

The Development of Ibn Sīnā's Formal Logic

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Preface with Acknowledgments

This book has two main aims. The first aim is to give, so far as possible, a chronology to the earlier logical writings of Ibn Sīnā, up to the time of his meeting his amanuensis al-Jūzjānī in Jurjān, probably in 1013 (in the Gregorian calendar, which I follow throughout the book). The second aim is to trace the development of Ibn Sīnā's logical systems, above all from his *Short Epitome in Logic* to his *Middle Summary in Logic*.

It might seem sensible to write two books, one on each of these two aims. In practice this is difficult. When we are writing on the development of the logical contents, we naturally look to whatever evidence we have about the dates at which the source texts were written. If text X was written before text Y, then we don't generally expect to find that a logical idea developed in X disappears from view in Y. And on the other hand, Ibn Sīnā's earlier logical works tend not to come with historical information about when they were written, so that we must deduce their relative dates from the state of development of their contents.

The outcome is a book in five parts. The first part is introductory; the other four parts all begin with material for dating the works, and then progress to examine the development of ideas during the relevant period. There are many cross-references between the dating arguments and the analysis of logical development.

The whole book was planned as one of four volumes on aspects of Ibn Sīnā's logic. The other volumes are (1) *Identifying Ibn Sīnā's Hypothetical Logics* (logics which are treated in the present book less thoroughly than the predicative logics); (2) on Aristotle's methods for proving non-entailment and the sterility of premise-pairs, and the understanding of these methods in the East up to Ibn Sīnā; (3) *Mathematical Background to the Logic of Ibn Sīnā*. A substantial amount of each of these books is already written.

This work calls on a wide range of skills, from historiography to mathematical logic. That puts me and my readers in debt to a wide range of my teachers from various parts of my career, and I should acknowledge what I think are my main relevant debts. As a student in Oxford from 1959 to 1966 I attended the classes of G. E. L. Owen on the Presocratics, then the seminars of Eduard Fraenkel on Aristophanes, and the lectures of Edgar Wind on religions of the eastern Roman Empire. My philosophy tutor David Wiggins joined Gilbert Ryle to persuade me to move into logic in the Greats faculty. This involved breaking a promise that Fraenkel had extracted from me, never to abandon classics in my career; I did in fact teach mathematics until my retirement. Happily the present book allows me to save my conscience by applying the skills of a classicist to medieval Arabic.

I always found the community of mathematical logicians friendly and welcoming, and in it I learned from more people than I can remember. I must mention here my first logic tutor John Crossley. In fact I followed his lead in moving into the early history of mathematics outside Europe. Meanwhile Saharon Shelah was hugely generous with logical help of various kinds.

When I retired, one of the chief causes of my switch to Arabic Logic was meeting Roshdi Rashed in Casablanca. To support the switch I attended evening classes in Arabic at SOAS in London, first with Salman al-Saad (who was completing a dissertation on Conditional Structures in Classical Arabic), then with Shaza Shannan of the British Council. Then came Charles Burnett's afternoon seminars in medieval Arabic science and philosophy at the Warburg Institute.

Today I particularly appreciate my co-authors in Arabic logic and related areas: Saloua Chatti, Thérèse-Anne Druart, Manuela E. B. Giolfo, Ahmad Hasnaoui, Spencer Johnston and Stephen Read. Among my correspondents in and around Tehran I chiefly value my contacts with Zia Movahed, Leila Qomiavili and Mohammad Esmaeili.

To all of these above and my thoughtful and loving wife Helen, my thanks!

Part I

Ibn Sīnā in his contexts

Chapter 1

Life, works, sources

A little over a thousand years ago a teenage boy called Abū ‘Alī bin Sīnā—known for short as Ibn Sīnā and in the West as Avicenna—lived in Bukhārā in present-day Uzbekistan. Abū ‘Alī had learned something about logic and geometry from his father’s lodger, and had been inspired by these lessons. He had also managed to get hold of Arabic translations of Aristotle’s logic and Euclid’s geometry, and he made a resolution to teach himself the contents of these works. At this stage he probably had no tutor in logic beyond the hundred-year-old textbook *Syllogism* of al-Fārābī [59]. After a year and a half of intense study Abū ‘Alī reckoned that he understood Aristotle’s logic. He also reckoned that he had identified several serious mistakes that Aristotle had made in modal logic.

Abū ‘Alī was now around eighteen years old. He wrote a short summary of what he had learned about Aristotle’s logic. It survives under the name of *Short Epitome in Logic*. It covers all of Aristotle’s logical works (known to us as the *Organon*), but its main emphasis is on the formal logic of syllogisms.¹

Short Epitome in Logic discusses three kinds of syllogism. The first are Aristotle’s basic syllogisms, commonly known today as ‘categorical syllo-

¹The adjective ‘formal’ means that we study the forms of inferences. Ibn Sīnā followed Aristotle in using letters to represent forms. For example the inference from ‘Some *A* is a *B*’ to ‘Some *B* is an *A*’ is formal because the inference holds if we take any two nouns and put the first for *A* at both occurrences and the second for *B* at both occurrences—so that for example we can infer ‘Some female is human’ from ‘Some human is female’. The word ‘syllogism’ is not always used consistently, but for our purposes we can say that unless otherwise stated, a syllogism is an inference with two premises and a conclusion, as in ‘Some animals are horses; no horse laughs. Therefore some animals don’t laugh.’ (Example from Paul the Persian.)

gisms'. The second are some 'hypothetical' syllogisms that Abū 'Alī had learned from al-Fārābī's textbook.² The third are a kind of syllogism of Abū 'Alī's own invention; we will describe them in Chapter 6 below under the name 'wholly *muttaṣil* syllogisms'.

Short Epitome in Logic says very little about modal logic. It does pinpoint a modal issue on which Abū 'Alī knew that he rejected Aristotle's teaching; but at this date he apparently didn't know enough about modal logic to offer any justification of his challenge to Aristotle. We will discuss this point of disagreement in Chapter 7 below and elsewhere, as the Modal Challenge that seems to have driven Ibn Sīnā's innovations in modal logic.

Young Abū 'Alī—let us call him Ibn Sīnā from now on—went on to a lifetime of new discoveries in logic, almost all of them consequences of questions raised already in *Short Epitome in Logic*. He was still working on issues thrown up by the Modal Challenge in *Pointers and Reminders*, his latest surviving logical work, in around 1030.

The brief introduction above is mostly taken from Ibn Sīnā's own *Autobiography*, which runs up to the year 1013 when Ibn Sīnā met Abū 'Ubayd al-Jūzjānī in Jurjān. Al-Jūzjānī became Ibn Sīnā's amanuensis and stayed with him to the end of Ibn Sīnā's life. Al-Jūzjānī also continued Ibn Sīnā's *Autobiography*, taking it up to Ibn Sīnā's death in 1037. Gohlman [117] publishes Ibn Sīnā's *Autobiography* together with al-Jūzjānī's continuation, in Arabic with English translation and many informative notes. This book should be read alongside a short essay by al-Jūzjānī, [118] 1.7–4.7, which is attached to the beginning of Ibn Sīnā's *Madkhal* as a preface to his encyclopedic *Cure*. This essay describes the circumstances of the writing of *Cure*. Gutas [74] pp. 29–34 publishes an English translation of it.

To illustrate how [117] and al-Jūzjānī's preface to *Cure* inform us about Ibn Sīnā's life, let us see what light they cast on the dates of Ibn Sīnā's birth and death. At [117] p. 88f al-Jūzjānī says:³

- Ibn Sīnā passed away into the presence of his Lord and was
(1.1) buried in Hamadhān in the year 428. The year of his birth was
370.

Gohlman's notes add that the 12th century historian Abū l-Ḥasan Bayhaqī

²Al-Fārābī also describes these hypothetical syllogisms as 'exceptive'; see Section 6.5 below.

³These dates are in the hijri calendar. Henceforth I give all dates in the Gregorian calendar.

(also known as Ibn Funduq) dates his death more precisely to the first Friday of Ramadān 428, which is 18 June 1037 in the Gregorian calendar. Our other historical sources say nothing to contradict this, so we join the consensus that Ibn Sīnā died in 1037.

On the other hand there is no consensus on when Ibn Sīnā was born. As Gutas [74] p. 108 points out, al-Jūzjānī gives inconsistent dates. The hijri year 370, which al-Jūzjānī names in (1.1) as the year of Ibn Sīnā's birth, is 980 in Gregorian years. But in [117] p. 66f al-Jūzjānī tells us that Ibn Sīnā finished writing *Cure* while accompanying his patron 'Alā' al-Dawla on a journey from Iṣfahān to attack Shābūr Khwāst. Historians record that 'Alā' al-Dawla made two such raids while Ibn Sīnā was based in Iṣfahān, one in 1027 and one in 1030. But in his preface to *Cure* al-Jūzjānī says that when Ibn Sīnā finished writing *Cure* he was forty years old ([118] 3.15–17, translated [74] p. 33). Arithmetic deduces that he was born not in 980, but in either 987 or 990, a discrepancy of at least seven years. As Ibn Sīnā's companion for over twenty years, al-Jūzjānī should have been a reliable source for the basic facts of Ibn Sīnā's life; but clearly something has gone wrong.

