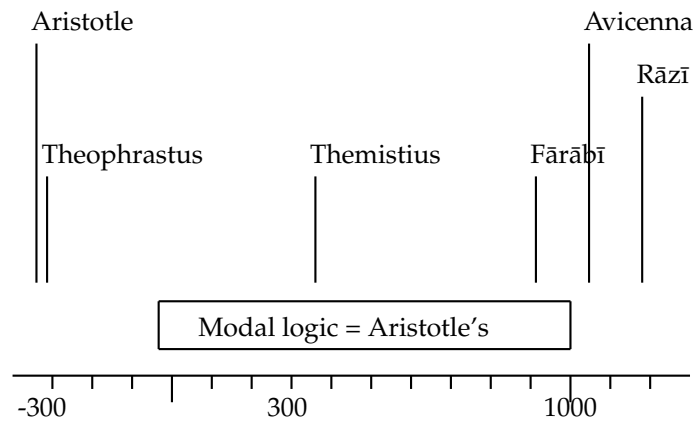


# Notes for 'The creation of two paradigms for modal logic: Avicenna and Rāzī'

Wilfrid Hodges

St Andrews, May 2019

## 1 Dates



- Aristotle 384–322 BC
- Theophrastus c. 371–287 BC
- Themistius c. 317–c. 385
- Al-Fārābī 870–950
- Avicenna (= Ibn Sīnā) c. 980–1037
- Fakhr al-Dīn al-Rāzī 1149–1209

## 2 Modal forms in Aristotle, Avicenna and Rāzī

The four quantified categorical sentence forms are (as normally written in Arabic logic from al-Fārābī onwards)

$$\begin{pmatrix} \text{Every} \\ \text{Some} \end{pmatrix} B \begin{pmatrix} \text{is} \\ \text{isn't} \end{pmatrix} \text{an } A.$$

$B$  is the subject term and  $A$  the predicate term. For simplicity we ignore the forms that are not quantified.

Before al-Fārābī the modal sentence forms were got from the categorical sentence forms by adding a modality to the ‘content’ part ‘is/isn’t an  $A$ ’. This modality is sometimes described as ‘on the predicate’.

Al-Fārābī in his *Long Commentary on the Prior Analytics* argued that Aristotle’s modal logic presupposes a second modality ‘on the subject’, i.e. qualifying  $B$ . For example

- [The dictum de omni states that:]  $A$  is predicated affirmatively or negatively, with whatever modality it happens to be of the modalities of
- (1) predication, of all of what is posited for  $B$ , and is described as a  $B$  by an affirmation only, with whatever modality it happens to be of the modalities of predication. ([4] 133.2–4)

Al-Fārābī’s book is mostly lost, but Averroes gives this and other quotations in one of his *Quaesita*, unfortunately not one of those translated into Latin. Other examples of al-Fārābī’s dictum de omni are at [4] 128.5–7, 129.6f and 146.9f. For further details see Hodges [2]. Note that at this date al-Fārābī was still normally writing the predicate  $A$  before the subject  $B$ .

The statement that al-Fārābī himself amplified the subject term seems to be a half-truth based on these passages quoted by Averroes. What al-Fārābī actually says in them is that Aristotle amplified the subject in dealing with some syllogisms. Avicenna foreshortened this, and all the statements by Rāzī, Ṭūsī and other later logicians attributing ampliation to al-Fārābī seem to be based on their reading of Avicenna’s remark. Bayhaqī reports that in the 12th century texts of al-Fārābī were hard to get hold of east of Syria.

### 2.1 Aristotle’s modalities

Since there is a difference according as something belongs, necessarily belongs, or may belong, ... it is clear that there will be different deductions for each of these, and deductions with differently related terms, one concluding from what is necessary, another from what is, a third from what is possible. (*Prior Analytics* i.8. trans. A. J. Jenkinson)

It’s generally agreed that under ‘possible’ Aristotle includes two cases, viz. the simple modality ‘possible’ and the Janus modality ‘possible and not necessary’, i.e. contingent. So there are really four modalities here. All except ‘what

is' are known as *alethic* modalities (after G. H. von Wright [17] p. 1f). Janus modalities are a conjunction of an affirmative and a negative.

