

## Early strands in Ibn Sīnā's formal logic

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This talk will be entirely about Ibn Sīnā's writings in formal logic, and almost entirely about those from the first half of his career, i.e. up to *Middle Summary in Logic* (المختصر الاوسط في المنطق) in 1013.

Michot has described *Middle Summary in Logic* as one of Ibn Sīnā's 'minor works on logic'.

I think no logician would describe it this way.

It is the central work where all the explorations and experiments of his early years finally come together as a coherent whole.

His more famous later works in logic do contain some new ideas, but mostly refinements.



Valuable consequence: Ibn Sīnā's early works in logic can mostly be seen as a progression of logical ideas, culminating in *Middle Summary in Logic*.

For logicians, that implies a temporal order, though we should allow for some false turns.

However, a logical progression may not always be visible to readers not fluent in logic.

So let me first draw out some more 'literary' evidence for an order in these early works.



### Ibn Sīnā's early logical works and their 'literary' order

≤ c. 1000	}	?? <i>Epitome on Foundations of Logic</i> (الرسالة الموجزة في أصول المنطق)
		<i>Short Epitome in Logic</i> (الموجز الصغير في المنطق)
		<i>Philosophy for 'Arūḍī</i> (الحكمة العروضية)
c. 1000		early syllogistic work consulted by Abū l'Barakāt
c. 1012		<i>Twenty Questions</i>
1013		<i>Middle Summary in Logic</i>





*Short Epitome in Logic* contains some strong original logical ideas characteristic of Ibn Sīnā.

But also multiple evidence that it is a very early work.

E.g. its account of categorical syllogisms contains text copied from al-Fārābī *Syllogism*, including at least one logical error of al-Fārābī.

Modes are identified with (Ammonian) matters, and no connection is made between modes and time.

‘Absolute’ (مطلق) simply means mode-free, and there is no hint of the Themistian absolute introduced in *Twenty Questions* (which later became ‘broad absolute’).

The new modalisation of categorical logic in *Short Epitome in Logic* contradicts Aristotle in all three figures.

It agrees with Ibn Sīnā’s later modalisations of categorical logic, up to and including *Pointers*.

So as far back as we can trace it, Ibn Sīnā’s modal logic was intended to replace Aristotle’s, not to interpret it.

We can trace the ideas that Ibn Sīnā introduced, mainly by 1012 (the time of *Twenty Questions*), in order to justify his new modalisation. They include new notions of ‘absolute’.

But in the rest of this talk I concentrate on hypothetical syllogisms.

### The hypothetical logics of *Short Epitome in Logic*

*Short Epitome in Logic* describes two separate origins for ‘hypothetical’ (شرطي) syllogisms.

They create two different ‘hypothetical logics’ with no sentence forms in common.

In later works Ibn Sīnā points out that they are ‘hypothetical’ in different senses, and he proposes a broader notion of ‘hypothetical sentence’ that excludes categorical sentences but includes the non-categorical sentences of both these logics.

The first logic is based purely on Ibn Sīnā’s analysis of categorical syllogisms.

Ibn Sīnā describes it as ‘recombinant hypothetical’ (اقتراني شرطي), but this name creates conflicts with later logics, so I replace it by ‘wholly *muttaṣil*’.

The second logic is al-Fārābī’s exceptive logic, using Stoic ideas.

WARNING: Not all modern commentators are aware of both logics.

Some commentators know only the first, some recognise only the second.

Chatti is fully aware of both, though her translations of the sentence forms of the first (following Rescher) are sometimes potentially misleading.

The conclusions of wholly *muttaṣil* syllogisms are always hypothetical.

The conclusions of exceptive syllogisms are normally categorical and hence not hypothetical.

In the near-doublets of *Twenty Questions* and *Middle Summary in Logic*, Ibn Sīnā claims that mathematical theorems are often hypothetical, and that he is the first to find inference rules that yield hypothetical conclusions.