One might be tempted to look for textual emendations that will remove the inconsistency. I think we can do a little better than that. At least one of al-Jūzjānī's dates is factually incorrect, and this may show up as allowing too much time, or too little time, for Ibn Sīnā's recorded activities at some stage of his life. A suitable correction of such a date may lead to a more convincing account of the course of Ibn Sīnā's career. For example take the stretch of Ibn Sīnā's life that ran from his reaching the age of 21 (*Autobiography* [117] 38f) to his having to leave Bukhārā and travel west to Gurgānj in present-day Turkmenistan (*Autobiography* [117] 40f).

Ibn Sīnā doesn't tell us why he had to leave Bukhārā, but historical events provide a clue. Since the late ninth century Bukhārā had been the capital of the region ruled by the Sāmānids, a family who portrayed themselves as Persian and loyal to the Islamic empire and its culture. Ibn Sīnā's father had been a village governor under the Sāmānids; so we are not surprised to learn from the *Autobiography* that Ibn Sīnā took an administrative post under the Sāmānid Emīr of Bukhārā—probably Abū l-Ḥārith Maṣṣūr, or possibly his father Nūḥ ibn Maṣṣūr who died in 997. Ibn Sīnā had already entered the service of Nūḥ ibn Maṣṣūr as a medical expert, and had been given permission to read in Nūḥ's extensive library. But in February 999 the deposition and blinding of Abū al-Ḥārith Maṣṣūr dramatically ended the Sāmānid regime in Bukhārā. For several years the regime had been under attack from an alliance of Qarakhānid Turks and local warlords.

After 999 Bukhārā was a Turkic city; the last of the Sāmānids fled and was killed in 1005. It's no surprise if after 999 Ibn Sīnā felt that the city where his employer had been executed and his employer's family thrown out was no longer a safe environment.

Autobiography [117] 38–41 describes several things that Ibn Sīnā did between reaching the age of twenty-one and leaving Bukhara. Two people who lived near him invited him to write books. Abū l-Ḥasan al-ʿArūḍī asked for a 'comprehensive book' on philosophy. Most of the book survives under the name *Philosophy for ʿArūḍī*, though unfortunately for us the part on syllogism is missing. Abū Bakr al-Baraḳī asked for a book too, so Ibn Sīnā wrote him two books, neither of which survives. The first was apparently a summa of theoretical philosophy, and Ibn Sīnā says that it was twenty volumes long. During this period Ibn Sīnā's father died, and it was after this that Ibn Sīnā took up an administrative post with the Sāmānid Emīr, which must have been in 998 at the latest. Allow Ibn Sīnā at least a year for the substantial literary efforts of this period, and we find that he was at least twenty-one in 997, and hence born in 976 at the latest. These dates are not robust. For example the age twenty-one may have had some symbolic rather than historical significance for Ibn Sīnā. It's also possible that Ibn Sīnā left Bukhārā before 999, anticipating the inevitable overthrow of the Sāmānids; this would push the date of his birth earlier. But to save further guesswork I will give him the birth date of ≤ 976 .⁴

One of the greatest obstacles to research on Ibn Sīnā's logic has been the large number of contexts in which it needs to be seen. Each context calls for a different expertise in the researcher. In the remainder of this chapter we will examine three contexts. In Section 1.1 we describe the chief sources available to Ibn Sīnā for learning Aristotelian logic. Then in Section 1.2 we will name and describe Ibn Sīnā's main surviving publications in logic, listing them in what the records suggest is the chronological order of their composition. For the works that he wrote in the second half of his career, our dating follows Gutas [74], except that in Chapter 10 we will examine the evidence that puts *Twenty Questions* in around 1012, some fifteen years earlier than Gutas's dating. Finally in Section 1.3 we approach the main task of this book, which is to trace the development through time of Ibn Sīnā's concepts and beliefs in formal logic. The section will sketch this de-

⁴Gutas [77] has argued for putting Ibn Sīnā's birth no later than 964, on the basis of the identification of Ismāʿīl the Ascetic mentioned in the *Autobiography* [117] 20f. But 964 is sixteen years earlier than the birth date of 980 stated by al-Jūzjānī; surely people who knew Ibn Sīnā would have noticed the misfit?

velopment and refer forwards to places where the evidence is given.

The main context missing from this list is the mathematical structures of the logics that Ibn Sīnā developed, and the relations between these structures. Experience teaches that some readers of the present book would find the mathematical details unwelcome, so these details have been diverted to two other books, if time's wingèd chariot allows: *Mathematical Background to the Logic of Ibn Sīnā* [98] mainly on predicative logic, and *Identifying Ibn Sīnā's Hypothetical Logics* [99] mainly on hypothetical logic. There will be plenty of cross-references.

Ibn Sīnā was also one of the chief contributors to a branch of Aristotle's logic that is widely neglected, namely the theory of sterility and nonentailment, as I hope to report in [100].

1.1 Ibn Sīnā's logical sources

Aristotle: Aristotle (384–322 BC) wrote the founding documents of logic in the mid fourth century BC. The version that Ibn Sīnā knew was an Arabic translation of a version edited for publication in the first century BC. We have no direct information about what the editor did to his material. But fortunately the part chiefly concerned with syllogisms, *Prior Analytics* i.1–22, reads as a continuous text and is generally assumed to be in the form that Aristotle himself gave it. The editor gathered together the works of Aristotle that were considered to be connected with logic; in the West this collection became known as the *Organon* during the Renaissance. One also speaks of the 'Arabic *Organon*', which is the equivalent collection in Arabic translation. It consisted of the following works in order:

Maqūlāt ('categories')
'Ibāra ('interpretation')
Qiyās ('syllogism' i.e. *Prior Analytics*)
Burhān ('demonstration' i.e. *Posterior Analytics*)
Jadal ('debate' i.e. *Topics*)
Safsata ('sophistics')
Khiṭāba ('rhetoric')
Shī'r ('poetics').

A translation of Porphyry's *Eisagōgē* was generally added at the beginning under the title *Madkhal* ('introduction').

Ibn Sīnā knew Aristotle from Arabic translations of his works. Most of the classical Arabic translations of Aristotle were due to a highly professional team of Syriac-speaking translators based in Baghdad, led by Ḥunayn b. Ishāq in the 9th century and Ibn Suwār in the late 10th century. The 11th century manuscript of the *Organon*, catalogued as Arabe 2346 in the Bibliothèque Nationale in Paris, was copied directly from Ibn Sūwar's manuscript; I will refer to this text as the 'Baghdad Standard text'. The translation of *Prior Analytics* in Arabe 2346, published in Jabre [139] pp. 169–412, is due to 'Theodorus'; Lameer ([151] pp. 3f) assembles evidence that points to this Theodorus being a member of the Syriac team somewhere around the mid 9th century. Very likely the translation was made first into Syriac and then into Arabic.⁵

The default assumption must be that Ibn Sīnā worked from a version of Theodorus's translation; though I know of no research to confirm this, and it might be difficult given Ibn Sīnā's preference for saying everything in his own words. For example Ibn Sīnā refers to 'the Philosopher's [i.e. Aristotle's] habit of saying *bi l-wujūd* (*Twenty Questions* [115] 94.6). To the best of my knowledge the phrase *bi l-wujūd* never appears in Theodorus's translation of *Prior Analytics*. But the word *mawjūd* (participle rather than verbal noun, from the same root) is very frequent there. Would Ibn Sīnā have counted this as close enough? My guess is yes, but you may disagree.

Theophrastus: Theophrastus (c. 371–287 BC) was a student of Aristotle, and then his successor as leader of the Peripatetic school in Athens. Theophrastus's own *Prior Analytics* was lost during antiquity, but at least some of it survived into the early Roman Empire, to be quoted by Alexander of Aphrodisias and Themistius. Theophrastus influenced Ibn Sīnā's logic through his 'triad' (*tathlith*, [124] i.3, 27.6; 1.6, 41.5, see Section 10.2 below) of ways of reading 'necessary' as a temporal condition, and through his study of 'metathetic' negation (which is incorporated inside terms and hence falls below the radar of the usual logical rules). At least for the first half of his career, Ibn Sīnā was unaware of Theophrastus's early work on hypothetical logic; see the end of Section 6.3 below. For Theophrastus's logic in general see Ierodiakonou [109].

⁵See Gutas [75] on the industry of translations from Syriac to Arabic, and Hugonnard-Roche [107] on the manuscript Arabe 2346.

Euclid: Euclid's *Elements* were first translated into Arabic in around AD 800, and by the time of Ibn Sīnā several translations were available (de Young [41]). We saw Ibn Sīnā analysing one of these texts during his teenage self-education in Bukhārā. As a result of those analyses, Ibn Sīnā felt he could describe the reasoning of Euclid as 'syllogistic' (*Syllogism* ix.3, 433.8)—he denied the same honour to Archimedes, explaining that logic 'was not yet available' in the time of Archimedes! (*Syllogism* i.2, 15.10.) He used the *Elements* as a resource to show the forms that syllogistic reasoning could take. Thus for example Euclid uses syllogisms with more than two premises (*Syllogism* ix.3. 433.8 again), he gives proofs that are not graph-circular (see Section ?? below on forms of proofs and on graph-circularity), and he delivers theorems that need to be analysed as hypothetical rather than predicative (*Twenty Questions* 102.1). Ibn Sīnā's account of reductio ad absurdum in *Syllogism* viii.3 makes a claim about how scientific works use conditional clauses without repeating them when required; Euclid is not mentioned, but any reader who guesses that Ibn Sīnā meant Euclid will find plenty of examples in the Arabic *Elements* to confirm the guess. Ibn Sīnā also uses the parallel postulate as an example of begging the question in geometry (*Syllogism* ix.16, 530.12–531.6); but disappointingly this example turns out to be lifted from Aristotle's *Prior Analytics* ii.16, 65a4–9, with Euclid's name gratuitously added.