## 2.2 Avicenna's modalities

Avicenna himself gives no list of what he considers the modalities used in his modal logic. The following is a *prima facie* list; in fact he uses all of these modalities except 8. The labels on the right are ours, though the first four are based on Avicenna's proposed terminology in his [9].

1. always during its existence	( <i>d</i> ) <i>arūrī</i>
2. all the time that it's a <i>B</i>	( <i>l</i> ) <i>āzim</i>
3. sometime while it's a <i>B</i>	( <i>m</i> ) <i>uḡāfiq</i>
4. sometime during its existence	<i>mu(t)laq</i> <sup><i>cāmm</i></sup>
5. necessarily	( <i>nec</i> )
6. possibly	( <i>pos</i> )
<hr/>	
7. sometime but not always during its existence	( <i>t</i> $\wedge$ $\neg$ <i>d</i> )
8. sometime but not always while it's a <i>B</i>	( <i>m</i> $\wedge$ $\neg$ <i>l</i> )
9. contingently	( <i>pos</i> $\wedge$ $\neg$ <i>nec</i> )

In *Easterners* [9] Avicenna gives a rather full list of modalities, including several that he never uses in modal logic; see Chatti [1].

## 2.3 Rāzī's modalities

Rāzī lists his modalities in his *Mulakhhaṣ* [12] 180.5–1284.8. Kātibī in his *Shamsīya* conveniently reports Rāzī's list as a list of six simple forms and seven 'compound' (i.e. Janus) forms:

1. necessarily	( <i>nec</i> )
2. always during its existence	( <i>d</i> )
3. necessarily given some condition on subject	( <i>cnd</i> )
4. all the time that it's a <i>B</i>	( <i>l</i> )
5. sometime during its existence	( <i>t</i> )
6. possibly	( <i>pos</i> )
<hr/>	
7. narrow conditioned	( <i>cnd</i> $\wedge$ $\neg$ <i>d</i> )
8. all the time that it's a <i>B</i> but not all the time it exists	( <i>l</i> $\wedge$ $\neg$ <i>d</i> )
9. sometime but not necessarily	( <i>t</i> $\wedge$ $\neg$ <i>nec</i> )
10. sometime but not always during its existence	( <i>t</i> $\wedge$ $\neg$ <i>d</i> )
11. just at some definite time	( <i>def</i> $\wedge$ $\neg$ <i>d</i> )
12. necessarily at some indefinite time, not permanently	( <i>indef</i> $\wedge$ $\neg$ <i>d</i> )
13. contingent	( <i>pos</i> $\wedge$ $\neg$ <i>nec</i> )

### 3 Avicenna on modal logic

Avicenna's works discussing modal logic, in probable chronological order of writing, are:

**Gems, logic part very early** *Uyūn al-ḥikma* (Gems—or possibly Springs—of Wisdom).

**Middle, before 1014** *Al-mukhtaṣar al-'awsaṭ fī al-manṭiq* (Middle Abridgment on Logic), [5].

**Deliverance, written c. 1014, published c. 1026** *Kitāb al-najāt*, [6].

**Guidance, c. 1023** *Kitāb al-hidāya*.

**Syllogism, c. 1023** *Al-qiyās*, [7], from the encyclopedic *Shifā'*.

**Easterners, c. 1027** *Manṭiq al-mašriqiyyīn*, [9].

**Pointers, c. 1030** *Al-išārāt wal-tanbīhāt* (Pointers and Indications), [10].

*Gems* uses only Aristotle's sentence forms. *Guidance* is too brief to be helpful.

### 4 Theophrastus's trio

Theophrastus, Aristotle's successor as leader of the Peripatetic School, wrote a *Prior Analytics* some of whose comments survive through Themistius, whose *Commentary on the Prior Analytics* survives in a Hebrew summary, probably translated via Arabic, [14].

Theophrastus says that the necessary is said of three things: the first [N1], and the most befitting the term [necessary], is that which exists perpetually all the time, as in our saying. "The heavens are ungenerated." ...

(2) The second [N2] is that which does not exist perpetually, but rather exists of necessity as long as the subject exists, like the existence of soul in man and heat in fire.

