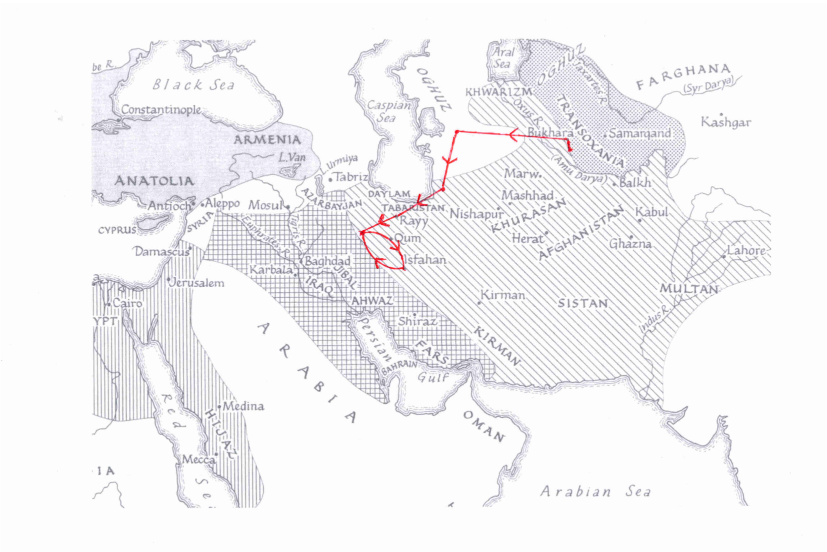
Avicenna sets up a modal logic with a Kripke semantics

Wilfrid Hodges
August 2017

`http://wilfridhodges.co.uk/arabic57.pdf`

Avicenna
(= Ibn Sīnā)
c. 980–1037



Ibn Sīnā's aim:

TO CREATE A MODAL LOGIC

Ibn Sīnā knew only one previous logic,
the categorical logic which he knew from the
'Arabic Aristotle' (Aristotle, 4th century BC,
translated into Arabic via Syriac).

The propositional logic of the Stoics
survived only as a few definitions with little context.

The Arabic categorical logic had four sentence types:

Every A is a B.
No A is a B.
Some A is a B.
Not every A is a B.

Aristotle catalogued all the logical relationships between two or three sentences of these forms.

He also chose some of these relationships as axioms, and derived the others from these axioms.

Aristotle also worked on a 'modal logic': the same four sentence forms with 'necessarily' or 'possibly' added.

The Syriac logicians (5th c. onwards) gave up trying to make sense of this. Ibn Sīnā's predecessor al-Fārābī (c. 870–c. 950) claimed to be the first Muslim logician to study the modal logic in Aristotle's *Prior Analytics*.

Today we can reconstruct a semantics for Aristotle's modal logic that validates his claims, but only by introducing some ontological notions. The current standard is Marko Malink, *Aristotle's Modal Syllogistic*, 2013.
Ibn Sīnā knew none of this reconstruction.

Avicenna took Aristotle's treatment of modal logic as a starting point.

Avicenna's view:

Aristotle made some bad mistakes,
and his successors tried to justify his mistakes
instead of looking directly at the problems.

Nevertheless Avicenna himself used earlier ideas whenever he could (like Gödel using Hilbert and Russell?).

Avicenna took a set of modal sentence types from the Arabic Aristotle:

Necessarily every A is a B.
Absolutely every A is a B.
Possibly every A is a B.
Necessarily no A is a B.
Absolutely no A is a B.
etc. etc.

We will call these forms *Mod*. Those with 'necessarily' or 'possibly' are the 'alethic modal forms'.

For simplicity I left out the modality 'Contingently'.

He also introduced a new set of sentence forms got by adding temporal quantification to the categorical forms.

For example

Every A is a B for as long as it exists.
Every A is a B, at least once while it exists.
At the times when it exists, no A is a B.
Every A is at least once in its existence not a B.
etc. etc.

Following Mitchell (1885) we call these 'two-dimensional sentences'. We also write them as *Temp*.

Avicenna calls the *Temp* sentences with 'for as long as it exists' the 'necessary' sentences (*ḍarūrī*, (d) for short).

He calls the *Temp* sentences with 'at least once while it exists' the '(broad) absolute' sentences.

Note the double use of 'necessary' and 'absolute', as *parts* of some *Mod* sentences and as *names* of some *Temp* sentences.
The 'absolute' *Mod* sentences won't concern us today.