Galen: Galen (AD 129–c. 200) is best known as a medical writer, but he maintained an interest in medical applications of logic (as discussed in his book *De Demonstratione*), and we know his Greek introductory logical text *Institutio Logica* from one poor medieval copy. Though both al-Fārābī and Ibn Sīnā liked to criticise Galen's remarks on logic, it is not clear that they had access to any logical works of his. (On this question see Carmona Tabja [31].)

Alexander of Aphrodisias: Alexander (Titos Aurelios Alexandros, *floruit* AD 200) was a Peripatetic philosopher. From a recently discovered inscription by Alexander himself, we can infer that he was the leader of the Peripatetic school in Athens. (See Frede and Martijn [64] on this, Alexander's life—so far as known—and his works.) He wrote the first surviving commentary on Aristotle's *Prior Analytics*. It covered book i, but we know it had Arabic translations that went only as far as the listing of syllogisms, and there is evidence that Ibn Sīnā had access to one such copy. There is also an essay of Alexander *On*

Conversion [14] which survives only in al-Dimashqī's Arabic translation; the essay is interesting for its partial anticipation of Ibn Sīnā's use of the 'incorporation' of modes into terms. It may have influenced Ibn Sīnā either directly or through al-Fārābī. Alexander was also one of the very few logicians after Aristotle who took pseudoconclusions seriously enough to get them right. Ibn Sīnā, having been very careless with them in the spirit of al-Fārābī, made a belated attempt in his *Syllogism* to treat them rigorously in hypothetical logic. But the issue quickly becomes technical, so I have discussed it more fully in [100].

Themistius: Themistius (c. 317–c. 390) was an independent philosopher with a particular interest in Aristotle including his logic. Ibn Sīnā refers quite often to Themistius in connection with formal logic. The *Catalogue* [172] of the Baghdad bookseller Ibn al-Nadīm, from the late tenth century, lists a translation of a commentary on *Prior Analytics* by Themistius. This Arabic translation may have served al-Fārābī, Ibn Sīnā and Ibn Rushd as their main source of information about the logical views of Themistius himself and also those of Theophrastus; but both the original and the translation are lost. In 1988 Rosenberg and Manekin [190] published a Hebrew manuscript, twelve pages in translation, that describes itself as excerpts from Themistius's *Commentary on Prior Analytics*. It does fit the bill as a selective précis of the *Commentary*, and there are signs that it was translated from Arabic. But it falls short of substantiating everything that Ibn Sīnā attributes to Themistius. Besides [190] see Hasnawi [78].

Al-Fārābī: Al-Fārābī (c. 870–950) devoted his career to making the ideas of Aristotle available to Arabic readers, through a range of commentaries and introductory essays. His main works on logic are *Syllogism* ([59], English translation [36]), *Short Syllogism* ([60], English translation [186]) and *Long Commentary on Prior Analytics* ([57]). Ibn Sīnā's readings of *Syllogism* and *Long Commentary on Prior Analytics* had a profound effect on Ibn Sīnā's own logic, *Syllogism* through its broad viewpoints and *Long Commentary* through the tools that it provided for the analysis of meaning (on which see Section 11.4 on semantic expansions). Unfortunately all of the syllogistic part of *Long Commentary on Prior Analytics* is lost, except for some brief but valuable quotations by Ibn Sīnā, Maimonides and Ibn Rushd (on which see [94]). See also Street [205] on Ibn Sīnā's debt to al-Fārābī, and

Chatti [34] on al-Fārābī's theory of syllogisms. In his later writings on the ontological prerequisites of formal logic, Ibn Sīnā signals a debt to al-Fārābī's adventurous philosophical work *Hurūf* [61].

1.2 Ibn Sīnā's logical works

The present book and *Identifying Ibn Sīnā's Hypothetical Logics* [99] both seek to trace the historical development of Ibn Sīnā's logics, with the present book concentrating on the predicative logics and *Identifying Ibn Sīnā's Hypothetical Logics* concentrating on the hypothetical logics. For completeness I include some information on the development of the hypothetical logics in the present book.

Epitome of the Principles of Logic (*Al-risāla l-mūjaza fī uṣūl al-manṭiq*): My comments are based on the only relevant part of this work that I currently have access to, namely the partial Arabic text and its English translation in [42]. This work shows no awareness of the logical breakthroughs in *Short Epitome in Logic* or *Middle Summary in Logic*. The one exception is that it does mention the syntactic forms of the hypothetical sentences as Ibn Sīnā spells them out in *Short Epitome in Logic* and later works; but the semantic argument given for them in *Short Epitome in Logic* is completely absent, and the new syllogistic rules for them described in *Short Epitome in Logic* are missing too. The natural explanation is that when he wrote *Epitome of the Principles of Logic* Ibn Sīnā was not yet aware of the wholly *muttaṣil* logic reported in *Short Epitome in Logic*. Since we will have reason to place *Short Epitome in Logic* in 994 at the latest, this puts *Epitome of the Principles of Logic* even earlier; we will date it to 990 or earlier.

Short Epitome in Logic (*Al-mūjaz al-ṣaghīr fī l-manṭiq*): Arabic texts are [113] pp. 2–13 (edited by Ülken) and [112] pp. 1–14 (edited by Badawī); a partial translation is in Chapter 3 below. Some of the formal logic is very rudimentary, but at the same time there are clear expositions of characteristic features of Ibn Sīnā's logic, including his wholly *muttaṣil* logic and an early statement of where he disagreed with Aristotle on alethic modal logic. He states these opinions on alethic modal logic without any supporting arguments and apparently a confused understanding of what a mode is. One of these dissenting opinions will serve as a lighthouse for much of Ibn Sīnā's later work, even up to his exploration of shapes of proofs in *Philosophy for 'Alā' al-Dawla* in the

late 1020s. So this opinion deserves a label; we will call it the 'Modal Challenge'. The work also contains a slightly altered version of exceptive logic, which Ibn Sīnā continues to use in later works. Chapter 4 below dates *Short Epitome in Logic* to approximately 994 or earlier, and summarises its contents.

Al-Barakāt's *Al-kitāb al-muṭtabar* This work is by Abū l-Barakāt al-Bagh-dādī and is from the middle-to-late 12th century. The reason for introducing it here is the evidence (mostly collected by Janssens [143]) that al-Barakāt based the logic section of his major work *Al-kitāb al-muṭtabar* on work of Ibn Sīnā from the first decade of the 11th century. For example al-Barakāt allows wholly *muttaṣil* sentences to be based on *muhmal* predicative sentences, and he illustrates (without naming them) hypothetical syllogisms 'in an incomplete part'. These are two innovations that we know Ibn Sīnā made later than *Short Epitome in Logic* but before *Middle Summary in Logic*. In this way *Al-kitāb al-muṭtabar* helps to fill out our picture of the rapid development that Ibn Sīnā made in hypothetical logics in the first decade of the 11th century; in Chapter 9 I discuss this development under the name 'the Hypothetical Breakthrough'.

Twenty Questions (*Al-ajwiba ʿan al-masāʾil al-ʿishrīnīya*, 'Answers to twenty questions'): Mohaghegh and Izutsu [167] pp. 81–103 publish a text established by Daneshpazhuh; the edition is non-critical (i.e. without information on manuscript readings). The work is not translated into any Western language. The version [115] edited by Dāneshpazhūh consists of nineteen sections each labelled 'Question' (*masʾala*); we label these questions i to xix. A few of the questions are divided into subquestions, so the numbers nineteen and twenty are not exact. All of the questions relate to formal logic, either predicative or hypothetical. In Section 8.1 below I give reasons for seeing the work as the author's preparatory notes for some sections of *Middle Summary*, and accordingly I date it to around 1012. The work may be the earliest surviving description of the Hypothetical Breakthrough by Ibn Sīnā himself; and in contrast to the account by al-Barakāt, it includes a motivation for the Hypothetical Breakthrough, namely to find all the rules of inference that are needed in order to derive hypothetical conclusions to syllogisms. *Twenty Questions* also contains the earliest surviving evidence of Ibn Sīnā's wrestling with the Modal Challenge. This takes the form of choosing temporal sentences, including some

that are said to be ‘Themistian absolute’, to replace categorical sentences.

Middle Summary in Logic (*Al-mukhtaṣar al-awsaṭ fī l-manṭiq*): An exposition of logic using as template the first five books of the *Organon* (*Eisagōgē* to *Posterior Analytics*). The book is dated to 1013 thanks to historical information in [117] REF. It contains what is probably the earliest surviving account by Ibn Sīnā himself of the new hypothetical logics of his Hypothetical Breakthrough. These include three extensions of wholly *muttaṣil* logic: (1) Contrary to *Short Epitome in Logic*, Ibn Sīnā now allows sentences of wholly *muttaṣil* logic to be based on *muhmal* (unquantified indeterminate) categorical sentences; this is an early sign of blurring between wholly *muttaṣil* logic and exceptive logic. (2) Ibn Sīnā now experiments with generalisations of wholly *muttaṣil* logic that extend to *munfaṣil* logic; see BELOW. (3) Ibn Sīnā begins to break through a major red line of Peripatetic logic by using inference rules that apply to subterms of terms of sentences. His attempt to use such rules was unsuccessful, and the challenge remained open until Frege’s *Begriffsschrift* in 1879, in spite of attempts on it by Leibniz. *Middle Summary in Logic* also contains innovations essential for Ibn Sīnā’s 2-Dimensional (2D) logic and its support for Ibn Sīnā’s alethic modal logic. These include (4) the sixfold classification of kinds of temporal necessity, (5) a format (borrowed from al-Fārābī) for defining what is quantified over in 2D sentences, and (6) a formula for translating sentences of alethic modal logic classified as ‘possible’ into *t* sentences of 2D logic. With (4)–(6) Ibn Sīnā’s contribution to alethic modal logic was essentially finished.

