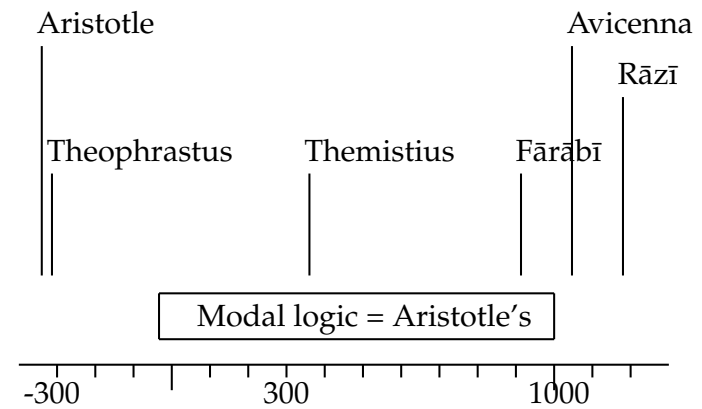


The creation of two paradigms for modal logic: Avicenna and Rāzī

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Plan of the talk

What's a paradigm? (Answer in terms of modalities.)

What modalities did Aristotle, Avicenna and Rāzī use?

Where did Avicenna get his modalities from?

What aims did Avicenna and Rāzī have in choosing their modalities?

What's a paradigm?

I will assume (because this assumption works in practice) that Avicenna's modal logic is *formal* in the sense that he studies and classifies *validity of forms of argument*.

For a formal logician a logic is mainly determined by the *sentence forms* used in it.

So a major change in sentence forms marks a paradigm shift.

For all the logicians we are discussing, modal sentence forms are got by adding *modalities* to categorical sentence forms:

$$\begin{pmatrix} \text{Every} \\ \text{Some} \end{pmatrix} B \begin{pmatrix} \text{is} \\ \text{isn't} \end{pmatrix} \text{an } A.$$

So different modal logics are distinguished by their modalities.

Fārābī argued (*Long Commentary on Prior Analytics*) that Aristotle's modal logic presupposes modalities added in two places,

- (i) to qualify *B* (the subject modality),
- (ii) to qualify 'is/isn't an *A*' (the main modality).

He said that the same modalities are available for (i) as for (ii).

He described a format for defining modalities, using universal affirmative sentence forms; he called it *maqūl 'alā al-kull* (= dictum de omni).

What modalities
did Aristotle, Avicenna and Rāzī use?

Aristotle's modalities

According to the Arabic Aristotle, 'premises differ as absolute and necessary and possible' (*Prior Anal* i.8).

In fact Aristotle has four modalities, since he uses 'possible' both for possible and for contingent. 'Contingent' is a Janus modality, got by taking two simple modalities X , Y and combining them as 'X but not Y'. Thus 'contingent' is 'possible but not necessary'.

Following von Wright 1951, we will call 'necessary', 'possible' and 'contingent' the *alethic* modalities. 'Absolute' means without an alethic modality; the Arabic logicians count this as a kind of modality.



Rāzī's modalities

Rāzī says he has thirteen modalities, and lists them carefully. A standard version of Rāzī's list, in the later *Shamsīya* of Kātibī, lists them as six simple modalities and seven Janus (or 'compound') modalities.

Example of simple modality: 'conditioned',
i.e. necessarily whenever some given condition holds.

Example of Janus modality: 'non-permanent conventional',
i.e. all the time it's a B , but not all the time it exists.



Avicenna's modalities

These are less easy to pin down than Aristotle's or Rāzī's, probably because he was feeling his way.

Also serious study of the relevant texts of Avicenna has only begun within the last decade, e.g. Chatti 2019.

Nusseibeh 2018 p. 95f rightly notes that investigation of Avicenna's logic should not just determine the separate forms but also 'delimit the boundaries of [his] logical system'. This will be an important issue.



But a preliminary list gives Avicenna nine modalities, three of them Janus:

- | | | |
|-------|--|------------|
| 1. | always during its existence | (d) |
| 2. | all the time that it's a B | (ℓ) |
| 3. | sometime while it's a B | (m) |
| 4. | sometime during its existence | (t) |
| 5. | necessarily | (nec) |
| 6. | possibly | (pos) |
| <hr/> | | |
| 7. | sometime but not always during its existence | |
| 8. | sometime but not always while it's a B | |
| 9. | contingently | |

Score: Aristotle 4, Avicenna 9, Rāzī 13.