Avicenna says that the necessary *Temp* sentences are the 'meanings' of the corresponding *Mod* sentences.

He could mean either:
(A) Conceptual analysis: The *Temp* sentence analyses the meaning of the concept 'necessary'; or
(B) Logical methodology: To understand the logic of the *Mod* sentence we should first translate it into the *Temp* sentence.

(A) probably describes the historical background to Avicenna's work.
But we'll see that (B) describes how he actually uses the *Temp* sentences.

Avicenna explains (many times) that in the *Temp* sentences the quantification over A is over things that actually are As at least once, not things that are possible As.
(I.e. he denies ampliation to the possible.)

This allows us to symbolise the *Temp* sentences in a two-sorted modal language.
A symbolisation of this kind was first proposed by Rescher and Van der Nat in 1974, except that they ignored Avicenna's 'existence' predicate, E below.

Thus we can write

'Every A is a B as long as it exists'

$$\forall x\,(\exists\tau A x\tau \to \forall\tau(Ex\tau \to Bx\tau)).$$

'Every A is not a B at least once during its existence'

$$\forall x\,(\exists\tau A x\tau \to \exists\tau(Ex\tau \wedge \neg Bx\tau)).$$

(For simplicity these formulas ignore existential import.)

The Arabic Aristotle lists the syllogisms (two-premise inferences) according to the modes of the premises, starting with those not involving possibility.
The schedule starts:

Both premises absolute
Both premises necessary
One premise absolute, the other necessary.

When Avicenna follows this schedule,
does he mean the *Mod* sentences that include 'Necessarily' or 'Absolutely',
or does he mean the *Temp* sentences called necessary or absolute?

Until recently it was taken for granted that he meant the *Mod* sentences.
But the evidence points to the *Temp* sentences.

(a) In his fullest mature treatments (the books *Syllogism* and *Easterners*), Avicenna starts by explaining the *Temp* sentences before the *Mod* sentences, and he repeatedly says that 'necessary' and 'absolute' mean the *Temp* sentences.

So for a reader who starts at the beginning and follows the definitions, *Temp* is the default.

(b) The *Temp* sentences have clear, objective and unambiguous meanings, unlike the *Mod* sentences.
So the *Temp* sentences are the natural place to start.

(c) Read for the *Temp* sentences, Avicenna's arguments all make good sense and the calculations are all correct.
Some are very delicate and sophisticated.
This is much less clear for the *Mod* reading.

In short, Avicenna's theory of the necessary and absolute *Temp* sentences is (after filling in a few gaps from hints he himself gives) both correct and complete.

He copies Aristotle's idea of trying to justify valid modal inferences by adapting the categorical scheme of axioms and derivation from axioms.
He identifies places where a new argument is needed, and shows how to find arguments to do the job.

So he creates a logic of 2D sentences,
just as Aristotle created a logic of categorical sentences.

But what about the logic of alethic modal sentences, *Mod*?

This emerges when Avicenna follows the second half of the listing of syllogisms in the Arabic Aristotle, which takes them in the order

Both premises possible
One premise possible, the other absolute
One premise possible, the other necessary

Since Avicenna doesn't use 'possible' as a name for any *Temp* sentences,
this has to be where he discusses the logic of *Mod*.

I will leave aside the possible + absolute, since they improperly mix *Mod* and *Temp*.

For the others, Avicenna has two methods for justifying the valid syllogisms,
both of them forms of reduction from *Mod* to *Temp*.

Method One: Since the theory of *Temp* is axiomatic,
Avicenna can derive all the valid inferences of *Mod* by showing that *Mod* sentences satisfy the same axioms.
(Not an issue of formal logic, but his proofs that they do satisfy these axioms are unsatisfactory.
Like some justifications of the axioms of ZFC?)

Method Two: He sets up a translation

necessary *Mod*	$\mapsto$	necessary *Temp*
possible *Mod*	$\mapsto$	absolute *Temp*

He proves an entailment in *Mod* by showing that the translated entailment in *Temp* holds.

Sometimes he describes this method as
'making the possible absolute',
but sometimes he uses it without any explanation.

What is Method Two about?

Formally, Method Two is exactly the same as giving a certain S5 Kripke-style semantics for *Mod*, and then proving entailments in *Mod* by giving model-theoretic proofs of them.

One way to show this is to describe the structures for the formulas we used to formalise *Temp* sentences.

