Barakāt's logical diagrams

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Abū al-Barakāt bin Malka al-Baghdādī

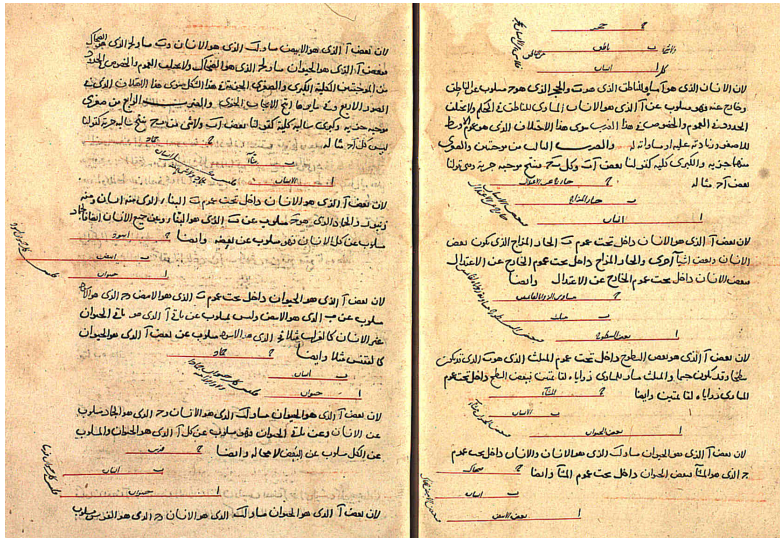
'A very original thinker who proceeds meticulously with a logical method.' (Moshe Pavlov)

Was recently identified with the well-respected Talmudic authority and poet Rabbi Baruch ben Melekh.

Lived c. 1080–c. 1165, chiefly in Baghdad.
Converted to Islam around 1163, for unknown reasons.

Book: *Kitāb al-Muʿtabar*, 'Book of conclusions I came to'.
It begins with about 300 pages on knowledge and logic, roughly 25 of them on categorical syllogisms.

The treatment of categorical syllogisms makes perfect sense in modern terms,
but is radically different from all other known treatments of logic before the 19th century
(Gergonne 1816/7, Bolzano 1837, Tarski 1936),
including all later Arabic logic.



1. What is new in Barakāt's approach to syllogisms?
2. How does Barakāt's approach to syllogisms work?
3. Where did he get the idea, and how does it fit with his other ideas?
4. Why was his approach not understood until (it seems) two years ago?

1. What is new in Barakāt's approach to syllogisms?

Aristotle has four kinds of categorical sentence:

> Every *B* is an *A*.
> No *B* is an *A*.
> Some *B* is an *A*.
> Some *B* is not an *A*.

(Or with other letters.)

A 'premise-pair' is two categorical sentences with one letter in common.

Aristotle lists 48 premise-pairs, grouped into three 'figures'. He classifies them under two heads.

A premise-pair is 'productive' if it entails one or more conclusions.
(Its 'conclusion' is the strongest entailed sentence.)

Example:

> Every *C* is a *B*. No *A* is a *B*.
> *Productive with conclusion* No *C* is an *A*.

Otherwise it is 'unproductive'
(called 'sterile' by Ibn Sīnā from 1010 or earlier).

Example:

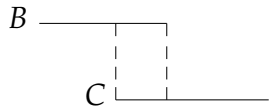
> Some *B* is a *C*. Some *B* is not an *A*.
> *Sterile*

Like Aristotle, Barakāt classifies the premise-pairs as productive or sterile.

Unlike all other logicians before Leibniz,
Barakāt uses pictures such as

$$\frac{C \quad \text{black}}{A \quad \text{animal}}$$

Similar diagram from Leibniz unpublished notes,
representing 'Some B is a C' (late 17th century):



Leibniz's picture represents a sentence, just as the later circle diagrams of Euler and Venn. (Actually two equivalent sentences: 'Some B is a C' and 'Some C is a B'.)

Barakāt is *not* doing the same as Leibniz. Barakāt's diagram needs three separate sentences to express it: 'Some C is an A', 'Some C is not an A', 'Some A is not a C'. His diagrams represent *interpretations*, not sentences.

Aristotle did use interpretations in his logic,
but only for proving sterility.
Barakāt uses his diagrams to prove productivity. !!

For Aristotelian logicians this must be wrong. What can be proved for one interpretation is not necessarily provable for *all* interpretations, so not logically necessary.

2. How does Barakāt's approach to syllogisms work?

Barakāt assumes a new definition of entailment, which works only for formal logic (i.e. with letters). Thus:

Premises Φ entail conclusion θ
if and only if
every interpretation that makes Φ true also makes θ true.