Where did Avicenna get his modalities from?

Avicenna discusses modal logic in seven books,
in probable chronological order:

- ▶ *Gems of wisdom* (very early, uses Aristotle's modalities)
- ▶ *Middle abridgment*
- ▶ *Deliverance*
- ▶ *Guidance* (too brief to be useful)
- ▶ *Syllogism*
- ▶ *Easterners*
- ▶ *Pointers and indications*

Dropping *Gems* and *Guidance* leaves five accounts.

We begin with two features of Avicenna's descriptions of modal logic that are prominent in all five accounts, namely *semantic expansions* and *the sextet*.

The semantic expansions are adaptations of al-Fārābī's *maqūl 'alā al-kull*, a device for explaining meanings of modalised universal affirmative sentence forms.

The sextet is a list of (usually) six modalities that involve time and can be counted as 'necessary'. It is Avicenna's adaptation of a shorter list given by Theophrastus (the trio or *tathlīth*).

Examples of semantic expansions:

(*Middle* 108.7–11) The meaning of the sentence 'Every *B* is an *A*' is that it is what is described as actually a *B*, definitely but we don't know when, given that it has that actually at some time, regardless of whether it is the time of the content or another time that is not determined—that thing is described as being an actual *A* also, but we don't know when (etc. etc.)

(*Pointers* 93.9–14) Each of the things that are described as a *B*, regardless of whether it is described as a *B* in mental supposition or in the world, and regardless of whether it is so described permanently or non-permanently, but rather however it is (*kayfa kāna*), is a thing described as an *A* without any addition that it is so described at such-and-such a time or in such-and-such a case.

The pattern (in both Fārābī and Avicenna) is that we say ‘Every *B* is an *A*’ and pack it out with a variety of comments on what is or is not supposed to be understood by the parts of this sentence.

The comments vary from one example to another, according to what the author wants to mention in the given context.

The chart below illustrates the way comments come and go in over twenty examples of semantic expansions in Avicenna.

passage	mode	actual	permanent	however	mind	exist	assume
<i>Mid</i> 100.16–101.1a	-	Y	N	N	Y	N	Y
<i>Mid</i> 100.16–101.1b	-	Y	N	N	Y	N	Y
<i>Mid</i> 102.17–19	-	Y	N	N	Y	N	N
<i>Mid</i> 103.7–11	-	Y	Y	N	N	N	N
<i>Mid</i> 103.12–16	-	Y	Y	Y	Y	N	N
<i>Mid</i> 108.7–11	(t)	Y	N	N	N	N	N
<i>Mid</i> 108.21f	(d)	N	Y	Y	N	N	N
<i>Mid</i> 131.2–6	<i>nec</i>	N	Y	Y	N	Y	Y
<i>Mid</i> 132.10–12	-	N	N	Y	N	N	Y
<i>Del</i> 42.10–12	(d)	N	Y	N	Y	N	N
<i>Del</i> 43.12–14	<i>pos</i>	N	N	Y	N	N	Y
<i>Del</i> 66.6–9	<i>nec</i>	N	Y	N	N	N	Y
<i>Syl</i> 26.18–27.2	-	Y	Y	N	N	N	Y
<i>Syl</i> 31.15–32.1	(d)	Y	Y	N	N	N	N
<i>Syl</i> 33.11–14	<i>con</i>	N	Y	N	N	N	N
<i>Syl</i> 33.15–34.6	<i>pos3</i>	Y	Y	N	N	N	N
<i>Syl</i> 127.11f	<i>nec</i>	N	Y	N	N	N	N
<i>Eas</i> 64.3–6	-	Y	Y	N	N	Y	Y
<i>Eas</i> 64.14–16	-	Y	N	N	N	N	N
<i>Eas</i> 68.6–8	(d)	Y	N	N	N	N	N
<i>Eas</i> 69.12–14	(d)	N	N	Y	N	N	N
<i>Poi</i> 72.17–73.3	-	N	N	N	Y	Y	Y
<i>Poi</i> 93.9–14	(t)	N	Y	Y	Y	N	Y

Most of Avicenna’s comments relate to the subject part *B*. The consistent message is that ‘Every *B*’ means ‘Everything that was, is or will be an actual *B*, including abstract mental objects as well as physical objects’.

