

How the teenage Avicenna planned out several new logics

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Abū 'Alī Ibn Sīnā
known in the West today as
Avicenna;
father Bactrian from
Northern Afghanistan,
mother probably Sogdian

This talk is on an episode in the history of logic, where one logician working on his own decided to make a paradigm shift in logic by changing not just the laws but also the foundations.

The logician was Avicenna, born in around 976 in what is now Uzbekistan. We will look at three works of his:

- ▶ *Short Epitome* c. 994, arguing that Aristotle got the laws of logic wrong.
- ▶ *Twenty Questions* c. 1011, arguing for new foundations.
- ▶ *Middle Summary* 1013, creating the new system.

None of the three works mentioned above are available in Western translation.

An authoritative critical edition of *Middle Summary* was first published in Tehran in 2017.

Short Epitome and *Twenty Questions* were incorrectly dated until a few months ago.

So we will be relying on recent scholarship, particularly on studies by Gutas and Janssens of interrelations between various works.

I will take this background scholarship for granted and concentrate on the logic.

Short Epitome is fifteen pages long. It is a brief report on Aristotle's logical writings, but very much concentrated on syllogisms (i.e. two-premise inferences) and their rules.

The contents of *Short Epitome* are of three kinds:

- (i) Taken from Aristotle.
- (ii) Taken from al-Fārābī's textbook *Syllogism*, early 10th century.
- (iii) Known to be innovations by Avicenna which appear in his logic throughout his career.

The author knew Aristotle and al-Fārābī's *Syllogism*, but possibly no other sources.

The modal logic is extremely primitive.

Avicenna tells us in his *Autobiography* that in his early teens he learned some elementary logic from his father's lodger. At age sixteen he resolved to teach himself Aristotle's logic thoroughly.

This took him about a year and a half, and involved checking Aristotle's arguments (and likewise those of Euclid) line by line, filling the gaps and arranging the components in the right order.

Short Epitome exactly fits his situation at the end of this period, before he had visited the famous library of Nūḥ with its hundreds of manuscripts.

In the list of categorical syllogisms in *Short Epitome*, Avicenna adds some claims about which syllogisms remain valid when modes are attached to some sentences. Some of these claims directly challenge Aristotle.

In particular he claims against Aristotle that the following is valid:

No C is a B.

Every A is necessarily a B.

Therefore every C is necessarily not an A.

This is one of a small group of his claims that we call the 'Second Modal Challenge'.

Short Epitome says nothing to justify the challenge.

In *Middle Summary* in 1013 Avicenna gives his first (or at least first surviving) proof of the Second Modal Challenge. This was at the end of a gap of at least nineteen years from *Short Epitome* to *Middle Summary*. What was he doing during the gap?

Twenty Questions lists changes that need to be made in the foundations (*ʿuṣūl*) of Aristotle's logic in order to prove the Second Modal Challenge and some subsidiary claims. *Twenty Questions* reads like Avicenna's preparatory notes for the relevant part of *Middle Summary*.

We list five moves, four of them sketched in *Twenty Questions* and worked out in *Middle Summary*.
(He made the remaining move later.)

Every one of them is based on ideas found in earlier Aristotelian logicians.
Avicenna had nowhere else to look for ideas to build a new logic.
But in every case his use of his predecessors' ideas is highly original.

2. 'Necessary predications'.

If there are useful sentences with 'sometimes',
then there should also be useful sentences with 'always'.

Theophrastus in his lost *Prior Analytics* listed three different connections between being necessary and being permanent.

Avicenna refines the list to six 'necessary predications', and this list of six appears in all his main logic writings from *Middle Summary* onwards.
But in *Twenty Questions* he concentrates on just two cases.

1. Fluents. In the real world things may have a property at one time and not at another,
Traditional logicians call these properties separable accidents, modern situation theorists call them fluents.

Avicenna broadens the class of basic sentences to include the 'broad absolutes' which have the quantifier 'Sometimes'.
It's clear that 'No *B* is an *A*' entails 'No *A* is a *B*'.

But Avicenna notes that 'Every *B* is sometimes not an *A*' doesn't entail 'Every *A* is sometimes not a *B*'.
This blocks one of Aristotle's arguments against the Second Modal Challenge.