The third [N3] is the existent thing when it exists, of which it is said that it exists of necessity, like the sitting of Reuven when he sits, or the walking in that which walks, when it walks. ([14] p. 94)

Avicenna knows this classification, but as a classification of absolute (non-modal) propositions. He calls it the 'trio' (*tamthīl*).

(3) Some people ... recognise just three cases [of absolute proposition]: one is that the *B* is an *A* permanently, the second is that it is [an *A*] while it continues to be described as *B*, and the third is [that it is an *A*] while it continues to be described as an *A*. (*Syllogism* [7] 27.3–5)

The ambiguity about whether this is a list of necessary or absolute propositions goes a long way back; it is present in Themistius. The issue seems to be that the modalities of these propositions are temporal rather than alethic (and this makes them absolute rather than necessary), though what they express are kinds of necessity. Averroes [4] 118 refers to ‘necessary premises that are not necessary’!

## 5 Avicenna’s semantic expansions and sextet

### 5.1 Semantic expansions

These follow a standard Peripatetic recipe for removing ambiguities in a phrase. The phrase is repeated with comments added. The comments usually don’t relate to truth-conditions; rather they explain what is supposed to be happening or not happening in the mind of a person who thinks the phrase. In over twenty places Avicenna uses this recipe for clarifying the meaning of a modalisation of ‘Every *B* is an *A*’, following examples laid out by al-Fārābī under the name ‘said of all’ (dictum de omni).

Typical examples:

(4) Those things that are described, in act in the intellect, as being a *B*, in any way and at any time, given that they are described as being a *B*, no such thing is described as being an *A*, but we don’t know whether it is [not] an *A* at a particular time or permanently, regardless of whether it is together with this description of it at some specific time or at all times and that time together with its being a *B* permanently, or together with its being a *B* at some time of its being a *B*, or before that or after that. (*Middle* [5] 103.12–16)

(5) Each one of the things described and assumed to be actually (*bil-fiʿl*) a *B*, permanently or not permanently, in fact is also described as being an *A* without referring to when, and to which of the three segments of time it is in. (*Syllogism* [7] 26.18–27.2)

(6) Each one of the things that are described as a *C*, regardless of whether it is described as a *C* in mental assumption or in the world, and regardless of whether it is so described permanently or non-permanently, but rather however it is (*kayfa kāna*), is a thing described as a *B* without any addition that it is so described at such-and-such a time or in such-and-such a case. (*Pointers* [10] 93.9–14)

passage	mode	actual	permanent	however	mind	exist	assume
<i>Mid</i> 100.16–101.1a	-	Y	N	N	Y	N	Y
<i>Mid</i> 100.16–101.1b	-	Y	N	N	Y	N	Y
<i>Mid</i> 102.17–19	-	Y	N	N	Y	N	N
<i>Mid</i> 103.7–11	-	Y	Y	N	N	N	N
<i>Mid</i> 103.12–16	-	Y	Y	Y	Y	N	N
<i>Mid</i> 108.7–11	( <i>t</i> )	Y	N	N	N	N	N
<i>Mid</i> 108.21f	( <i>d</i> )	N	Y	Y	N	N	N
<i>Mid</i> 131.2–6	<i>nec</i>	N	Y	Y	N	Y	Y
<i>Mid</i> 132.10–12	-	N	N	Y	N	N	Y
<i>Del</i> 42.10–12	( <i>d</i> )	N	Y	N	Y	N	N
<i>Del</i> 43.12–14	<i>pos</i>	N	N	Y	N	N	Y
<i>Del</i> 66.6–9	<i>nec</i>	N	Y	N	N	N	Y
<i>Syl</i> 26.18–27.2	-	Y	Y	N	N	N	Y
<i>Syl</i> 31.15–32.1	( <i>d</i> )	Y	Y	N	N	N	N
<i>Syl</i> 33.11–14	<i>con</i>	N	Y	N	N	N	N
<i>Syl</i> 33.15–34.6	<i>pos3</i>	Y	Y	N	N	N	N
<i>Syl</i> 127.11f	<i>nec</i>	N	Y	N	N	N	N
<i>Eas</i> 64.3–6	-	Y	Y	N	N	Y	Y
<i>Eas</i> 64.14–16	-	Y	N	N	N	N	N
<i>Eas</i> 68.6–8	( <i>d</i> )	Y	N	N	N	N	N
<i>Eas</i> 69.12–14	( <i>d</i> )	N	N	Y	N	N	N
<i>Poi</i> 72.17–73.3	-	N	N	N	Y	Y	Y
<i>Poi</i> 93.9–14	( <i>t</i> )	N	Y	Y	Y	N	Y