Street (2002 and often since) called attention to the semantic expansion at *Pointers* 93.9–14, remarking that ‘I would tend to take the [*Bs*] “in mental supposition” as licensing a kind of ampliation to the possible’.

Street has never put this passage in the context of Avicenna’s other semantic expansions.

We turn to the sextet.

All five books contain essentially the same list, in the same order, of six temporal modalities that could be regarded as forms of necessity.

In those books (*Middle*, *Deliverance*, *Syllogism*) that track Aristotle’s *Organon*, the list appears in different places, indicating that Avicenna considers it as his own.

Also a list that stays almost constant from early *Middle* to late *Pointers* has to be regarded as a structural element in Avicenna’s thinking.

The sextet summarised:

1. Permanently throughout time.
2. Permanently for as long as the thing exists.
E.g. every human is an animal as long as he exists.
(Avicenna says he calls this modality 'necessary' (*ḍarūrī*) and 'permanent' (*dā'im*).
We abbreviate to (*d*).)
3. All the time that the thing is a *B*.
E.g. every white thing dazzles while it stays white.
(In one place Avicenna suggests the name *lāzim*.
The name didn't stick, but we abbreviate it to (*ℓ*).)



4. Necessarily given that it satisfies a given condition.
E.g. Every person is necessarily moving while he is moving.
(This evolved into Rāzī's 'conditioned'.)
5. Under a necessity that holds at some definite time.
E.g. every deciduous tree has to grow leaves in the spring.
6. Under a necessity that holds at some unspecified time.
E.g. every person sometimes has to breathe.

Nusseibeh 2018 p. 96 adds a further two modalities.
But they are not forms of necessity and are not included in Avicenna's listings of the sextet.



Where did the sextet come from? We know the answer.
It came from a shorter list that Avicenna calls the *trio* (*tathlīth*), which was proposed by Theophrastus in his *Prior Analytics* and reported by Themistius:

- i. Necessarily throughout time.
- ii. Necessarily whenever the subject exists.
- iii. Necessarily whenever the existent thing exists.

The sextet clears up ambiguities in ii, iii of the trio.
Thus 2, 3 of the sextet are alternative ways of reading ii.
4, 5, 6 are alternative ways of reading iii.



Syllogistic example. According to Aristotle the following syllogism holds:

Every *C* is a *B*.
Every *B* necessarily is an *A*.
Therefore every *C* necessarily is an *A*.

Themistius disagrees. By the second premise, being an *A* follows from being a *B*. Since the first premise is not necessary, it allows that some *C* could sometimes fail to be a *B*. At this time, the thing is not a *B* and so doesn't have to be an *A*.



Avicenna's response: No problem about rewriting the modalities as temporal conditions; the correct rewrite is

Every sometime-*C* sometimes while it exists is a *B*.
 Every sometime-*B* is throughout its existence an *A*.
 Therefore every sometime-*C* is throughout its
 existence an *A*.

(This already in *Middle* 131.2–7.)

But Themistius has chosen the wrong temporal modality. He chose (*ℓ*), i.e. 'whenever it's a *B*', when he should have chosen (*d*), i.e. 'throughout its existence'. (Spelled out in *Syllogism* 126.5–127.2.)

What aims did Avicenna and Rāzī have
 in choosing their modalities?

Avicenna is explaining the syllogism in terms of two members of the sextet, (*d*) and (*ℓ*), together with another modality (*t*) that he calls 'broad absolute', namely

at least once during its existence.

The Arabic Aristotle lists this syllogism as having absolute minor premise and necessary major premise and conclusion.

Avicenna is reading 'absolute' as (*t*) and 'necessary' as (*d*).

A and R Aim One: To include all of Aristotle's modalities (possibly adjusted).

This accounts for the alethic (*nec*) and (*pos*) and contingent in both Avicenna's and Rāzī's lists.

As we saw, Avicenna reads Aristotle's 'absolute' as (*t*). Rāzī follows him in this.

Avicenna's Aim Two: Make the modalities unambiguous.

Rāzī's Aim Two: Make the names of modalities unambiguous.

We have seen Avicenna using semantic expansions and the sextet to remove ambiguities.

By contrast Rāzī seems to allow ambiguity in the modality on the subject.