A *categorical sentence* like

Some A is a B.

describes a structure M with a domain and subsets A^M and B^M for A and B. The sentence is true in M iff $A^M \cap B^M \neq \emptyset$.

A *2D sentence* describes a structure M with two domains, $\mathscr{O}^M$ of objects and $\mathscr{T}^M$ of times. Each symbol A, B etc. is interpreted by a subset of $\mathscr{O}^M \times \mathscr{T}^M$.

This structure can be repackaged as a family of structures N_τ of the categorical type, indexed by times $\tau \in \mathscr{T}^M$, and all with the same domain $\mathscr{O}^M$.
Aa holds in structure N_τ if and only if (a,τ) is in A^M.

Avicenna acts exactly as if he is using these Kripke-style structures as a semantics for *Mod*.
In Method Two he justifies inferences in *Mod* by translating them into statements about the models, and then proving these statements.

Avicenna has very little set theory. He can define ordered pairs and infinite sequences, but that's about it.

So instead he develops a proof theory of statements about the models. This is the theory of *Temp*.

Although Avicenna normally uses temporal language when discussing *Temp*, he drops hints that 'times' could more generally be situations. He also talks of 'possible times', so he is not limited to actual times.
(But he never speaks of possible worlds.)

Formally, generalising to possible situations is not needed, because the real world with actual times already contains enough interpretations of A, B etc. to fit the logic.

What does this semantics imply about *Mod*?

1. It gives the modal operators narrow scope. For example

   Necessarily every A is not a B.

is interpreted as

   Everything that is an A in at least one world
   is a non-B in all worlds where it exists.

(This is sometimes called a *de re* reading of the modalities.)

Since 'possibly an A' is read as 'an A in at least one situation', this semantics can be described as ampliating to the possible in all sentences of *Mod*.
Avicenna himself never accepts this description.
He consistently says he doesn't ampliate to the possible.

I think (though not everyone will agree) that his point is that the whole theory of *Mod*, including its semantics, can be set up without using any non-extensional notions.
We never need to mention 'possible entities'.

There are different interpretations of 'necessary'. Some of these do involve possible entities, but this belongs to the theory of interpretation, not to the formal logic of *Mod*.

2. 'Necessarily' comes out as 'in all worlds where it exists',
so the accessibility relation is universal and
the S5 laws hold (e.g. possibly $\Rightarrow$ necessarily possibly).

3. The universe is the same in all situations,
so the Barcan formula holds:

   If for all x necessarily $\phi(x)$
   then necessarily for all x $\phi(x)$.

Both 2 and 3 were noticed by Zia Movahed
in a paper of 2006.
BUT: the sentences of *Mod* never contain nested modalities, and they don't include any sentences of the form 'For all x necessarily $\phi(x)$'.

The nearest we get to the Barcan formula within Avicenna's own sentence forms (though we have to go outside *Mod*) is to consider what he calls 'modality on the quantifier', i.e. wide scope modality.

Then in his semantics we can show:

   If every sometimes-A is a B whenever it exists,
   then at all times every A is a B.

Moreover it seems Avicenna knew this and stated it.
Some people call the Barcan formula 'Ibn Sīnā's formula'.

Since Avicenna didn't have the language to explain what he was doing in modal logic, unsurprisingly his immediate successors didn't understand any of it.

Thus Averroes, early 12th c.: 'One should avoid the books of Avicenna if one wants to make a start in these arts; it would be wise to avoid them so as not to be misled by them.' And a little later Ibn Ghaylān: 'I exposed the errors of Avicenna in a field where no one would imagine he might err, namely logic, in numerous places therein'.

Ibn Ghaylān may have been part of the move (led by Rāzī) to consider alethic modalities and Avicenna's temporal modalities as different modalities that could be combined in the same sentence.

References

Nicholas Rescher and Arnold van der Nat, 'The theory of modal syllogistic in medieval Arabic philosophy', in Nicholas Rescher, *Studies in Modality*, Blackwell, Oxford 1974, pp. 17–56.

Zia Movahed, *Reflections on the Logic of Ibn-Sina and Suhrawardi*, Hermes, Tehran 2016.

Wilfrid Hodges and Spencer Johnston, 'Medieval modalities and modern methods: Avicenna and Buridan', *IfCoLog Journal of Logics and their Applications* 4 (4) (2017) 1029–1073 (volume in memory of Grisha Mints).