One of the two cases is where we say that a thing has a property 'at all times when the thing exists'.
He calls this 'strict necessity'.

In the twelfth century Fakhr-al-Dīn al-Rāzī complained correctly and vigorously that Avicenna used 'necessary' sometimes to mean the alethic mode 'inevitable', and sometimes to mean the temporal mode 'permanent' (i.e. with strict necessity).

In practice Avicenna reads 'necessary' as 'permanent' in contexts where there are also broad absolute sentences.
He reads it as alethic 'necessary' when the context contains alethic 'possibly'.

3. ‘Said of all’. Al-Fārābī had pointed out that in a sentence ‘Every *B* is an *A*’ we can modalise both *B* and *A*. For example

Everything that is possibly a B is necessarily an A.

He devised a format for expressing the meanings of variants of ‘Every *B* is an *A*’, including these two modes.

For Avicenna, disagreements about logical laws mainly result from disagreements about what the words mean. So in a reformed logic, the meanings of the sentence forms must be completely unambiguous. For this he frequently gives Fārābī-type statements about the meanings; he sometimes calls them ‘said of all’. I found 27 examples from *Twenty Questions* onwards.

A typical ‘said of all’, from *Middle Summary*:

... the meaning of the sentence ‘Every B is an A’ is that is what is described as actually a B, definitely but we don’t know when, given that it has that actually at some time, regardless of whether it is the time of the content or another time that is not determined—that thing is described as being an actual A also, but we don’t know when the A is described as being a B or whether it is at another time before it or after it or simultaneous with it in some interval and not at another or in every interval during which it exists, because our phrase ‘It is an actual A’ includes all of those.

Innovations 1–3 lead Avicenna to eight sentence forms:

- ▶ Every sometime-*B* is always an *A*.
- ▶ Every sometime-*B* is sometimes an *A*.
- ▶ No sometime-*B* is ever an *A*.
- ▶ Every sometime-*B* is sometimes not an *A*.
- ▶ Some sometime-*B* is always an *A*.
- ▶ Some sometime-*B* is sometimes an *A*.
- ▶ Not every sometime-*B* is sometimes an *A*.
- ▶ Not every sometime-*B* is always an *A*.

‘Sometime-*B*’ is our abbreviation for ‘thing that is sometimes an actual *B*’.

We call the logic of these eight sentence forms the ‘(*dt*) fragment’.

Within the (*dt*) fragment, one typical Second Modal Challenge syllogism takes the form

*Every sometime-C is sometimes not a B.
Every sometime-A is always a B.
Therefore no sometime-C is ever an A.*

Note: because the first sentence is broad absolute, ‘necessary’ in the second and third sentences is read as strict necessity.

For Avicenna as for Aristotle, a logic needs to have a proof calculus.

Aristotle's proof system doesn't work for the Second Modal Challenge syllogisms.

One standard modern account of Avicenna's logic says that Avicenna doesn't give a proof for the syllogism above. This is false. He gives two proofs for it, but neither of them is Aristotelian.

This is a common phenomenon in accounts of Avicenna's logic. Departures from Aristotle are not understood and are either ignored or treated as mistakes.

But what about genuinely alethic modal syllogisms? For example one form of the Second Modal Challenge is the syllogism

Every C is possibly not a B.
Every A is necessarily a B.
Therefore every C is necessarily not an A.

Here the first premise has an alethic mode, so 'necessarily' in the other two sentences is read as alethic necessity. In this case there is no Aristotelian formal proof, and Avicenna doesn't attempt to construct one. Instead he translates wholesale from alethic to the (*dt*) fragment.

4. Incorporation in terms (sometimes called Morleyisation today, though the modern version is due to Skolem). In the mid 1020s Avicenna pointed out that we can prove the syllogism above by replacing *A, B, C* by terms that incorporate the time quantifiers:

No C^{sometime} is a B^{always}.
Every A^{sometime} is a B^{always}.
Therefore no C^{sometime} is an A^{sometime}.

This device is one of the few innovations by Avicenna that were regularly used by his successors.

5. Making the possible absolute.

In this method, Avicenna translates a possibility sentence to the corresponding broad absolute sentence. This automatically causes any 'necessary' sentence to be interpreted with strict (not alethic) necessity. The resulting syllogism is checked in the (*dt*) fragment.

