Ibn Sīnā states and applies properties of temporal logic

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SIHSPAI, Paris October 2014

http://wilfridhodges.co.uk/arabic45.pdf

We consider Ibn Sīnā, *Qiyās* i–iv from his *Šifā'*.

In these books Ibn Sīnā introduces two forms of logic.

The *second* (mainly in *Qiyās* iii, iv) is Aristotle's logic of 'mixed syllogisms' as reported in the Arabic Aristotle.

This logic uses three alethic modalities: 'necessary' (*ḍarūrī*), 'possible/contingent' (*mumkin*) and 'absolute' (*muṭlaq*).

The *first* logic is one built around
some sentences that Ibn Sīnā introduces in *Qiyās* i.3
and in the parallel passage of *Mašriqiyyūn*,
written a little later than *Qiyās* but before *Išārāt* as we have it.

I will call these sentences *two-dimensional*, following Oscar Mitchell who studied similar sentences in the 1880s.
The second dimension is *time*. Example:

> Everybody who writes moves his hand all the time
> he is writing.

Formalised examples:

(a-d) Every (sometime-)B is an A all the time it exists.
(a-ℓ) Every (sometime-)B is an A all the time it's a B.
(a-m) Every (sometime-)B is an A sometime while it's a B.
(a-t) Every (sometime-)B is an A sometime while it exists.
(e-d) Every (sometime-)B is throughout its existence not an A.
(i-ℓ) Some (sometime-)B is an A all the time it's a B.
(o-t) Some (sometime-)B is sometime in its existence not an A.

'd', 'ℓ' etc. are based on names suggested by Ibn Sīnā.
In order of decreasing strength:
d = *ḍarūrī*, ℓ = *lāzim*, m = *muwāfiq*, t = *muṭlaq al-ʿāmm*.

Major Problem: To disentangle the two-dimensional logic from the alethic modal logic in *Qiyās* iii, iv
and show how Ibn Sīnā relates the two logics.

Most (all?) discussions in print solve this problem by ignoring or downgrading the two-dimensional logic.

One possible reason is that *Mašriqiyyūn* is generally not taken seriously.

Ibn Sīnā himself compares *Mašriqiyyūn* with *Šifā'* in prefaces to both:

> *Šifā'* is more detailed, but biased towards the Peripatetics. *Mašriqiyyūn* removes that bias.
> (More details in Gutas' book.)

That account seems exactly right. Will somebody please get us a properly edited text of *Mašriqiyyūn*?

I think I can solve the Major Problem in broad framework.
A scientific account will take time and effort (in progress!), and I won't attempt it here.

It depends crucially on understanding what Ibn Sīnā means by the *qawānīn* (rules) of logic,
a topic he emphasises in *Qiyās* i.2.

If I am right, there will always be work to do on fitting particular passages into the framework.

We'll concentrate on one passage, *Qiyās* iii.2, pp. 140–144.
In this passage Ibn Sīnā uses the two-dimensional logic to solve a previously unrecognised problem in Aristotle's text, and to show a novel fact about the possible shapes of modal inferences—a fact that he will develop in *Išārāt*.

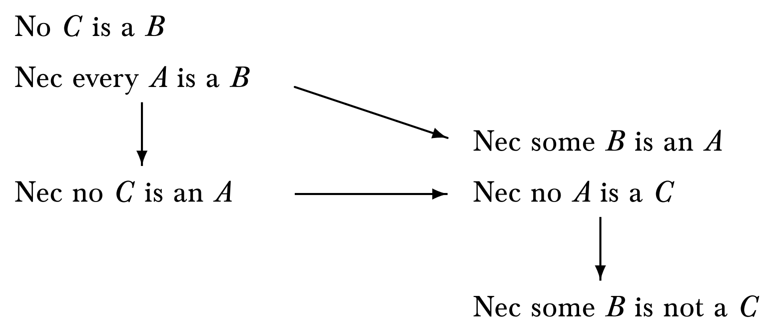
His use of two-dimensional logic in this passage is sophisticated and accurate to the fine detail.
It could still be accepted as a research contribution in a modern logic journal.

Aristotle claims that the following argument (modal *Camestres*) can't have 'with necessity' added to the conclusion.

No C is a B.
Every A is a B, with necessity.
Therefore no C is an A.

This is at *Prior Analytics* i.10, 30b20–31.
(Aristotle has B, A for A, B. We follow Ibn Sīnā.)

Aristotle's argument



'But nothing prevents one from choosing a B so that possibly every B is a C.'

So if the conclusion was valid 'with necessity',
then we could derive a false conclusion from true premises.