These semantic expansions, taken as a whole, send the message that Avicenna imposes the same modality on the subject term in all cases, viz. that ‘every *B*’ means everything that is, was or will be an actual *B*, regardless of whether it is or is not necessarily a *B*, and regardless of whether it is an object in the external world or an abstract mental object. The various parts of this explanation correspond to columns in the chart above.

This needs some comments.

1. Street [16] presents Avicenna as choosing between two accounts of the subject term, which Street calls essentialist and externalist. But as the chart shows, Avicenna is choosing from among quite a large range of options, fighting on several fronts. As a result, Street conflates temporal issues (e.g. ‘now’ or ‘sometimes’) with ontological ones (e.g. in the external world or not). This reduction to just two accounts comes from Rāzī *Mulakhkhaṣ* [12] 140.11–143, though on my reading Rāzī is not trying to impose it on Avicenna. It should be added that in Street’s descriptions of Avicenna’s logic it’s often unclear whether he is intending to explain what Avicenna meant or what logicians from Rāzī onwards understood Avicenna to mean.

2. The second column in the chart indicates what modality Avicenna says he is explaining. (In several cases he is explaining the universal affirmative form in general, so no modality is indicated.) The chart shows that he never includes ‘actual’ when he is talking about an alethic modality. (The apparent counterexample *pos3* is his ‘narrowest possibility’, which is actually a temporal modality referring to the future.) He never says anything to confirm this dis-

tion, so it might be pure fluke, given the random variations in the chart. But the question has to remain open until we have some broader consensus about his attitude to propositions with alethic modality. On the account in Hodges and Johnston [3], he *ought* to say that the modality on the subject term in an alethic proposition is the modality that translates to  $(t)$ , and this is ‘possibly’. But he never says this, and we need to understand why.

3. Our list of Avicenna’s modalities, and the modalities in the chart, fail to mention one modality that Avicenna certainly does discuss. This is the modality ‘at a given time’, which includes the case ‘now’. At *Syllogism* [7] 136.5–7 Avicenna explains why he thinks this modality is too hard to handle in modal logic. Buridan did handle it in his divided modal logic, and the complexity of Buridan’s results illustrates what Avicenna was afraid of. Note that Street [16] p. 20f misquotes Avicenna *Pointers* [10] 95.11–14 as attacking this modality. In fact Avicenna here and in a parallel passage at *Syllogism* [7] 82.13–83.1 is dissociating himself from the different view that quantification is over ‘the present and the past’, a view that al-Fārābī had suggested in the light of *De Interpretatione* 18b28–33. (Street omits ‘and in the past’.)

4. The semantic expansions are not the only explanations that Avicenna offers for his reading of subject terms. See *Syllogism* [7] 20.1–21.12 for his clearest and fullest account.

## 5.2 Sextet

The sextet appears already in the early work *Middle* [5] 105.9–21:

An example of strict necessary is the sentence ‘The human with necessity is an animal’, meaning that for as long as the essence which is the human is satisfied, it is an animal. [1] One case of this is what is necessary permanently because the essence is permanent, like the sentence ‘Allah exists’. But there are other cases of this that are not absolutely necessary, but rather, [2] for as long as the essence of the described thing remains satisfied; the human is not an animal permanently, because the human doesn’t exist permanently. ... As for other things, (7) that are necessary under a condition other than that the essence of the subject exists: ... besides the two aforementioned subdivisions, there are four subdivisions of these necessities. [3] One is where the condition is that the essence of the subject [is satisfied] according to some description; [4] another is where the condition is that the essence of the predicate is satisfied; [5] another is where the condition is a cause at a determinate time compelling the content [to apply to the subject]; [6] another is where the condition is a time when [the content applies] inevitably but the time is not determinate.