So the alethic syllogism above is proved by converting it to the preceding (*dt*) syllogism which we have already proved.

But why on earth should these proofs work?

We can see why if we construct a model M for the (dt) sentences. It will have two kinds of individual: objects and times.

At any given time t the objects have properties that constitute a ‘world at time t ’, say M_t . The property ‘exists at time t ’ will be represented in M_t by a relation E^{M_t} .

The structures M_t combine to form a Kripke structure with times as worlds, and a single universe of objects with an existence predicate E .

Accessibility is universal, but the interpretation of ‘necessary’ involves E .

Avicenna’s procedure of ‘making the possible absolute’ is formally identical with what we do when we give a semantic proof of the alethic syllogism, using the Kripke structures above as the semantics.

The fit between proof theory and semantics means that Avicenna’s ‘making the possible absolute’ is equivalent to giving an S5 proof—though Avicenna himself never gives formal proofs using nested modes.

These formal relationships were much clarified by recent work of Spencer Johnston giving semantic proofs for the divided modal logic of Buridan (later than Avicenna).

We return to *Twenty Questions* for another look.

Recently Janssens has collected evidence that the book *Kitāb al-Mu^ctabar* of Abū al-Barakāt al-Baghdādī (mid 12th century) uses material from Avicenna’s gap period (994–1013) in non-syllogistic areas of logic.

So it is interesting that *Twenty Questions*, at the end of the gap period, discusses another connection between necessity and permanence, which is also discussed in *Mu^ctabar*.

This is the notion of ‘necessary throughout a time’, where the time is when a cause exists.

E.g. for al-Barakāt the planets have to set all the time while a cause of their setting exists.

Also al-Barakāt is known to have had a strong influence on al-Rāzī, and one of the chief differences between al-Rāzī’s logic and Avicenna’s is the use of temporary necessities in Rāzī’s logic.

So Janssens’s observation and *Twenty Questions* together make it very likely that this feature of al-Rāzī’s logic (and of logics that follow al-Rāzī’s) is based on early work of Avicenna that hasn’t survived in detail.

This deserves to be investigated.

Summing up

Avicenna started his career in logic as a rather nerdy teenager interested in obscure formal questions that interested nobody else.

But he was very determined, and we have noted five foundational innovations that he made to defend his logic.

In each case the innovations were designed to ensure that the result would be a new logic that we can all agree on. (Not what happened, but only partly through his fault.) We have noted two new logics that he created—one temporal, the other alethic modal, both of them supported by proofs that meet modern standards.

Lots more, but no time for it today. Thank you!



- ▶ Dimitri Gutas, *Avicenna and the Aristotelian Tradition: Introduction to Reading Avicenna's Philosophical Works, Second Edition*, Brill, Leiden 2014.
- ▶ Jules Janssens, 'Abū al-Barakāt al-Baghdādī and his use of Ibn Sīnā's *al-Ḥikma al-ʿArūḍiyya* (or another work closely related to it) in the logical part of his *Kitāb al-Muʿtabar*', *Nazariyat—Journal for the History of Islamic Philosophy and Sciences* 3 (1) (2016) 1–22.
- ▶ Avicenna, *Al-mukhtaṣar al-awsaṭ fī al-manṭiq*, ed. Seyyed Mahmoud Yousofsani, Muassasah-i Pizhūhishī-i Ḥikmat va Falsafan-i Īrān, Tehran 2017 (hij. 1396).
- ▶ Alexander Kalbarczyk, 'The *Kitāb al-Maqūlāt* of the *Mukhtaṣar al-awsat fī l-manṭiq*: a hitherto unknown source for studying Ibn Sīnā's reception of Aristotle's *Categories*', *Oriens* 40 (2012) 305–354.



- ▶ Saloua Chatti and Wilfrid Hodges, *Al-Fārābī, Syllogism: An Abridgement of Aristotle's Prior Analytics*, Ancient Commentators on Aristotle, Bloomsbury Academic, London 2020.
- ▶ Wilfrid Hodges and Spencer Johnston, 'Medieval modalities and modern methods: Avicenna and Buridan', *IfCoLog Journal of Logics and their Applications* 4 (4) (2017) 1029–1073.
- ▶ Avicenna, *Short Epitome*, sections on syllogisms (translation), wilfridhodges.co.uk/arabic70.pdf.