Shorter Summary in Logic (*Al-mukhtaṣar al-aṣghar fī l-manṭiq*): This work was written in about 1015 but first published in 1027 as the logic part of a three-part work *Deliverance* (*Al-najāt*), after the completion of *Cure*. (See Section 14.1 for references.) As far as we know, the text in *Deliverance* is a faithful copy of that in *Shorter Summary in Logic*, so we can refer to it under either name. I follow the usual custom and cite it as *Deliverance*.

We will follow the text edited by Daneshpazhuh, [131]. Asad Q. Ahmed [5] gives an English translation from *Deliverance*, and like Daneshpazhuh he uses the manuscriptr Hunt 534 in the Bodleian Library as his primary source. His ‘Translator’s Preface’ (pages xxxi

to xxxvi of [5]) is interesting and valuable. It is largely a critique of Daneshpazhuh's editorial policy.⁶

Guidance (*Kitāb al-hidāya*, 'Book of Guidance'): ʿAbduh [121] (1974) is a critical edition with Arabic introduction. The work is in three parts: Logic, Physics, Metaphysics. The Logic part (pages 61–131 in [121]) is not yet translated into any western language. Al-Jūzjānī reports that Ibn Sīnā wrote *Guidance* while in prison in 1023 (*Autobiography* [117] 60). See Gutas [74] pp. 419f on general characteristics of the work.⁷

Cure (*Al-shifāʾ*): Ibn Sīnā completed this enormous encyclopedia in Iṣfahān in 1027, having begun it maybe seven years earlier. The first nine volumes correspond to the nine volumes of Aristotle's logical works (now known as the *Organon*) as they were generally listed in the East: 1. *Eisagōgē* (Ptolemy's 'introduction' to the *Organon*). 2. *Categories*. 3. *De Interpretatione*. 4. *Prior Analytics*. 5. *Posterior Analytics*. 6. *Topics*. 7. *Sophistical Refutations*. 8. *Rhetoric*. 9. *Poetics*. There is a critical edition of the whole of *Cure*, first published in Cairo under the general supervision of Madkūr and commonly referred to as the 'Cairo edition'; the volumes corresponding to the *Organon* were printed between 1952 and 1970 under the name *Al-manṭiq* 'Logic'. I list below the volumes of *Al-manṭiq*.

1. *Eisagōgē* (*Al-madkhal*): Arabic text [118] ed. M. El-Khodeirī et al. (1952).
2. *Categories* (*Al-maqūlāt*): Arabic text [122] ed. M. El-Khodeirī et al. (1959).
3. *De Interpretatione* (*Al-ʿibārā*): Arabic text [123] ed. M. El-Khodeirī (1970).
4. *Syllogism* (*Al-qiyās*): Arabic text [124] ed. S. Zāyid (1964). There are a number of partial translations of *Syllogism*; the most substantial

⁶Daneshpazhuh departs from Hunt 534 in many places, but gives inadequate notice that he is doing so, and inadequate reasons for these departures. Ahmed is careful to annotate and explain his own departures from Hunt 534. But I note that the reasons that he lists for departing from Hunt 534 never mention the cogency of the logical content. This suggests that either Ahmed doesn't consider logical cogency as relevant to the choice of readings, or he does regard it as relevant but includes it under what he calls 'consideration of sense'. An example that favours the first of these two possibilities is at paragraph iii on his page 31, where Ahmed prefers a meaningful but logically incoherent reading in Hunt 534 to the logically coherent reading of six other manuscripts. Ahmed is probably in good company when he takes this view of the logical content.

⁷In 1971 a Beirut publisher marketed in three volumes what it called *Al-hidāya fī l-manṭiq* by Ibn Sīnā. This work is not *Guidance*; it is mainly an extract from Ibn Sīnā's *Syllogism*, a completely different work.

known to me is Shehaby [197] who translates into English nearly all of the section on hypothetical logic. See also Nusseibeh [174], who translates some passages of *Syllogism* and makes comparisons with the logic of Quine.

The Arabic text runs to over 550 pages, and is easily the most complete of Ibn Sīnā's surviving accounts of his logics. Here is a quick comparison of the contents of Ibn Sīnā's *Syllogism* with Aristotle's *Prior Analytics*:

Ibn Sīnā	contents	<i>Prior Analytics</i>
i, ii.1,2,3	syllogism, contradiction, conversion	i.1–3
ii.4	predicative syllogisms	i.4–7
"	", both premises necessary	i.8
iii.1,2,3,4	syllogisms with one premise necessary	i.9–12
iii.5,6,7, iv	syllogisms with a possible premise	i.13–22
v	recombinant hypothetical syllogisms	
vi, vii	wholly <i>muttaṣil</i> logic + generalisations	
viii.1,2	exceptive syllogisms	
viii.3	syllogism of absurdity	i.23
ix	theory of analysis	i.24–46. ii

The main innovations in *Syllogism* seem to me to be the following:

subject	<i>Syllogism</i>	this book
1. Identifying contradictory negations of 2D sentences	i.5	
2. Proof by incorporation	iii.1	
3. Refuting Aristotle's third argument against <i>Camestres</i> (ABS,NEC,NEC)	iii.2	
4. Addition of <i>munfaṣil</i> to wholly <i>muttaṣil</i> logic; productivity and sterility with negative terms	vi.2	
5. Syllogism of absurdity made direct	viii.3	
6. Proof search algorithm for predicative logic	ix.6	
7. Induction as proof by cases	ix.21	

5. *Demonstration* (*Al-burhān*): Arabic text [125] ed. A. Afīfī (1956).

The difference between *Syllogism* and *Demonstration* becomes important at some points in this book. *Syllogism* is about formal syllogisms

and their rules. The key part of the definition of 'syllogism' is that if one assumes the premises of a syllogism then one has to accept that the conclusion is true too. This is a striking requirement, because the premises of a formal syllogism are formal sentences, so that in general they are not either true or false; but somehow we know what counts as assuming that they are true. By contrast *Demonstration* is about demonstrative syllogisms, also known as apodeictic syllogisms; these syllogisms are by definition material and not formal, and it is required that we are sure that their premises are true. The rules of syllogism, borrowed from *Syllogism*, then guarantee that their conclusions are certainly true too. SEE WHERE?

6. *Debate* (*Al-jadal*): Arabic text [126] ed. A. al-Ahwānī (1965).
7. *Sophistics* (*Al-safsaṭa*): Arabic text [127] ed. A. al-Ahwānī (1958).
8. *Rhetoric* (*Al-khiṭāba*): Arabic text [128] ed. M. Sālim (1954).
9. *Poetics* (*Al-shiʿr*): Arabic text [129] ed. A. Badawī (1966).
10. *Easterners* (*Al-mashriqīyūn*): Arabic text [116]. Its main contributions are a full and integrated discussion of definitions, and the best-organised presentation of what below we will call Ibn Sīnā's two-dimensional logic.

For further details we need to examine the contents of *Easterners*. Unfortunately the book as a whole was lost during Ibn Sīnā's lifetime, and all we have of the logical section runs from the introduction to the discussion of contradiction; it stops short of any treatment of syllogisms. Examination of what we do have in this logical part shows that it was very much a continuation and rounding off of a programme in *Syllogism*, which was to build up a self-standing logic of temporal modalities. In effect this marks the completion of Ibn Sīnā's logical research. In terms of its content the logical part of *Easterners* should be regarded as the last part of what the diagram above calls the 'Mature period' of Ibn Sīnā's logical work. Gutas [74] p. 132 suggests a date between 1027 and December 1028 for the composition of *Easterners*.

11. *Philosophy for 'Alā' al-Dawla* (Persian *Dāneshnāme-ye 'Alā'ī*): The French translation [1] by Mohammad Achena and Henri Massé has the merit of including the entire work, which gives the logic a context in Ibn Sīnā's philosophical thought. F. Zabeeh [225] has translated the logic section into English. At the beginning of the work Ibn Sīnā explains

that he wrote the work at the request of his patron ‘Alā’ al-Dawla as a brief introduction to the main sciences, for the education of ‘Alā’ al-Dawla’s courtiers. So we expect an elementary work, and indeed most of the logic section is devoted to elementary logic and is written in a pleasantly down-to-earth style. But a deeper inspection shows that Ibn Sīnā has used this work to launch a new branch of logic, namely the study of the shapes of proofs, not limited to categorical logic. It seems highly likely that Ibn Sīnā was alerted to the questions answered by this new branch of logic through his final justification in *Syllogism* iii.2 of the Modal Challenge. This justification appears in *Syllogism* iii.2, which Gutas dates to 1024. Probably Ibn Sīnā wrote this passage of *Philosophy for ‘Alā’ al-Dawla* not long afterwards. See [89] section 3.

12. Notes (*Al-ta‘līqāt*): Arabic text [138] ed. Seyed H. Mousavian (2013).

The part most directly connected with formal logic—it describes the architecture of formal logic as a science—is translated into English by Kalbarczyk [147] pp. 51–55.

