations. (*ʿIbāra* 43.9f.)

9 Scope of quantification

There is a very interesting discussion at *Qiyās* section 1.5, where Ibn Sīnā aims to find the negations of some of his two-dimensional statements from section 1.3. Take for example the sentence

> Every horse sleeps.

We know from the discussion in section 1.3 that both subject and predicate have a time argument, and that the time argument in this sentence is existentially quantified. We could ignore the time argument on 'horse', because nothing that was a horse ever becomes anything else. But for technical reasons it will be helpful to keep it in this example. The subject quantifier is universal. So we have quantifiers of different types to compare:

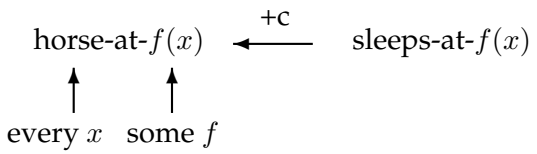


Before we start on the negation, we need to ask: how does the diagram determine the meaning 'Every horse sleeps sometimes' rather than 'There is a time at which every horse sleeps'? Recall that Ibn Sīnā has no notion of scope. He has to settle the matter in a different way. What's more, he knows he is in trouble:

> We land ourselves in the following difficulties. (*Qiyās* 39.15)

So a good deal of what he says is frankly casting around. Thus he observes (*Qiyās* 44.12) that the two quantifiers can't be combined into a single quantifier over both horses and times. Also he notes (*Qiyās* 39.3) that if we knew when each horse slept, we could without loss of information incorporate this information into the predicate and lose the need for a time quantifier. (As a modern logician would say, we can add a definable Skolem function.)

His next idea (*Qiyās* 39.8ff) is that we might adopt an indeterminate function, making the times depend on the horses in an undefined way. The next diagram is clearly beyond anything in Ibn Sīnā's text, but I believe it represents his thinking to the extent that his thought is coherent:



Now there is no longer a problem about how to relate the quantifiers. One of them is a quantifier over objects, the other is a quantifier over functions. They can be taken in any order, or independently, but the sentence is interpreted by fitting the individuals given by the first quantifier into the argument slots of the functions given by the second quantifier. People who know the Henkin-Hintikka theory of partially ordered quantifiers will find all this entirely familiar. Moreover it is the right direction to go in if you want to operate quantifiers without a notion of scope.

However, we wanted the negation of the sentence. At this point Ibn Sīnā becomes completely lost. In fact this is where he makes his remark about landing in difficulties. His problem seems to be this. Suppose we start by attaching negation to the sentence as a whole. Then the sentence apart from the negation should mean exactly the same as it did before, so the analysis in the diagram above should still work. But for some reason that he can't identify, the function f no longer makes sense. We can identify the reason for him. There is no sense in quantifying an existential quantifier within the scope of a single negation. But to explain this to him, we

would first have to explain to him the notion of the scope of a negation. And again, how could we do this within the framework of his assumptions about sentence structure?

10 Modality

Ibn Sīnā says that a modality can be attached to a simple subject-predicate sentence in either of two places:

> The modality should be adjacent to the copula if there is no quantifier. If there is a quantifier there are two places [for the modality], namely [adjacent to] the copula and [adjacent to] the quantifier ... You have the choice of attaching the modality in the first place or the second. (*ᶜIbāra* 113.1–3)

We can refer to modalities in these two places as copula modalities if they are attached to the copula, and quantifier modalities if they are attached to the quantifier. Ibn Sīnā is not completely consistent in his terminology: he sometimes (e.g. at *Qiyās* 30.16) refers to copula modalities as 'predicate modalities'. This is as if he attached the modalities to the predicates; but there will be scope problems if he moves a modality to the predicate and from a negative copula.

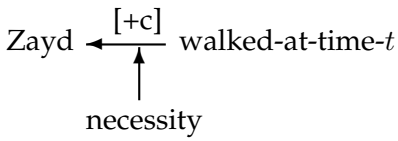
Ibn Sīnā says:

> But we will make it clear in what follows that this second view which arose [viz. that the modality goes on the quantifier] is not the appropriate one, and that the consideration of necessity and possibility is just in terms of the relationship of predicate and subject. The quantifier comes into play after that relationship, which has nothing to do with the quantifier. (*Qiyās* 31.3–5)

Ibn Sīnā seems to be saying that quantifier modalities are a late development and are not appropriate to use in exposition of the commentator tradition on Aristotle, which uses copula modalities throughout. (Also probably relevant: *Qiyās* 137.17f 'we find them ignoring the quantifier and paying no attention to it'. At *Qiyās* 207.5 Ibn Sīnā says that Aristotle never takes possibility as a quantifier modality.)