He claims also that he has classified all such inference rules.

Immediately after this in *Middle Summary in Logic* he lists several types of syllogism with hypothetical conclusions, beginning with the wholly *muttaṣil*.

These are presumably the fruit of the Hypothetical Breakthrough mentioned above.



The section on hypothetical syllogisms in Abū l-Barakāt al-Baghdādī's *Book of Things Considered* (كتاب الاعتبار, mid 12th century) has an intermediate list, again beginning with wholly *muttaṣil* syllogisms.

It includes but doesn't name the important logic of 'incomplete parts' that is mentioned in *Middle Summary in Logic*.

This agrees with Janssens's claim that the logic of *Book of Things Considered* uses work of Ibn Sīnā from around 1000. It may also throw light on the order in which Ibn Sīnā found the syllogisms of the Hypothetical Breakthrough.



At *Syllogism* vi.6, 356.9–11 (written probably no later than 1024) Ibn Sīnā tells us that he came across a book on hypothetical syllogisms

*about eighteen years after we had worked out this part of science.*

بعد أن كنا إستخرجنا هذا الجزء من العلم بقريب من ثمانى (عشر سنة)

He presumably means the Hypothetical Breakthrough, which he had described in very similar language.

Since he doesn't say he came across the book 'recently', we can presume it reached him at least two years earlier. Then it reached him not later than 1022, and he had worked out his own version of hypothetical logic at least eighteen years before that, i.e. in or before 1004.



Note: since the double verb *كنا إستخرجنا* is perfective in both parts, it should refer to the completion of the breakthrough, not the duration of the work. So I follow Gutas (AAT p. 109) and not Di Vincenzo (and my thanks to Manuel Sartori for discussion).

In 1022 Ibn Sīnā is not likely to have counted the breakthrough as complete before 'incomplete parts' were included.

So we have approximate dates:

- ▶ < c. 1002, *Short Epitome in Logic*
- ▶ ≤ c. 1002, early syllogistic work of Ibn Sīnā consulted by Abū l'Barakāt
- ▶ ≤ c. 1004, Hypothetical Breakthrough complete, including syllogisms 'in an incomplete part'
- ▶ c. 1012, *Twenty Questions*
- ▶ 1013, *Middle Summary in Logic*



## Logical content

We need at least to describe (i) the wholly *muttaṣil* syllogisms of *Short Epitome in Logic*, (ii) the inferences ‘in an incomplete part’ in *Middle Summary in Logic*.

Both have major importance in the history of logic.

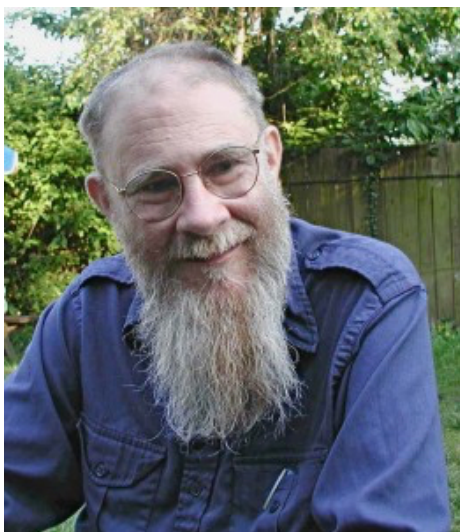
(i) The wholly *muttaṣil* syllogisms follow the same formal rules as the categorical syllogisms, for example there are fourteen valid moods grouped in three figures.

The ‘terms’ of categorical logic are usually nouns, while those of wholly *muttaṣil* logic are *sentences P*, whose individuals are the situations, times or assumptions under which *P* is true. Ibn Sīnā writes this class of individuals as ‘when *P*’ or ‘if *P*’. We quantify over this class by saying ‘Sometimes when *P*’, ‘Never when *P*’, etc.