13. Pointers and Reminders (*Al-ishārāt wa-l-tanbīhāt*): Arabic text [135]; the logic section of *Pointers and Reminders* is available in an English translation by Shams Inati [110]. Gutas [74] dates the work to 1030 or soon after, which makes it the latest of Ibn Sīnā’s works to contribute to his logic. But at this date there are few signs that Ibn Sīnā was doing any fresh thinking in logic. Some significant developments in *Syllogism* are overlooked altogether.

Pointers and Reminders is written in a clipped and telegraphic style, which makes it in a way intermediate between an exposition and a checklist. One result is that Ibn Sīnā has deprived himself of his usual way of resolving ambiguities, which involves expanding the length of the text (cf. Chapter 10 below on semantic expansions). As Inati says:

- (1.2) ... the Arabic text is written in a highly difficult style. Its statements are concise and many of its sentences are elliptical. Much clarification and much explanation is needed before the reader can grasp its ideas. ... The best method for dealing with the difficulties that arise in the course of the work proved to be a constant checking of IS’s views in his other writings. (Inati [110] pp. 2, 4)(NOTE: For an example of a misleading elliptical statement in *Pointers and Reminders*, see Street on contr neg of (ℓ) REF.)

Inati's very welcome last sentence should be put alongside Ibn Sīnā's own advice on reading *Pointers and Reminders*, in his semi-poetic Epilogue to the work:

- (1.3) ... protect this truth from the ignorant, the vulgar, those who are not endowed with sharpness of mind, with skill and experience (Trans. Inati [111] pp. 4f, cf. Gutas [74] pp. 48f.)

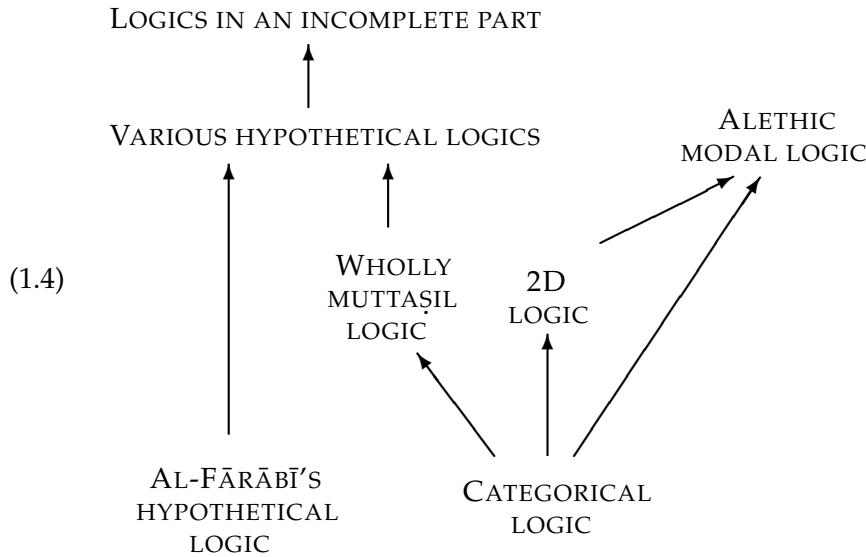
In other words, readers of *Pointers and Reminders* need to be alert and to have done their homework. Study of Ibn Sīnā's other writings is an obvious candidate for that homework. As it stands, *Pointers and Reminders* is a constant temptation for scholars who prefer to interpret Ibn Sīnā rather than read him (either in Arabic or in translation). (NOTE: Also I think Gutas oversimplifies when he says: "What [Ibn Sīnā presented in *Pointers and Reminders*] was neither Peripatetic nor Eastern philosophy but, as he claimed, just the truth and philosophical points: 'Dear Friend: I have churned for you in these Pointers the cream of truth (*al-ḥaqq*) and have fed you choice morsels of philosophy (*ḥikam*) in pithy sayings' " (Gutas [74] p. 141; the quotation from Ibn Sīnā is from the Epilogue to *Pointers and Reminders*, cf. [74] p. 48). In fact the logic part of *Pointers and Reminders* has many examples of the kind of 'indirection' that Ibn Sīnā claimed in (2.12) to have eliminated in *Easterners*; we study one such example in Section 2.2 below.)

The following chart summarises the dates assigned in the discussion above.

≤ 990	<i>Epitome of the Principles of Logic</i>
≤ 994	<i>Short Epitome in Logic</i>
1012	<i>Twenty Questions</i>
1013	<i>Middle Summary in Logic</i>
1015	<i>Shorter Summary in Logic</i>
1023	<i>Guidance</i>
1024	<i>Syllogism</i>
1027–1028	<i>Easterners</i>
1027–1029	<i>Philosophy for 'Alā' al-Dawla</i>
1027–1037	<i>Notes</i>
≥ 1030	<i>Pointers and Reminders</i>

1.3 A life in logic

As we list the works, we will sketch what contributions they make to the logics in the following diagram:



The two bottom logics in the diagram, namely al-Fārābī's hypothetical logic and (Aristotle and al-Fārābī's) categorical logic, were available to Ibn Sīnā when he started work in logic. All the other logics in the diagram are due to Ibn Sīnā. Al-Fārābī also refers to the syllogisms of his hypothetical logic as 'exceptive' (*istithnāʾī*), and Ibn Sīnā follows him in this. For the meaning of 'exceptive' see *Identifying Ibn Sīnā's Hypothetical Logics* [99] Subsection 1.3.4.

Our most direct source for Ibn Sīnā's life story is his own *Autobiography*. It starts with his birth (undated) and runs as far as his first meeting with Abū ʿUbayd al-Jūzjānī, which we will date to 1013. Al-Jūzjānī became Ibn Sīnā's amanuensis, and among many other services he continued Ibn Sīnā's *Autobiography* up to Ibn Sīnā's death in 1037. Gohlman [117] publishes together Ibn Sīnā's autobiography and its continuation by al-Jūzjānī, with an English translation and many informative notes.

While we are discussing Ibn Sīnā's birth, we note that his father was a Bactrian from what is now Northern Afghanistan ([117] 16.2). His mother's origins are not recorded, but she could have been a Sogdian since Bukhārā was the capital of the region of Sogdia. So Ibn Sīnā was not an Arab. He wrote in Arabic because Arabic was the common language of the Islamic empire. Nor was he an Uzbek—the Uzbek Turks took over the region only

in the sixteenth century.

Ibn Sīnā's father was a senior administrator for the Sāmānid rulers ([117] 16.2–6). With this background it was natural for Ibn Sīnā to find employment as an administrator—though his medical skills also increased his value to some employers. He served as administrator for several of the families, mostly presenting themselves as Persian, who claimed control of various towns of present-day Iran, Afghanistan, Turkmenistan and Uzbekistan where Ibn Sīnā spent his life. These families were constantly feuding, so that Ibn Sīnā several times had to change allegiance or even go into hiding; sometimes he was imprisoned. At least being in hiding or in prison gave him time to write.⁸

Three periods of Ibn Sīnā's life account for nearly all of his surviving writings on logic.

The first period consisted of his teenage years in Bukhārā. Chapter 3 will collect the evidence that puts *Short Epitome in Logic* within this period. In fact we will date it to no later than 994, when Ibn Sīnā was around eighteen years old.

The second period was Ibn Sīnā's stay⁹ in Jurjān; we discuss it in Chapter 11 below. Statements of Ibn Sīnā and al-Jūzjānī allow us to date this period to 1013–1015. The logical works associated with the period are *Twenty Questions*, *Middle Summary in Logic* and *Shorter Summary in Logic*. If I am right in regarding *Twenty Questions* as Ibn Sīnā's notes to himself in preparation for writing *Middle Summary*, then *Twenty Questions* may have been written a year or two before Ibn Sīnā had settled in Jurjān. I will place it in 1012.

The third period, discussed in Chapter 14 below, can usefully be taken to begin with Ibn Sīnā's imprisonment in the castle of Fardajān in 1023. In 1023/4 he made his final move to Iṣfahān where he received the protection of the warlord 'Alā' al-Dawla. Ibn Sīnā remained based in Iṣfahān for the rest of his life. Ibn Sīnā had written *Guidance* while in prison in Fardajān, and then in Hamadhān before his move to Iṣfahān he had begun some writing which he completed in Iṣfahān. The main logical writings completed while he was in Iṣfahān were *Syllogism* and the logical sections of *Easterners*, *Pointers and Reminders* and the Persian work *Philosophy for 'Alā' al-Dawla*. The work *Notes* may contain a student's notes of tutorial sessions with Ibn Sīnā in Iṣfahān.

⁸Frye [67] is a good source on the confused political history of this period.

⁹Strictly this was his second stay in Jurjān ([117] 42.2–5), but it would be tedious to keep saying so.

Almost as important as the periods when Ibn Sīnā was writing logical works that have come down to us are the periods when he was *not* writing these works. There are three such periods: between Bukhārā and Jurjān, between Jurjān and Iṣfahān, and some six or seven years after 1030 when Ibn Sīnā was still in Iṣfahān but apparently not writing about logic. Each of these three silent periods has its own character.

The first silent period, the gap of at least nineteen years that runs from *Short Epitome in Logic* in around 994 to *Middle Summary in Logic* in 1013, is particularly significant, and we will refer to it simply as the Gap. We know that this period was not really silent; during it Ibn Sīnā wrote several works on logic, but not all of them have survived. There are a few logical manuscripts from the Gap that are still sitting unpublished in libraries. In Chapters 9 and 12 we discuss what is at present reconstructable from the Gap. Work of Janssens [143] unexpectedly makes al-Barakāt relevant to this task, and may indirectly connect the Gap to aspects of the logic of al-Rāzī.