Here is an example of Ibn Sīnā's that illustrates the relationship between a copula modality and an attachment to the predicate. Consider the sentence 'Zayd walked with necessity'. With copula modality this analyses

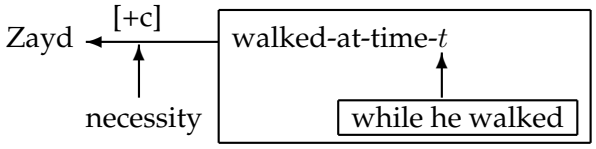
as



Ibn Sīnā remarks that we change the sense if we add an attachment to the predicate.

> I just mean that the [predicate's] being satisfied is not necessary in terms of its criterion without the addition of a condition … So there is a difference between saying 'Zayd walked with necessity' and saying 'Zayd walked with necessity while he walked'. ([c]Ibāra 74.11–13)

His intention seems to be that the predicate becomes 'walked while he walked', we need a box around the new predicate:



Note that we are inferring the box on the predicate from Ibn Sīnā's commentary, not from the facts of the sentence itself.

11 Quantifier modality

Virtually all (perhaps all) of Ibn Sīnā's examples of quantifier modality are with two-dimensional sentences. So we might ask which of the two quantifiers the modality is supposed to be attached to. At first sight it must be the subject quantifier, because the other is not always explicit. But on a closer look it seems that in cases of quantifier modality, Ibn Sīnā always takes the sentence to have the form 'There could be a time …' or 'Necessarily for

every time . . . '. Thus:

> This gave rise to a different view of modalities, which considered necessity and possibilitiy as applied to a proposition itself in terms of whether its quantifier is true or false, ignoring the question of how the predicate is related to the subject. Thus the sentence 'Every animal is a human' is possibly true, because one could imagine a time when there are no animals except humans, and in this case it would be true that 'Every animal is a human'. So this would be a true two-dimensional (*wujūdī*) premise. (*Qiyās* 30.8–12)

In this explanation, Ibn Sīnā emphasises that the time arguments of subject and predicate are united: a single time is considered for both. With the benefit of modern terminology we can add that he takes the time quantifier to have wider scope than the subject quantifier.

In the case of necessity too, modality on the quantifier implies wide scope for the time quantifier – in fact it has to have wider scope than the subject quantifier.

> Suppose 'Something white is with necessity not alive' is read with quantifier modality. Then this has to be true permanently; it would never be true that every white thing is alive. The quantifier-modality reading requires that this can be true, as you know. Whereas if the sentence is read not with quantifier modality, but as saying that some white things have 'animal' permanently false of them, then it follows from this that in the same way 'human' is false of these things permanently. (*Qiyās* 151.14–152.3)

This and many similar passages strongly suggest that in quantifier modality Ibn Sīnā intends the modality to be attached *to the time quantifier*.

This conclusion is supported by some remarks he makes about a contrast of 'absoluteness on the predicate' and 'absoluteness on the quantifier', for example:

> Or else what is being used is the absolute proposition whose absoluteness is not in the predicate but in the quantifier, since the quantifier makes it universally true at some time. (*Qiyās* 113.9f)

Note that here he is talking about something in the commentator tradition; he seems to imply that while the commentators never attached *modalities* to

the quantifier, they did sometimes attach *absoluteness* to it. This implication is correct. Bearing in mind that he is talking here about negative universal statements, uniting the time references is what we would describe as having a time quantifier with both subject and predicate in its scope. Thus:

$$\forall x \forall t \ (A(x,t) \rightarrow \neg B(x,t)).$$

This is precisely the statement that a thing that is an A at some time fails to be a B throughout the time in which it is an A. It is exactly the kind of statement that some of Ibn Sīnā's successors described as *waṣfī*, and it does appear already in Ammonius. In short it looks as if 'absoluteness on the quantifier' may be an alternative name for *waṣfī*. (Also *Qiyās* 152.9–11, 159.12–14; the second passage attributes 'absoluteness of the quantifier' to some unnamed 'they'.)

Now there is a problem of interpretation. What does it mean to say that there 'could be' a time at which ϕ? Given what we saw above about Ibn Sīnā's views on possible individuals, it would be unwise to suppose he is thinking about possible times. But there is no need to suppose this. He could simply mean 'There is no contradiction in supposing that there is a time at which ϕ'.