So wholly *muttaṣil* logic has four sentence forms corresponding to the scholastic forms *a, e, i, o*:

- (*a*) Whenever (i.e. always when) *P, Q*.
- (*e*) Never when *P, Q*.
- (*i*) Sometimes when *P, Q*. (Equivalent: Sometimes *P* and *Q*.)
- (*o*) Not always when *P, Q*. (Equivalent: Sometimes *P* and not *Q*.)

This use of ‘when/if *P*’ was pointed out independently in modern logical semantics by the philosopher David Lewis (in English, not Arabic, though Ibn Sīnā’s use of the phrase closely matches that of Lewis).



David Lewis (1941–2001)

Example from *Short Epitome in Logic*:

*Sometimes when C is I, G is T.*  
*It is never the case, when G is T, that A is B.*  
 [This] yields that it is not the case that whenever C is I,  
 A is B.

This is the wholly *muttaṣil* equivalent of mood *Ferio* in first figure.

Ibn Sīnā worked with wholly *muttaṣil* syllogisms almost throughout his career. They form the backbone of his ‘hypothetical logics’.

The idea of adapting term logic to a logic of sentences that are read as true or false in various ‘cases or conjunctures of circumstances’ also occurred to Boole in 1854 (*Laws of Thought*, end of Chapter XI). Boole’s followers went on to tidy up the resulting logic so as to create propositional calculus.

Saloua Chatti and I semi-independently examined Ibn Sīnā’s texts for evidence of how he thought about justifying syllogisms like that above. We reached closely similar results. Ibn Sīnā noticed the presence of a smaller syllogism inside the one above:

*Every C is a D.*

*Some C is an H.*

*[It yields:] Some D is an H.*

Note: As in Aristotle’s syllogisms, in the inside syllogism all parts are underlined, i.e. complete. Also the inside syllogism is standard categorical.

(ii) Syllogisms ‘in an incomplete part’.  
Example (*Middle Summary in Logic* 154.3):

*Whenever A is a B, every  $\bar{C}$  is a  $\bar{D}$ .*

*Some C is an H.*

*[It yields:] Whenever A is a B, some  $\bar{D}$  is an  $\bar{H}$ .*

Underlined parts are *complete parts*, i.e. maximal terms inside their sentences. Overlined parts are *incomplete parts*, i.e. smaller terms inside the complete parts. An inference ‘in an incomplete part’ applies a logical rule to a part at second or lower syntactic level.

The second step of justification is to restore the two ‘whenever’ clauses that we removed. This is a non-Aristotelian step. How can we justify it?

‘But it’s obvious!’

Yes maybe. But unfortunately it’s also false in general, since the outside syllogism is valid only under suitable conditions.

The first premise and conclusion are about ‘every time when A is a B’, but the second premise has no time quantifier and so can be read as referring at most to the present.

In *Review of Symbolic Logic* 2017 I gave (without proof) sufficient conditions for this second step to be justified. But these conditions require the notions of bound and free occurrences of variables, which in modern logic arrived with Frege around 1900.

A paper in preparation will prove the conditions.

In fact Ibn Sīnā never pointed out the need for such conditions. It seems he was sidetracked onto the different question how we can make deductions from self-contradictory premises. My own impression is that Ibn Sīnā left it to the reader to find an appropriate adjustment of the text; he does similar things elsewhere.

# Thank you!



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- ▶ Dimitri Gutas, *Avicenna and the Aristotelian Tradition: Introduction to Reading Avicenna's Philosophical Works, Second Edition*, Brill, Leiden 2014.
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- ▶ David Lewis, 'Adverbs of quantification', in *Formal Semantics of Natural Language*, ed. Edward L. Keenan, Cambridge University Press 1975, pp. 3–15.
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- ▶ Hilmi Ziya Ülken, *Les opuscules d'Ibn Sīnā: Uyūn al-ḥikma et l'opuscule d'Abū'l Faraj et la réfutation d'Ibn Sīnā*, Türk Tarih Kurumu Basımevi, Ankara 1953.