In *Short Epitome in Logic* Ibn Sīnā had set out some points on which his formal logic diverged from that of Aristotle; this put him under an obligation to defend his views against those of Aristotle. In *Middle Summary in Logic* we find him still defending exactly the same challenges to Aristotle, but now he has a collection of new ‘foundations’ for attacking Aristotle and defending his own position. These new ‘foundations’ are all in turn based on ideas that we can identify in Ibn Sīnā’s known sources. From this information we can go some way towards reconstructing his research during the Gap. But nineteen years is a long time, and we should expect that he followed a few false leads as well. Study of the evidence will find probable traces of some of those false leads.

There is a gap of at most eight years between Ibn Sīnā’s logical writings in 1015 and his picking up his pen again before his move to Iṣfahān. But the character of the work in this shorter interlude is different from that in the Gap. In 1013 his main logics were up and working; they were generally sound to modern standards, so that there was little in them that he needed to rewrite later. The advances that he wrote up while in Iṣfahān were all on more specialised questions; they range from the adaptation of wholly *muttaṣil* logic so as to allow the logical rules to handle negations of terms, to regularising the operation of incorporation of modes into terms in modal logic. Also Ibn Sīnā became more interested in the ontological underpinnings of logic. All of these advances are documented in the writings of the Iṣfahān period itself, so that there is no need for added speculations about the contents of the gap.

There is some evidence that Ibn Sīnā’s interest in formal logic was be-

ginning to weaken towards the end of his time in Iṣfahān. One indication is that much of the logic of *Pointers and Reminders* is frankly retrospective. For example its treatment of hypothetical logic could have been written already before *Middle Summary in Logic*. Another indication emerges in a letter that Gutas ([74] p. 60) dates to not long after 1030, in which a student of Ibn Sīnā comments:

- (1.5) We constantly used to hear him say: 'In analysis, do not spend too much time taking into account the forms of syllogisms, for that's one of the easy parts, and a sound instinct rarely makes a mistake about it. You should rather practice examining in detail the matters [of syllogisms].' (Gutas [74] p. 65f)

I won't explore the reasons for this change of mood. Perhaps having passed the age of fifty, Ibn Sīnā was turning to concentrate on weightier topics such as the metaphysics of the rational soul. Alternatively his not entirely successful efforts in *Syllogism* vi.2 and vi.4,5 to answer some technical questions about his extensions of hypothetical logic may have brought home to him that he had uncovered problems that he was unlikely to be able to solve in his lifetime, and which had little hope of immediate application in the sciences.

Chapter 2

From mind to paper

By the ninth century the Chinese art of paper-making had made its way along the Silk Road and become naturalised in Central Asia and the Middle East. The more westerly Asian version of paper was made mainly from rags, as opposed to the woody material favoured by the Chinese. As a consequence paper for books was freely available and relatively cheap.¹ Ibn Sīnā was born into a culture which more and more was replacing memory by written record.

2.1 Ibn Sīnā as a writer

Ibn Sīnā expects his readers to work hard. He frequently tells them so:

- (2.1) The remainder of the compounds are left for you to test (*imtiḥān*).
(*Short Epitome in Logic* [112] 8.17)
- (2.2) It's your task to enumerate its possible premise-pairs.
(*Shorter Summary in Logic* [131] 86.2)
- (2.3) I leave it to you to list the remaining premise-pairs one by one.
(*Guidance* [121] 106.4)
- (2.4) You should look for terms [to prove these statements].
(*Syllogism* [124] ii.4, 117.5)

¹Bloom [25] pp. 8f.

- (2.5) Given the rule, you should test (*imtaḥana*) it in each one [of the other cases].
(*Syllogism* [124] vii.2, 379.16)
- (2.6) Apply exactly the same considerations to hypothetical premise-pairs.
(*Syllogism* [124] ix.6, 468.7)
- (2.7) I leave it to you to pursue the question of how *muttaṣil* sentences behave under quantification, *muhmal* form, contradictory negation and conversion, just as with predicative sentences.
(*Pointers and Reminders* [135] i.3.8, 82.2f)

Similar instructions to the reader appear in most of Ibn Sīnā's Arabic discussions of formal logic, in fact possibly all except *Twenty Questions*; they are particularly frequent in *Syllogism*. They are not typical of Arabic logical writings. We find nothing like them in al-Fārābī or al-Rāzī, though there are occasional examples in al-Barakāt.² They certainly contribute to the feeling you can get when reading some parts of Ibn Sīnā's logic, that you have been strong-armed and manhandled into a waiting taxi.

In Gutas [74] the 'Inventory of Avicenna's authentic works' that takes up pages 387 to 528 bears witness to Ibn Sīnā's huge output of writings. Al-Jūzjānī's biography puts some figures on how fast Ibn Sīnā could write. In composing the *Physics* and the *Metaphysics* of *Cure* Ibn Sīnā worked from memory (since at that point he had no access to writings that he could consult), and after planning the works as a whole he would write fifty pages a day (*Autobiography* [117] p. 59). On one occasion when he needed to supply fast answers to some questions about the logical work *Shorter Summary in Logic*, he caused some consternation by covering fifty sheets (i.e. presumably a hundred pages) during one night (*Autobiography* [117] p. 79). To write at these rates he must have had the material already worked in his head, and available to be discharged onto paper or dictated to a scribe.

His habit of keeping work in his head, ready to be supplied at short notice when needed, helps to account for another feature of his writings. Often several of his essays will cover similar ground, but he very rarely repeats himself verbatim. One consequence is that often his answers to the same question in different works will contain slightly different nuances.

²For example at [18] i.159.9ff al-Barakāt exhorts the reader to try out several variants of a wholly *muttaṣil* + *munfaṣil* syllogism.

For the modern reader, comparison of the two versions may be our best guide to what is essential and what is a mere accident of phrasing or a copyist's error.³

Ibn Sīnā did change his mind about some logical questions during the course of his career. The most conspicuous example is his change of view on the contradictory negations of absolute sentences, probably sometime in the late 1010s. (REF BELOW) Nevertheless in his logical writings the overall level of consistency, even between early works and late ones, is high. The main reason seems to be that Ibn Sīnā maintained—at least in his mind—a list of logical problems. When he had resolved one of these problems to his satisfaction, he tended to stay with the answer and repeat it in all later works.

Rather than contradictions between one work and another, readers of Ibn Sīnā are just as likely to meet apparent contradictions within a single work. These 'local' contradictions are worrying, because Ibn Sīnā gives an impression of being highly deliberate in his writing, and he is certainly not a paradox-monger. But they tend to fall into a very few patterns, raising the hope that we can account for them as responses to very specific difficulties; see Sections 2.2 and 2.3 below.

2.2 Aristotle versus truth

Almost the first thing that Ibn Sīnā did in formal logic was to contradict Aristotle. To give concrete examples we will need to use terminology that we haven't yet defined. For this I apologise, but it will allow me to introduce the broad forms of some important arguments.

In the early work *Short Epitome in Logic* Ibn Sīnā identified a group of second-figure modal syllogisms about which he made the following two statements:

- (2.8) [In the second figure] the mode of the conclusion is that of the negative premise.
(*Short Epitome in Logic* [112] 7.7)

- (2.9) The truth (*al-ḥaqq*) is that when one of the premises is necessary and the other is not, the conclusion is necessary.
(*Short Epitome in Logic* [112] 7.9)

³A central example is his semantic expansions; in Chapter TWELVE below we compare some two dozen versions scattered across almost all his logical works.

Remarkably these two sentences both contradict Aristotle, and contradict each other too, about the modes of the conclusions of the syllogisms under discussion.⁴

In Section 7.3 we will introduce a notation for moods of modalised categorical syllogisms. Taking that notation on trust for the moment, there are eight moods on which (2.8) contradicts (2.9), as follows. We write 'Conc 1' for the conclusion of the mood according to the statement (2.8) and 'Conc 2' for the conclusion of the mood according to the statement (2.9).

	Mood	Conc 1	Conc 2
(2.10)	<i>Cesare</i> (NEC, ABS)	ABS	NEC
	<i>Camestres</i> (ABS ,NEC)	ABS	NEC
	<i>Festino</i> (NEC, ABS)	ABS	NEC
	<i>Baroco</i> (ABS ,NEC)	ABS	NEC
	<i>Cesare</i> (NEC, CON)	POS	NEC
	<i>Camestres</i> (CON ,NEC)	POS	NEC
	<i>Festino</i> (NEC, CON)	POS	NEC
	<i>Baroco</i> (CON ,NEC)	POS	NEC

There are four modes involved: NEC = necessary, ABS = absolute, POS = possible, CON = contingent. In each of the eight moods of 2.10 there is one affirmative premise and one negative; I put in **bold** the mode on the negative premise. For each mood the mode of the conclusion under Conc 1 is the same as the mode on the negative premise, but with CON replaced by POS. The mode of the conclusion under Conc 2 is NEC in all cases. Aristotle happens to agree with (2.8) on the first four of these moods, but there is a further second-figure mood, *Baroco*(NEC,ABS), on which Aristotle disagrees with both (2.9) and (2.8). (See Footnote 4 to this section, and Section BELOW for further details.)

