Ibn Sīnā on structure and scope in propositions

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Ibn Sīnā on language generally

The following statement is almost completely false:

Ibn Sīnā claims that it is fruitless to hunt for rules of inference in ordinary language, which is so context-dependent. Instead, a technical language, based on philosophical analysis, is to be worked out, and logical rules to be devised for it. (Bäck, On Reduplication: Logical Theories of Qualification p. 95.)

خاصّی Bäck bases this conclusion on mistranslations of

and خواصّ on p. 108 of ^cIbāra as 'specialist(s)'. They should probably be translated as 'narrow (meaning)' and 'careful speakers' respectively. In fact Ibn Sīnā condemns the use of artificial languages, and emphasises respect for the مفهوم (the normal meaning) and the مستعمل (normal usage).

Sketch of Ibn Sīnā's theory of sentence construction

Meanings are structured, and the structure of a sentence copies (and hence transmits to the hearer) the structure of the speaker's meaning.

Present problem: To describe Ibn Sīnā's notion of the structure of meanings.

Notation (Jackendoff, *Semantic Structures*): The meaning of 'horse' is [HORSE]. The meaning of 'every horse' is [EVERY HORSE].

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... logic is not concerned with what happens in one language as opposed to another. ... It often happens that one language assigns an atomic expression to a complex meaning, ... while another language expresses this compound meaning only by a complex expression. (*°Ibāra* 19.16–20. 3)

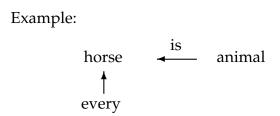
The form of a construction sometimes varies from one language to another. The element added to the other element can come first in one language and second in another. There is nothing in nature to make subject and predicate come in one particular order in a sentence. (*cIbāra* 31.2–4)

In particular Ibn Sīnā's notion of the structure of meanings *doesn't include linear ordering*.

What it does include is the notion of attaching one element *A* to another element *B*; I symbolise this by

 $A \longrightarrow B$

Also Ibn Sīnā sometimes says a construction (or a logical rule) takes a part of a sentence as a unit, without looking inside it. Then we put a box around this part.



The top arrow represents the copula, labelled 'is' to show it's affirmative. Ibn Sīnā also allows elements of meaning (e.g. negation, modalities) to be attached to the copula.

On the left, [EVERY HORSE] is formed. Ammonius rule: the sentence is true iff everything that satisfies [HORSE] satisfies [ANIMAL].

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Some things that Ibn Sīnā understood very well

Subjects and predicates generally contain many implicit parameters.

For example 'half' is a relational term, so implicitly it means 'half y'.

Also 'black' implicitly means 'black on the outside' or 'black all through' or indeterminately 'black at places z'.

Every real-world predicate contains an implicit time predicate:

'half of y at time t', 'black on the outside at time t', etc.

Ibn Sīnā gives famous examples of sentences which we can't understand without making explicit the time reference:

Everybody who travels from Rayy to Baghdad passes through Kermanshah.

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Everything that breathes in breathes out.

Distinguish the subject quantifier (the main quantifier) from the time quantifier(s). Ibn Sīnā points out we often have to study the relation between them.

For example in a sentence that is 'absolute on the quantifier', the time quantifier has widest scope.

He claims this form was used by the commentators before him. Presumably he means

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'Every *A* is a *B* for as long as it is an *A*', i.e. $\forall t \forall x \ (A(x,t) \rightarrow B(x,t)).$

Scope: something that Ibn Sīnā didn't understand at all

Ibn Sīnā has great difficulty forming negations.

He could always negate a sentence by putting ليس at the beginning. But he prefers to move negations inwards.

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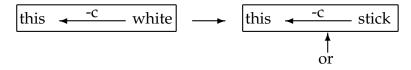
So why can't he negate 'white stick'? He says it's not 'white non-stick', but he doesn't tell us what it is. We can picture his problem. He has no disjunctive terms, so he has to negate 'This is a white stick' as a whole.

this
$$\stackrel{+c}{\longleftarrow}$$
 white $\stackrel{-}{\longrightarrow}$ stick

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Here '+c' stands for positive copula. We write '-c' for negative copula.

The negation:

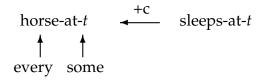


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What are the rules?

Much harder: negate the sentence

Every horse sleeps.



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To express the relationship between the quantifiers without using scope:

horse-at-
$$f(x) \leftarrow c$$
 sleeps-at- $f(x)$
 $\uparrow \qquad \uparrow$
every x some f

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Note the Skolem function f.

Henkin
$$\begin{pmatrix} \forall x \\ \exists f \end{pmatrix}$$
 equivalent to $\exists f \forall x$.

Ibn Sīnā thinks negation just adds a meaning element outside and leaves the remaining meaning unaffected.

But you can't Skolemise an existential quantifier within the scope of a single negation.

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Ibn Sīnā doesn't have the notion of negation scope, so he is very puzzled.

Modalities: Ibn Sīnā had insights about these

Ibn Sīnā says a modality is attached either to the copula or to the quantifier.

He adds that Aristotle and the commentators used only copula modality.

Some evidence that quantifier modality means modality attached to the time quantifier, with wide scope (though Ibn Sīnā doesn't have the words to say this).

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Ibn Sīnā's Barcan formula: If it's quantifier-possible that some A is a B, then some A is copula-possibly a B.

The premise probably means

'It's not contradictory that $\exists t \exists x (A(x, t) \land B(x, t))'$.

This avoids the notion of possible worlds, which seems to have been foreign to Ibn Sīnā's logic.

The conclusion probably means

 $\exists x \ (\exists t \ A(x,t) \rightarrow \text{possibly} \ \exists t' \ B(x,t')).$

(Attaching the modality to the copula means it's outside the time quantifier on the predicate.)

There is no immediate implication in either direction, unless *A* is a necessary property, like 'person' in Ibn Sīnā's example.

Ibn Sīnā's statement looks to me like cautious acceptance of his version of Barcan.

He accepts several modal statements with various degrees of reservation, and sometimes with a comment

Maybe this is true, but it's not a question of logic to determine whether it is.

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