# Dābiṭatu sharā'iṭ al-ashkāl al-arbaᶜa in al-Taftāzānī and al-Yazdī

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About ten years ago Khaled El-Rouayheb told me he had found a form of the syllogistic Laws of Distribution in Arabic logic. This has now appeared in his *The Development of Arabic Logic (1200–1800)*.

On page 81 he quotes al-Taftāzānī's ضابطة, commenting

> *This is akin to, even if not identical to, medieval Latin notions of "distribution".*

In 1998 I published a proof that the syllogistic Laws of Distribution are a consequence of a basic metatheorem of first-order logic (the Lyndon Interpolation Theorem) and hence can be used to place syllogistic logic on a solid foundation.

This last month I looked at al-Taftāzānī's formulation in his *Tahdhīb* and the exposition of it by Mullā ᶜAbdullāh al-Yazdī in his *Ḥāshīya*, to see what al-Taftāzānī was aiming to do with his ضابطة, and what he achieved by it.

I have no expertise in 14th to 16th century Arabic-speaking logic, but I will do my best to put these issues into the right broader context of logic. I will ignore what seem to be minor mistakes by Aristotle and others.
Apologies for my misunderstandings!

Nearly everything goes back to Aristotle *Prior Analytics* i.4–6, so let me start there.

Aristotle *Prior Analytics* i.4–6.
Four quantified categorical sentence forms

| | |
|---|---|
| Every *B* is an *A*. | (universal affirmative) |
| No *B* is an *A*. | (universal negative) |
| Some *B* is an *A*. | (existential affirmative) |
| Some *B* is not an *A*. | (existential negative) |

I use the Arabic (and English) orderings of the formal letter terms *A*, *B*, *C* and other ordered items.
These are often the reverse of Aristotle's Greek.
In the examples above, *B* is the *subject* and *A* the *predicate*.

Aristotle listed 48 *premise-pairs* (اقتران in Ibn Sīnā), i.e. pairs of sentences where the first sentence uses term letters *C*, *B* in some order, and the second uses *B*, *A* in some order.
A sentence with subject *C* and predicate *A* is called a *goal*, Arabic مطلوب, of the premise-pair.

If a premise-pair has a goal that follows logically from the pair of premises, we say that the premise-pair is *productive* (منتج),

or that it is a *syllogism* (قياس).

The strongest such goal is called the *conclusion* (نتيجة) of the premise-pair.

A premise-pair that is not productive is *sterile* (عقيم),
a name probably invented by Ibn Sīnā in his teens.

For each of his 48 premise-pairs Aristotle *either*

(1) showed that the premise-pair is productive and found its conclusion, *or*

(2) showed that we can describe a situation where the premises are both true and the goal 'Every *C* is an *A*' is true, and another situation where the premises are both true and the goal 'No *C* is an *A*' is true.

In case (2) the premise-pair is sterile, but Aristotle described the goals 'Every *C* is an *A*' and 'No *C* is an *A*' as *conclusions*. Ibn al-Muqaffa$^c$ refined this name to 'non-necessary conclusions', but al-Fārābī and al-Dimashqī went back to 'conclusions', and were followed in this by Ibn Sīnā and al-Taftāzānī among others.
Today I refer to non-necessary conclusions as *pseudoconclusions*.

Thus Aristotle gave 48 separate arguments,
one for each premise-pair,
to group the 48 premise-pairs into two lists,
one list of fourteen productive premise-pairs
and another list of thirty-four sterile ones.

| *muntij* (productive) | *ᶜaqīm* (sterile) |
|---|---|
| Every *C* is a *B*. | No *C* is a *B*. |
| Every *B* is an *A*. | Some *B* is an *A*. |
| So every *C* is an *A*. | ♯ |
| | |
| Etc. | Etc. |
| Etc. | Etc. |

In or before the early Roman Empire, an unnamed logician gave a short set of conditions that together are necessary and sufficient for a premise-pair to be *muntij*.
These became known in Arabic as *sharā'iṭu l-intāj* (conditions of productivity).

We don't know how people justified these conditions.
But given any formal premise-pair, we can easily see whether it obeys these conditions, and which list of premise-pairs it is in.
So it takes at most an hour to confirm that the conditions state facts already included in Aristotle's 48 arguments.