Aristotle in *Prior Analytics* gives what seem reasonable arguments to show that each of the first four moods in (2.10) entails an absolute sentence of the appropriate form. (We can't be more definite than this without establishing what Aristotle takes the premises to mean, which we are not yet ready to tackle.) But this doesn't show that the conclusion is absolute, because at least in theory there could be a different proof that yields a necessary sentence, and this necessary sentence would be the conclusion. (We

⁴(2.10) gives eight examples where (2.8) contradicts (2.9). Aristotle agrees with (2.8) on the first four examples, and hence (2.9) contradicts him on these. But on the mood *Baroco*(NEC,ABS), whose first premise is negative, (2.8) agrees with (2.9) in giving a necessary conclusion, contradicting Aristotle who makes the conclusion absolute.

come back to this point at BELOW.) Aristotle is aware of this, and so he gives what he thinks are counterexamples (i.e. pseudoconclusions) to any proof of a necessary sentence from any of these moods.

Starting in *Twenty Questions*, Ibn Sīnā will dismantle Aristotle’s counterexamples. But there is a catch: by the time he comes to write *Twenty Questions*, Ibn Sīnā has decided that Aristotle interpreted his modal sentences in an inappropriate way. In place of Aristotle’s absolute premises, Ibn Sīnā puts what he calls Themistian absolute or Alexandrine absolute premises. Both of these kinds of sentence contain a quantification over times, which is something completely missing from *Short Epitome in Logic*. (To emphasise the connection with time, Ibn Sīnā introduces a new word *wujūdī* to describe these new kinds of absolute sentence.⁵) Because he has changed the sentences involved, it’s not clear that we can reasonably say that from *Twenty Questions* onwards, Ibn Sīnā has a refutation of the Aristotelian condition (2.8). Before *Twenty Questions*, Ibn Sīnā says nothing at all to support (2.9) against (2.8).

Also starting in *Twenty Questions*, Ibn Sīnā provides his own new argument based on the notion of an ‘essential distinction’ (*farq dhātī*) to derive a necessary conclusion from any of the eight moods in (2.10). This is a sound argument, and Ibn Sīnā will continue to use it in *Middle Summary in Logic*, *Deliverance*, *Hidāya*, *Syllogism* and *Pointers and Reminders*.⁶ In *Syllogism* Ibn Sīnā will sketch a different and neater proof that *Camestres*(ABS,NEC) has a necessary conclusion, using the device of ‘incorporation in the term’.

Before I suggest some verdicts on this mass of material, there is one further point worth making. Street [204] p. 148 correctly reports that Ibn Sīnā claims that all of the first four moods in (2.10) have necessary conclusions, adding ‘without however giving the proofs’. This is wrong; Ibn Sīnā gave the required proofs, both the proof by essential distinction and (at least in sketch) the proof by incorporation in the predicate. But there is an insight in Street’s mistake: *neither the proof by essential distinction nor the proof by incorporation in the predicate is Aristotelian*. Street describes on his pages 139–142 what he takes to be the rules that Ibn Sīnā uses in proofs. All the rules that Street lists are Aristotelian, so these new methods of proof are overlooked.

⁵Usually translated ‘existential’, which is in a way justified since we will see BELOW that these sentences contain an existential quantifier over times. But because of better-established uses of ‘existential’, I will stay with the Arabic name *wujūdī*.

⁶It is missing in *Easterners*, but this may be because the whole of the section of *Easterners* covering syllogisms is lost (see ABOVE). It seems also to be missing in *Philosophy for ‘Alā’ al-Dawla*, for the different reason that the logic in *Philosophy for ‘Alā’ al-Dawla* is mostly elementary and doesn’t discuss modes.

Why does Ibn Sīnā repeatedly say that the eight syllogistic moods above have non-necessary conclusions, if he believes that all eight have necessary conclusions?

We can draw some answers to this question from Ibn Sīnā's own statements of his intentions.

(a) In *Madkhal*, the first book of his encyclopedic *Cure*, Ibn Sīnā tells us:

Anything reckoned important (*yūʿtaddu bih*) in the books of the
(2.11) ancients is included in this book of ours.
(*Madkhal* [118] i.1, 9.17–10.1)

This is not a sentiment that we expect to find at the start of a textbook today. As knowledge moves on, older perceptions are abandoned in favour of newer insights. But Ibn Sīnā was writing at a time when it was still expected that a textbook of logic, and particularly an encyclopedic textbook like *Syllogism*, would begin at the beginning, and it was agreed that the beginning was Aristotle. And of course 'reckoned important' is not the same as 'correct in all details'. Granted, some of the books in which Ibn Sīnā reports Aristotle's view on the moods in (2.10) are far from being encyclopedias (for example *Guidance*). But maybe we should see this fact as evidence of how much Ibn Sīnā (not the ancients) reckoned that the questions raised by (2.10) were important.

(b) In *Short Epitome in Logic* Ibn Sīnā offers no evidence to support his claim that the moods in (2.10) all have necessary conclusions. But he fills this gap from *Twenty Questions* and *Middle Summary in Logic* onwards. So why does he continue to contradict himself by first stating that these moods have absolute conclusions? The following passage from *Easterners* provides

a possible answer:

(2.12) Now since those who are occupied with Philosophy are forcefully asserting their descent from the Peripatetics among the Greeks, we were loath to create schisms and disagree with the majority of the people. We thus joined their ranks and Adhered in a Partisan spirit to the Peripatetics, since they were the sect among them most worthy of such an Adherence. We perfected what they meant to say but fell short of doing, never reaching their aim in it; and we pretended not to see what they were mistaken about, devising reasons for it and pretexts, while we were conscious of its real nature and aware of its defect. If ever we spoke out openly our disagreement with them, then it concerned matters which it was impossible to tolerate; the greater part of these matters, however, we concealed with the veils of feigned neglect. (*Easterners* 3.12–16, trans. Gutas [74] p. 38)

There are signs of feigned neglect in some of the versions of Ibn Sīnā's argument by essential distinction. This will be easier to document in BELOW when we have all the relevant details available. But here are some pointers.⁷

(i) The account at *Middle Summary in Logic* [114] 131.15–132.12 is careful and correct, giving both Aristotle's view and Ibn Sīnā's rival view that the moods all have necessary conclusions, adding that Ibn Sīnā's view can't be proved by the Aristotelian methods of conversion and absurdity. But while in some other accounts Ibn Sīnā says that his own view is 'the truth', he speaks more indirectly here and says that the 'verifying discourse' (*al-qawl al-muḥaqqiq*)⁸ supports his view.

(ii) In *Shorter Summary in Logic* ([131] 67.3–7) he tells us that it is 'generally accepted' (*masshūr*) and *zāhir* that the dominance belongs to the negative premise, but that 'the truth (*al-ḥaqq*) compels' that the conclusion is necessary. The dictionary offers several translations for *zāhir*, including 'manifest', and 'evident'. Ahmed [5] p. 53 goes to the opposite extreme and translates as 'superficial'. Was Ibn Sīnā's choice of this ambiguous word deliberate?

⁷See also *Madkhal* [118] i, 10.14, where Ibn Sīnā contrasts *Cure* with *Easterners*: '[*Cure*] is more elaborate and more accommodating to my Peripatetic colleagues. Whoever wants the truth [stated] without indirection, he should seek [*Easterners*].'

(Trans. Gutas [74] pp. 44f.)

⁸With a different vowelling it could be *al-qawl al-muḥaqqaq* 'the unquestionable account'. See Section 11.2 below/

(iii) in *Guidance* [121] 100.1–5 the proof by essential distinction is barely recognisable: the notion of essential distinction is not mentioned, and the statement 100.2 that the conclusion is negative necessary omits to say 'conclusion'.

(c) In *Pointers and Reminders* [135] i.7.4, 145.13–150.15 Ibn Sīnā gives a detailed examination of second-figure predicative syllogisms. At 147.8 he states that the dominance in the mode belongs to the negative premise. This is close to the Aristotelian position, and Ibn Sīnā says nothing to suggest that there is any doubt about it until over two pages later at 149.12f, where he mentions that there are 'extra syllogisms' (*ziyādatu qiyāsāt*). These extra syllogisms are those of (2.10) with necessary conclusions. Why does Ibn Sīnā regard these syllogisms as in any sense 'extra'?

I can only see one reason, and that is the fact mentioned above, that none of these eight syllogisms are proved by methods recognised by Aristotle. It seems that even near the end of his career, Ibn Sīnā is embarrassed by some of his logical discoveries that use non-Aristotelian proofs. At *Deliverance* [131] 67.6 he specifically tells us that there is nothing to be embarrassed about when we prove necessary conclusions in these cases. But why is this worth saying unless there was something to cause embarrassment? At least it seems from (2.12) that he saw potential difficulties between himself and other Peripatetic logicians.

In sum, it seems that the reasons why Ibn Sīnā kept contradicting himself over the conclusions of the syllogisms in (2.10) may not be straightforward. Already in his teens when he wrote *Short Epitome in Logic* he was challenging orthodoxy on the conclusions of modal syllogisms. He maintained these challenges to the end of his career, but he may well have been conflicted about how to present them.

Let me draw two conclusions. The first is that Ibn Sīnā never—at least after his mid teens—took the view that the task of a logician is to interpret Aristotle's logic.

The second is that there is evidence of how seriously he took the issues around the syllogisms mentioned in (2.10); they keep reappearing throughout his active career. So these issues deserve some names. I will refer to Ibn Sīnā's claim that the syllogisms in (2.10) have necessary conclusions as his Modal Challenge. We saw also that Aristotle anticipated the challenge by giving what he took to be reasons why the conclusions couldn't be necessary. I will call these his Modal Refutations. The group of three refutations at *Prior Analytics* 30b18–24, 30b24–31 and 30b31–40 seem to have been par-

ticularly significant for the development of Ibn Sīnā's modal logic.