12 The Barcan formula

The Barcan formula, applied to a subject-predicate sentence, says that if it's possible that some A is a B, then there is some A that is possibly a B. If these two sentences are read as different, then the first must have a quantifier modality and the second must have a copula modality. So on the conclusions above, the first says

> It's not contradictory that there is a time at which something is both an A and a B. (1)

The second says

> There is something which is at some time an A and could be at some time a B. (2)

The Barcan formula in this case says that (1) entails (2).

Before we go to Ibn Sīnā's text, it's not clear that either (1) or (2) entails the other. The implication from (1) to (2) is the Barcan formula itself, so it clearly needs justification. But even the other direction is not clear. If

(1) was false then it would be contradictory to suppose that something is both A and B; but it doesn't follow that it would be contradictory to suppose that something is A and could at some time be B. So there seems no implication from (2) to (1).

There is one case where (2) does entail (1), namely where A is a necessary property of everything that satisfies it. In this case (2) implies that there is something which could be at some time both an A and a B, and this does contradict the falsity of (1).

Did Ibn Sīnā believe the Barcan formula, read as above? The place where he addresses the question most directly is in a passage cited by Movahed [5].

> As for the sentence
>
> > Some people — possibly they aren't writers.
>
> it is perhaps equivalent in a way (*qad yusāwī min jiha*) to
>
> > It's possible that some person is not a writer.
>
> though the first may be different from the second (*qad yuḵālifuh*) even if it follows from the second ... (ᶜ*Ibāra* 116.8)

Here 'person' is a necessary property, so the argument above shows that the second quoted sentence does entail the first. What Ibn Sīnā about the converse implication is guarded. He suggests that it might depend on some 'aspect' (*jiha*), which might well be some special condition. I think the most we can say is that he doesn't rule it out.

Actually this is par for the course. Ibn Sīnā is normally guarded in accepting modal principles unless they can be read off directly from non-modal ones. I mention three examples.

In the first, Ibn Sīnā is considering true sentences of the form 'Some A can be a B and some A can fail to be a B', where he makes it clear that the

modality attaches to the copula.

> This must be something that is not self-evident to beginners. From their point of view the predicate [in such cases] doesn't have to be both true of some of the subject and false of some of it. But the beginner must be aware that [if the predicate wasn't true in some cases and false in others] this would be a remote and outlandish possibility. It would be like having some species that are never satisfied by any individual at any time. There is someone who gives a length proof [of what the beginner doubts], but he uses premises unknown to the logician. Maybe he could get away with this proof if he was proving as one does in practical arts and crafts. (^{c}*Ibāra* 47.17–48.5)

So Ibn Sīnā is denying that it is a truth of logic that if some A is possibly a B then some A is or was or will be actually a B; and he seems very doubtful that it is a truth of any theoretical science. He is here rejecting one form of the so-called Principle of Plenitude.

A second example in *Qiyās*:

> It's plausible (*yušabbah*) that it is not correct to say that
>
> > Something which is contingent for each individual could fail to be true of any of them ever.
>
> ... It is not for the logician as a logician to know the truth about this. (*Qiyās* 48.13–17?)

So he is prepared to tolerate the implication 'If every A is contingently a B' (with modality on the copula) then some A is at some time a B'. But again he makes it clear that he doesn't regard this as a truth of logic.

A third example is in his justification of conversion of possible i-propositions: From 'Some A is possibly B' deduce 'Some B is possibly A'. He says:

> This is on the basis that this has an aspect which seems to me rather close [to the truth], namely that we say: If a thing is permissible and possible, then what follows from it is possible. (*Qiyās* 96.4f)

Since he is discussing a classical principle, he must (as we saw above) be talking about copula modality and not quantifier modality. In any case it

could be argued that if the principle he quotes was meant for quantifier modality, then it would be quite obviously correct and there would be no need for his cautious words at the opening. Applied to the two sentences under discussion, the principle says exactly that if some A is possibly a B then some B is possibly an A; which is precisely the conversion that he is trying to justify. In short, he is saying that the conversion itself could, when looked at from the right angle, be regarded as having some truth in it. But he never says he believes it.

References

[1] Allan Bäck, *On Reduplication: Logical Theories of Qualification*, Brill 1996.

[2] Leonard Bloomfield, *Language*, George Allen and Unwin, London 1933.

[3] P. H. Matthews, *Syntax*, Cambridge University Press, Cambridge 1981.

[4] Zia Movahed, 'Ibn-Sina's theory of conditional syllogism re-examined'.

[5] Zia Movahed, 'Ibn-Sina's anticipation of the formulas of Buridan and Barcan', in *Logic in Tehran*, ed. Ali Enayat et al., Association for Symbolic Logic and A. K. Peters, Wellesley Mass. 2006, pp. 248–255.