Robin Smith (commenting on *Prior Analytics* i.9, 30a25–28, a parallel argument):

> 'Aristotle's technique is sophisticated and flawless.'

Ibn Sīnā reckons that 'all the time it exists' is a kind of *necessity*,
and 'sometime in its existence' is a kind of *possibility*.

So if Aristotle's modal arguments work at all,
they should still work if we put d sentences for 'Necessarily' and t sentences for 'Possibly'.

In his *Qiyās* iii.2 Ibn Sīnā tries this with the argument that Aristotle rejected above.

No C is a B.
Every A is a B, with necessity.
Therefore no C is an A, with necessity.

Two-dimensional version, using weakest possible (t) for the absolute premise:

(e-t) Every sometimes-C is sometimes not a B.
(a-d) Every sometimes-A is always a B.
(e-d) Therefore every sometimes-C is always not an A.
VALID.

So Aristotle's refutation must be wrong. Ibn Sīnā checks it:

If every sometimes-C is always not an A,
then every sometimes-A is always not a C.
VALID.

If every sometimes-A is always a B,
then some sometimes-B is always an A.
INVALID. BUT ...

If every sometimes-A is always a B,
then some sometimes-B is sometimes an A.
VALID, and moreover

(i-t) Some sometimes-B is sometimes an A.
(e-d) Every sometimes-A is always not a C.
(o-d) Therefore some sometimes-B is always not a C.
VALID, AND IT'S EXACTLY ARISTOTLE'S CONCLUSION.
!!!

It seems that

- *Camestres* with necessary conclusion is valid.
- The steps in Aristotle's refutation of *Camestres* with necessary conclusion are also valid.

Do we have a paradox?

Aristotle claims that his data show we can choose B and C so that a false conclusion is derivable from true premises.

Ibn Sīnā checks what happens if we try to do this, using two-dimensional sentences.

Ibn Sīnā's analysis: we can choose B, C so that
(1) Every sometimes-C is at least once not a B, but
(2) every sometimes-B is at least once a C.

Example:
(1) Every human is at least once not laughing, but
(2) every laugher is at least once human.
Both true.

Now add the other premise 'Every A is always laughing'. (No matter what A is.)

This creates an inconsistency:
every A must be sometimes human by (2),
hence sometimes not laughing by (1).

Ibn Sīnā's conclusion:

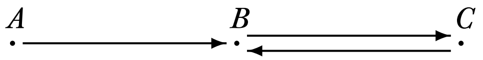
"So [Aristotle's] statement that 'nothing prevents this' is not true. The fact is just that nothing prevents it if one takes [the pair of sentences with terms B and C] on its own."

Paul Thom 1996 reaches the same conclusion—
apparently the first Westerner to do so:

"Aristotle's mistake was to conclude that because ab^a is compatible with the denial of Lab^i, the conjunction of ab^a with Lbc^a must be compatible with the denial of Lab^i."

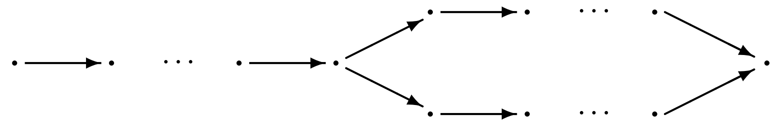
Why did Aristotle make his mistake?

Probable answer: the minimal inconsistent configuration

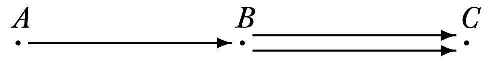


(where an arrow from A to B represents a sentence with subject term A and predicate term B)
can't occur with assertoric sentences.
Every minimal inconsistent set of assertorics has a circular configuration.

With 2D sentences the minimal inconsistent configurations all look like



which allows the above configuration and also



Ibn Sīnā knew this second configuration.
In his later *Išārāt* i.7 he gives a minimal inconsistent set illustrating it:

(a-d) Every A is a B throughout its existence.
(a-ℓ) Every B is a C throughout the time while it's a B.
(e-t) No B is a C throughout its existence.

Note the use of an ℓ sentence. Ibn Sīnā is right; nothing weaker than an ℓ will work for this configuration.

Broad observation: Ibn Sīnā is here using his ***extensional*** (‘***bil fiʿl***’) two-dimensional logic as a testbed for Aristotle's ***intentional*** alethic modal logic.

Following Ibn Sīnā involves understanding the two-dimensional logic itself.
That includes inference rules for multiple quantification, a topic barely touched in the West before the 19th century.

Further details at
`http://wilfridhodges.co.uk/arabic44.pdf`
a book in progress.