In his subsequent discussion he gives as examples:

For [3]: Everything white has to have a colour dispersed for the eye as long as it is white.

For [4]: Zayd is sitting, necessarily as long as he is sitting.

For [5]: The moon has to be eclipsed under certain conditions (i.e. when the earth lies between it and the sun).

For [6]: Each person has to breathe at some time.

Essentially the same list of six forms of necessary statement is given at *Deliverance* 35.2–26.13, *Syllogism* 31.14–33.10, *Easterners* 69.6–20, *Pointers* 88.9–89.11. (In *Easterners* [1] is missing.) These lists keep appearing throughout Avicenna's mature career. They are clearly a basic part of his understanding of 'necessary'.

A list in *Demonstration* (*Burhān* [8], written around same time as *Syllogism*, 120.15–121.23) is interesting:

Let us list the aspects from which a thing can be called 'necessary'. ... [0] meaning that the thing couldn't ever fail to be the case at any time at all. ... [1] It is said to be necessary when [the assertion is understood to be that] something is, or is not, the case permanently, and it always was so and always will be. ... [2] It is said to be necessary when [the assertion is understood to be that the predicate] holds, or fails to hold, ... permanently in the sense 'so long as the essence of the subject is satisfied'. ... [3] It is said to be necessary when [the assertion is understood to be that the predicate holds, or fails to hold,] ... for as long as its essence continues to carry the description expressed by the meaning of the subject term. ... [4] Or the necessity is under the condition 'so long as the predicate continues to be satisfied'. ... [5+6] Or the necessity is adjoined to a temporal condition where the time is specifically not that of the condition that the subject or predicate is satisfied. ... An example is: Some trees shed their leaves [in the autumn] with necessity and come into leaf in the spring with necessity.

Note here:

In [1]-[4] it is now made explicit that the statements are purely temporal and that these temporal conditions count as kinds of necessity. So these are temporal non-alethic modes of necessity. By contrast [5+6], not distinguishing definite from indefinite time, is where at certain times there is a non-temporal necessity.

Avicenna also adds [0] where 'necessarily' means 'not possibly not, regardless of the time'. This is alethic necessity, as opposed to the temporal necessities of the sextet.

Themistius on a modal first-figure syllogism:



- [30a15-30b7] He said: It is evident that all the concludent species [of premise-pairs] in this mixture will yield an assertoric [conclusion]. Let us assume first that the major is necessary and the minor assertoric, as is our saying, “Every *C* is *B* assertorically” and “Every *B* is *A* necessarily”; if so, “Every *C* is *A* assertorically.” This is so because every *B* is *A* necessarily, and it is evident that *A* will not be separated from *B* at any time. Now, since *B* is predicated of *C*, and its predication is not necessary, it is possible that [*B*] will be separated from [*C*] at some time, [whereas] *A* and *B*, which exist together, will not be separated. If so, in the same mode that *B* will be predicated of *C*, *A* will be predicated of it also, i.e. of *C*], etc. ([14] page 100)
- (9)

## 6 Ambiguity of ‘necessary’

Besides using *ḍarūrī* for Aristotle’s alethic necessity, Avicenna redefines it as the temporal modality (*d*). In fact he counts all the modalities in the sextet as *ḍarūrī* in some sense, but only one modality in the sextet is *ḍarūrī* ‘unqualified’ (*mursal*), and this is (*d*). Thus *Syllogism* [7] 33.8–10, supported by *Easterners* [9] 68.6–8 and 71.5. So Rāzī is absolutely right to accuse his predecessors (meaning Avicenna) of using *ḍarūrī* both for the inevitable and for the permanent.

However, Rāzī at [13] 304.8–13, followed by Street [15] p. 136, tells us that by *ḍarūrī* Avicenna means alethic necessary. In the details of proofs, this has the effect that both Rāzī and Street misread (*d*) sentences as alethic necessary, so they have Avicenna deducing ‘necessary’ and ‘permanent’ from each other. My strong impression is that Avicenna doesn’t do this.