(So nothing is said about inferences or derivations.)

Using this definition, he gives a single procedure which, when applied to a premise-pair,
either gives the answer 'Productive' and supplies a conclusion,
or gives the answer 'Sterile'.

In other words, he supplies a decision procedure for the logic of categorical syllogisms.

Logical decision procedures were not consciously adopted by later logicians until around 1920
(Post, Behmann, Bernays etc.).

In theory, to apply the definition of entailment we have to look at *every interpretation* (and there are infinitely many).

But Barakāt noticed that the sentences made true by an interpretation are determined by the diagram of the interpretation.

So we need only look at *every diagram that makes both premises true*, and check whether there is a further sentence θ that they all make true.
This is a finite task and entirely feasible for humans.

3. Where did he get the idea, and how does it fit with his other ideas?

More questions than answers.

The idea of a single procedure that finishes up with either a proof of productiveness or a proof of sterility is radically new.
But Paul the Persian in his *Logic* (mid 6th century, survives only in Syriac) taught a treatment of syllogisms that makes the proofs of productiveness *look like* the proofs of sterility, though it's really smoke and mirrors.

Paul was writing six hundred years before Barakāt and in a different culture.

Janssens has assembled evidence that later parts of Barakāt's logic in *Kitāb al-Muᶜtabar* were based on Ibn Sīnā's early work *Ḥikmat al-ᶜArūḍīya* (AD 1001, when Ibn Sīnā was about twenty-one and still immature in logic).

The part of *Ḥikmat* about categorical syllogisms is lost.
But Barakāt never uses the term 'sterile',
suggesting that he is working from a source earlier than Ibn Sīnā's introduction of this term.

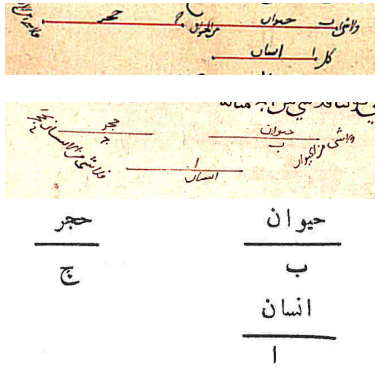
Absolutely nothing in Ibn Sīnā's later logic compares with Barakāt's new method of using interpretations for proofs of productivity.

There is evidence that Barakāt had not read Aristotle's treatment of syllogisms (though he claimed he had).

Aristotle showed, and illustrated many times, that for a proof of sterility only *two* interpretations are needed.
Barakāt always gives *three* interpretations for the sterile cases, though always one of the three is redundant.

Also to illustrate the Aristotelian proofs that he doesn't need, he gives a proof that is not in fact in Aristotle, but was used by Ibn Sīnā and goes back to Galen.
Analysis of his modal syllogisms may shed more light.

4. Why was his approach not understood until recently?



Two manuscripts and a print version. The classes are (top left) stone, (top right) animal, (below) human.

Even logical experts were bewildered.
Ṭūsī in his *Asās al-iqtibās* mentions Barakāt's diagrams and correlates features of them with universal, particular and negative,
i.e. with features of *sentences* rather than interpretations.

Unsurprisingly he has no explanation of diagrams of the form

$$\frac{C}{A}$$

There is also evidence that medieval Arabic writers lacked understanding of the basic ideas of algorithmic procedures. Two great examples:

- The language search algorithm of al-Khalīl bin Aḥmad, 8th century, was understood as a principle for organising a dictionary, but the broader idea of a search algorithm was not extracted.
- Ibn Sīnā's proof search algorithm, 11th century, likewise left no trace as a recursive search algorithm.

So it shouldn't be entirely surprising that the idea of a decision algorithm never registered, particularly since Barakāt had no vocabulary to support this notion.

Wilfrid Hodges, 'Two early Arabic applications of model-theoretic consequence', *Logica Universalis* 12 (1–2) (2018) 37–54.

Jules Janssens, 'Ebü'l-Berekāt el-Bağdādī'nin Kitābü'l-Muᶜteber'inin Mantık Kısmında İbn Sīnā'nın el-Hikmetü'l-Arūziyye'sini (veya onunla yakından ilişkili diğer bir eseri) Kullanımı', *Nazariyat İslām Felsefe ve Bilim Tarihi Araştırmaları Dergisi* 3/1 (2016) 1–22.

Thanks also to Robert Wisnovsky for access to manuscripts, and Roshdi Rashed and Karine Chemla for discussion of early algorithms.