SO: There is no new scientific content in the *sharā'iṭu l-intāj*. They simply repackage what Aristotle proved.

BUT: The *sharā'iṭu l-intāj* are easier to remember than Aristotle's bare lists, so they are a good *mnemonic* for students to remember the *muntij* premise-pairs.

Al-Fārābī, *Ḥurūf* trans. Khalidi para. 143:

> *[During Aristotle's time] theoretical science is completed, the mathematical methods are all distinguished, theoretical philosophy and universal practical philosophy are perfected . . . . It becomes an art that is only learned and taught.*

The *sharā'iṭu l-intāj* illustrate al-Fārābī's claim.
They are an excellent research result, but it is educational research, not logical research.

After the Stoics, genuine logical research began with Ibn Sīnā. He introduced new sentence forms, looked for the inference rules that they obey, and studied how to justify these rules.

The old *sharā'iṭu l-intāj* were not adequate for the new sentence forms.
Al-Taftāzānī's *ḍābiṭa* was (as far as I know) the most serious attempt to fill this gap, though he looked only for *sufficient* conditions. He said:

> *When you take care to satisfy [the ḍābiṭa] in any recombinant predicative [premise-pair], it is muntij and covered by the sharā'iṭu l-intāj decisively.*

I.e. the *ḍābiṭa* gives a sufficient condition for productivity of predicative premise-pairs (i.e. categorical with modes allowed).

Al-Yazdī's comments on the *ḍābiṭa* in his *Ḥāshīya* support al-Taftāzānī's claim by considering each of the ways in which a premise-pair can satisfy the *ḍābiṭa*, and showing that in each of these cases the premise-pair must also satisfy the *sharā'iṭu l-intāj* (with new clauses for the new non-categorical sentence forms).

Thus he shows that the information in the *ḍābiṭa* is already contained in the *sharā'iṭu l-intāj*, and we saw earlier that the information in the *sharā'iṭu l-intāj* for categorical premise-pairs is already contained in Aristotle's 48 arguments.
This is still a repackaging of old logical science, not new logical science. (I can't comment on other work of al-Yazdī.)

Also the restriction to sufficient conditions for productivity is suitable only for students who need to know the valid syllogisms, but not to know that they are all the valid ones. Other remarks of al-Taftāzānī in his *Tahdhīb* show that he didn't understand Aristotle's proofs of sterility.

However, al-Taftāzānī makes a clear and clever educational improvement by dividing the premise-pairs into just two cases: (a) where the middle term is subject of a universal premise, and (b) where the major term is subject of a universal premise.

*ḍābiṭa* seems to be a metaphor; the literal meaning is a channel for rainwater.
Al-Taftāzānī is claiming to have brought the conditions into a single 'channel' (though it seems to me more like two).

By contrast the *sharā'iṭu l-intāj* divided the premise-pairs into three, and later four, cases for the four syllogistic figures (*al-ashkāl al-arba$^c$a*).

The western Laws of Distribution didn't divide into any subcases.
In this narrow sense the *ḍābiṭa* was a move in the same direction as the Laws of Distribution.

Finally we have come round again to the Laws of Distribution.
I should give my opinion that no significant connection has been shown between these laws and the notions used by al-Taftāzānī in his *ḍābiṭa*.

The subject term in a universal categorical sentence is distributed in a classification of terms as ‘distributed’ or ‘undistributed’ that gives rise to new basic information (cf. above on Lyndon interpolation).
I guess that this is why El-Rouayheb thought he saw a connection between the *ḍābiṭa* and distribution.

But this classification is only known to give useful information if it is defined in terms of positive and negative occurrences, without any restriction to subject terms.
For a predicate term, being distributed in this useful sense has nothing to do with quantifiers.
(Peter Geach showed this in his *Reference and Generality* chapter 1, though he didn't show how to correct this common error.)

In sum: al-Taftāzānī's *ḍābiṭa* was a good achievement with a limited aim, namely to adapt the conditions of productivity to give memorable sufficient conditions of productivity for predicative premise-pairs (including some with modes).
This was a help for teaching and learning, but probably not for a deeper understanding of the logic.

Thank you!