In fact Avicenna keeps temporal arguments distinct from alethic ones, except when he considers syllogisms that have one premise absolute and the other possible—which he has to consider because he is following Aristotle’s schedule. But in the case of these syllogisms Avicenna’s methods give him nothing sensible to do, so he can only follow his nose.

## References

- [1] Saloua Chatti, ‘The logic of Avicenna between *al-Qiyās* and *Manṭiq al-Mashriqiyyīn*’, *Arabic Sciences and Philosophy* 29 (2019) 109–131.
- [2] Wilfrid Hodges, ‘Remarks on al-Fārābī’s missing modal logic and its effect on Ibn Sīnā’ (submitted).
- [3] Wilfrid Hodges and Spencer Johnston, ‘Medieval modalities and modern methods: Avicenna and Buridan’, *IfCoLog Journal of Logics and their Applications* 4 (4) (2017) 1029–1073.
- [4] Ibn Rushd (= Averroes), *Maqālāt fī al-manṭiq wa-al-ilm al-ṭabīʿī* (*Essays on logic and physics*), ed. Jamāl al-Dīn al-ʿAlawī, Casablanca 1983.

- [5] (*Middle*) Ibn Sīnā, *Al-mukhtaṣar al-'awṣaṭ fī al-manṭiq*, ed. Seyyed Mahmoud Yousofsani, Muassasah-i Pizhūhishī-i Ḥikmat va Falsafan-i Īrān, Tehran 2017 (hij. 1396).
- [6] (*Deliverance*) Ibn Sīnā, *Kitāb al-naḡāt*, ed. M. Danishpazhuh, Tehran University Press, Tehran 1945 (hij. 1364). The logic part is translated: Asad Q. Ahmed, *Avicenna's Deliverance: Logic* with introduction by Tony Street, Oxford University Press, Oxford 2011.
- [7] (*Syllogism*) Ibn Sīnā, *Al-qiyās*. Ed. S. Zayed, Cairo 1964.
- [8] (*Demonstration*) Ibn Sīnā, *Al-burhān*, ed. A. Afīfī et al., Cairo 1956.
- [9] (*Easterners*) Ibn Sīnā, *Manṭiq al-maṣriqiyyīn*, Al-Maktaba al-Salafiyya, Cairo 1910.
- [10] (*Pointers*) Ibn Sīnā, *Al-iṣārāt wal-tanbīhāt*, ed. Mojtaba Zāre<sup>c</sup>ī, Būstān-e Ketab-e Qom, Qom 2000. The logic part is translated: S. C. Inati, *Ibn Sīnā, Remarks and Admonitions, Part One: Logic*, Pontifical Institute of Mediaeval Studies, Toronto 1984.
- [11] Sari Nusseibeh, *Avicenna's Al-Shifā'*, Routledge, London 2018.
- [12] Fakhr al-Dīn al-Rāzī, *Manṭiq al-mulakhkhaṣ*, ed. Ahad Farāmarz Qarāmalekī and Ādīneh Asḡarīnezhād, Tehran 1961.
- [13] Fakhr al-Dīn al-Rāzī, *Sharḡ al-ishārāt wa-al-tanbīhāt*, ed. <sup>c</sup>Alī Riḡā Najafzādah. Amjuman-i Āṣār va Mafākhir-i Farhangī, Tehran 2005 or 2006 (hij. 1384).
- [14] Shalom Rosenberg and Charles Manekin, 'Themistius on Modal Logic: Excerpts from a commentary on the *Prior Analytics* attributed to Themistius', *Jerusalem Studies in Arabic and Islam* 11 (1988) 83–103.
- [15] Tony Street, 'An outline of Avicenna's syllogistic', *Archiv für Geschichte der Philosophie* 84 (2), 129–160 (2002).
- [16] Tony Street, 'Al-<sup>c</sup>Allāma al-Ḥillī (d. 1325) and the early reception of Kātībī's Shamsīya: notes towards a study of the dynamics of post-Avicennan logical commentary', *Oriens* 44 (3–4) (2016) 267–300.
- [17] Georg Henrik von Wright, *An Essay in Modal Logic*, Studies in Logic and the Foundations of Mathematics, North-Holland, Amsterdam 1951.