For example at *Mulakhkhaṣ* 286.10–288.1 Rāzī seems to suggest at least three different readings of the subject term as possible ways of justifying syllogisms in mood *Barbara* with both premises possible.

This leaves one in doubt whether it's possible to give a consistent and coherent semantics for the whole of Rāzī's logic, and hence whether it always makes sense to ask whether his claims of validity or invalidity are correct. But I may do him an injustice; more work is needed.

Rāzī complains in several places that Avicenna uses names of modalities ambiguously.

For example at *Mulakhkhaṣ* 150.2–5: 'The logical literature has found itself stumbling around as a result of using the expression *ḍarūrī* sometimes for what is inevitable and sometimes for what is permanent. ... We adopt the convention that by *ḍarūrī* we mean only what is inevitable.'

But Rāzī at *Sharḥ* 304.8–13 and Street 2002 (throughout) both assume that Avicenna has also adopted this convention. Hence they read as alethic necessary some sentences that Avicenna almost certainly intended as (*d*), generating false arguments.

But Rāzī got it right in *Mulakhkhaṣ*. Avicenna does use 'necessary' both for alethic necessary and for 'permanent', i.e. (*d*), and often tells us so.

This allows him to read all syllogisms with 'absolute' and 'necessary' as temporal, viz. (*t*) and (*d*).

This logic of 'absolute' and 'necessary' is then completely unambiguous, and he can give proofs of all valid syllogisms by suitable adjustments of Aristotle's categorical proofs. This is a major achievement, but it would have been better without the ambiguity.

Avicenna's Aim Three: Eliminate modalities that are redundant or useless, and those where we have no objective logical basis for saying what rules of inference they obey.

This aim is proposed to explain why several modalities mentioned by Avicenna never reach his inference rules, though they do reach those of Rāzī.

An objection to sentences mixing alethic and temporal could also explain why Avicenna doesn't use Janus modalities with this mix, but Rāzī has three:

- ▶ necessarily given some condition, not permanently
- ▶ sometimes but not necessarily
- ▶ necessarily at some indefinite time, not permanently

Rāzī doesn't have such an objection.

For example at *Mulakhkhaṣ* 189.3f he says that 'everything that can be at some time can also be at every time'. (What parsings of this statement are correct, and why?)

Items in the sextet but not in Avicenna's inference rules:

Avicenna says (1.) 'at all times' is redundant because it is just 'at all times when it exists' for the particular case of things that always exist.

Avicenna says that (4.) (as in 'sitting when he is sitting') is useless in practice.

There remain two modalities (5.), (6.) that mix alethic and temporal. Avicenna says logicians have no basis in logic for saying how permanent relates to alethic necessary.

Avicenna's Aim Three requires that he has an objective basis for finding logical laws for the alethic modalities. In fact he uses three methods, two reliable and one unreliable. The unreliable is to rely on intuitions of what 'close to' being true. The two reliable methods are both ways of translating alethic syllogisms into temporal by the translations

alethic necessary $\mapsto (d)$
 alethic possible $\mapsto (t)$
 alethic contingent $\mapsto (w)$.

An underlying justification is explained for example in Hodges and Johnston 2017. But Avicenna can only have had a vague idea of it.

Avicenna's Aim Four: Close the modalities under logical operations.

For example the proposition

Every B is an A all the time it is a B .

with main modality (ℓ) has contradictory negation

Some B is, at some time when it is a B , not an A .

with main modality (m).

Avicenna correctly records this in *Easterners* and *Pointers*.

Avicenna doesn't in fact use the Janus modality ' (m) and not (ℓ) ' which we listed for him. Presumably he never saw a use for it.

This accounts for all of the modalities that Avicenna uses in his modal logic.

But there are still things to explain in Rāzī's list.

For example he doesn't include (m) , though he certainly uses it in inference rules.

I don't have a good explanation of this.

On the other hand Rāzī does include the non-permanent conventional modality, for which there is no obvious use:

all the time it's a B , but not always.

This can be explained. Avicenna had shown that we have an inconsistent set:

Every sometime- B is an A whenever it's a B .

No sometime- B is an A throughout its existence.

Some sometime- C is a B throughout its existence.

When Avicenna reported this result he ran the first two sentences together, producing the non-permanent conventional. Rāzī put it in his list because he found it in Avicenna's text.