Some apparent contradictions in Ibn Sīnā's text call for other explanations. Thus at *Middle Summary in Logic* [114] 114.13 Ibn Sīnā asserts that if every *B* is an *A* with necessity then some *A* is a *B* with necessity, but five pages later he gives a counterexample ([114] 119.3f).⁹ Checking back, we find that at [114] 115.13f he told us that his first statement was Aristotle's view, but that it is highly problematic and 'the truth' is quite different.

2.3 The *khabṭ*

The Arabic word *khabṭ* means a state of confusion in which one bumps into things, as for example in the dark or in a stupor. Fakhr-al-Dīn al-Rāzī in his *Mulakhkhaṣ* (mid to late twelfth century) observes:¹⁰

- (2.13) The logical literature has found itself stumbling around (*qad waqa'a khabṭ*) as a result of using the expression 'necessary' (*darūrī*) sometimes for what is inevitable (*lā budda*), and sometimes for what is permanent (*dā'im*). This has led [logicians] to stumble around in connection with the parts of propositions and their contradictory negations, and not least the *wujūdī* propositions. We adopt the convention that by 'necessary' we mean only what is inevitable.
(Al-Rāzī *Mulakhkhaṣ* [178] 150.2–5)

Al-Rāzī is right to speak broadly about 'the logical literature'; the tradition of passing from necessary to permanent or vice versa with giddy freedom goes back to Aristotle himself. But al-Rāzī's references to problems with contradictory negations and to problems in the use of *wujūdī* sentences¹¹ make it clear that in this passage al-Rāzī has Ibn Sīnā in the centre of his

⁹'Every writer is a human with necessity'. This example reminds me (if I remember rightly) that David Kaplan told me that in his elementary logic class at UCLA he once wrote 'Theorem' on the board and wrote a theorem, then he added 'Proof' and wrote a proof of the theorem, then he added 'Counterexample' and gave a counterexample to the theorem. Nobody even raised a hand.

¹⁰See also al-Rāzī *Commentary on Pointers* [180] 188.10–16 and *Lubāb* [179] 185.8–186.4 for other places where al-Rāzī expresses the same view, again speaking of *khabṭ*.

¹¹REFS to Ibn Sīnā's discussions of contradictory negation and definition of *wujūdī*. As I hope my comments at these references make clear, I don't believe that Ibn Sīnā's struggles with the definition of contradictory negation or *wujūdī* sentences have anything to do with the *khabṭ*.

sights.¹²

Before asking whether al-Rāzī was right to imply that Ibn Sīnā operated in this state of *khabṭ*, we need to clarify just what al-Rāzī was accusing Ibn Sīnā of. If al-Rāzī was claiming only that Ibn Sīnā used the word *ḍarūrī* sometimes in the sense of inevitable and sometimes in the sense of permanent, then this is hardly a criticism. In Chapter 11 below we will find Ibn Sīnā listing and carefully distinguishing between six different meanings of *ḍarūrī* in connection with predication. Some dictionaries offer more than six different meanings of 'necessary', and nobody accuses them of inconsistency for doing this.

Or is al-Rāzī's complaint that Ibn Sīnā uses the word *ḍarūrī* with two different meanings as part of his own technical vocabulary? That would be harder to justify, unless Ibn Sīnā's usage also makes clear to the reader which sense Ibn Sīnā intends on each occasion.¹³

Note that it is going to be difficult to establish that Ibn Sīnā does use the word *ḍarūrī* in al-Rāzī's two senses if we can't point to places where he uses the word to mean 'inevitable' and other places where he uses it to mean 'permanent'. If we can do that, then we have already established that Ibn Sīnā has ways to distinguish the two senses, and that he sometimes uses these ways. Al-Rāzī's criticism may still be justified if Ibn Sīnā is forgetful or unreliable about making the distinction—this is something we will need to consider with texts in our hand in Chapter ?? below. But meanwhile let us see how he does sometimes make the distinction.

In Chapter 10 we will quote twenty-seven passages that we call 'semantic expansions', in which Ibn Sīnā describes sentence forms that he will use in syllogisms. In many of these passages he also proposes names for the sentence forms. These passages are an important part of his account of modal logic. But they need careful analysis, because of variations in the text of the semantic expansions and in the names that Ibn Sīnā proposes for them. We will need to return to these issues more fully in Chapter ?. But meanwhile here is a relatively straightforward example in which Ibn Sīnā

¹²I sometimes dream that al-Rāzī is calling attention to Ibn Sīnā's bad *khabits*.

¹³For example modern logicians use the same symbol $\models$ sometimes for satisfaction and sometimes for a form of logical consequence, but in practice it is always clear which meaning is intended.

proposes the name ‘strict necessary’ for a kind of sentence form.¹⁴

- And the paradigmatic necessary (*al-ḍarūrī al-ḥaqīqī*) is [the sentence form] whose meaning is: Everything described as a *C* ...
 (2.14) is described as [an *A*] permanently at every time, for as long as the thing described as being a *C* continues in existence.
 (*Twenty Questions* [115] x, 90.6–8)

The sentence form that Ibn Sīnā is labelling here is defined in purely temporal terms as a particular kind of permanence, with no mention of any form of inevitability. Here he qualifies *ḍarūrī* as ‘paradigmatic’, but in practice he will often omit this qualification. In this way Ibn Sīnā does use *ḍarūrī* as a technical term of his logic with the meaning ‘permanent’.

To show that Ibn Sīnā uses *ḍarūrī* in the sense of ‘inevitable’, it will be helpful to note some Arabic rough synonyms of ‘inevitable’. Besides *lā budda*, literally ‘there is no escaping it’, two other phrases with similar meanings are *lā maḥāla* and *lā yakhlū*, which might in some contexts be read as ‘necessarily’ though they have other uses too. Another way of saying that something is inevitable is to say that it is not possibly not so, for example *laysa mumkinan an lā*. Dually if we find a writer using ‘not necessarily not’ as a synonym for ‘possibly’, then we can generally assume that ‘necessarily’ is being used in the sense of ‘inevitably’. I will use the term ‘alethic’ as a broad term for this kind of necessity, and also for its dual notion of possibility.¹⁵

Using this criterion for the use of *ḍarūrī* to mean alethic necessity, we can straight away find examples in Ibn Sīnā’s discussions of syllogisms that involve sentences with the mode ‘possibly’. He analyses several of these syllogisms using a *reductio ad absurdum* that takes a sentence with the mode ‘possibly’ to its contradictory negation with the mode ‘necessarily’—or in some cases vice versa. In these cases the ‘necessarily’ can be safely read as alethic. EXAMPLES.

We will pick up these points again in Chapter ?? below. Meanwhile let me state my general impression that there may be a few places where Ibn Sīnā uses the word *ḍarūrī* and sends confusing signals about whether he means alethic necessity or permanence, but there are not many and they

¹⁴This is SE1 in REF. I omit irrelevant parts etc, which are mainly devoted to saying what is *not* expressed by the sentence form.

¹⁵I think I am following the usage of Movahed [170] in his book. The word was introduced to modal logic by Von Wright [223], who used it to distinguish ETC from ontological or epistemic or deontic necessity CHECK. Von Wright and Movahed

are quite hard to find.¹⁶ Consider also the passage:

- (2.15) You have already met the difference between permanent (*ḍā'im*) and necessary (*ḍarūrī*). One has to take care (*yurā'īya*) of this in all the cases that we are talking about. These are fundamental points that you will have to keep in mind for yourself, because people don't bother with them.
(*Syllogism* [124] ii.2, 90.5f)

Here at least, Ibn Sīnā is explicit that he expects his more diligent readers to be able to make the distinction between alethic necessary and permanent.

What did al-Rāzī propose as a remedy for the *khabṭ*? He gave us his answer already in (2.13) above:

- (2.16) We adopt the convention that by 'necessary' (*ḍarūrī*) we mean only what is inevitable.

This admirable suggestion was taken up by al-Rāzī's successors, among them al-Khūnajī and al-Ṭūsī. It left the name *ḍā'im* available for sentences that are necessary in the sense 'permanent'.¹⁷

¹⁶For example

We will prove that the conclusion is necessary permanent when *C* and *B* are different, so that for one of them, all the individuals under it are said to be necessarily and inescapably *As*, or else the same with 'not *As*'; and the other term is such that every individual under it is of one of two kinds: either it has necessarily the opposite property to those of the first term, or it is possible (*yumkinu*) that it has the opposite property ... (*Middle Summary in Logic* [114] 132.4–7)

My own reading is that this is all temporal; *lā yakhlū* has an implied quantification over individuals rather than over possibilities, and 'it is possible' is a clumsy choice of words. This passage is probably one of Ibn Sīnā's first attempts to formulate a non-Aristotelian justification for some claims that he makes in contradiction to Aristotle; SEE BELOW.

¹⁷Street, in his survey of Ibn Sīnā's modal syllogistic [204] p. 136, shows no awareness of Ibn Sīnā's double usage of *ḍarūrī*. In practice it seems that he counts Ibn Sīnā's *ḍarūrī* as always meaning 'inevitable' (which he writes as *L*) rather than 'permanent' (his *A*). I believe the decision to count *ḍarūrī* as 'inevitable' in all cases first appears in al-Rāzī and was adopted by al-Rāzī in deliberate opposition to earlier logicians including Ibn Sīnā. The result is that in a number of cases where Ibn Sīnā says *ḍarūrī*, Street reads this as *L* although the evidence suggests *A*. People using Street's transcriptions of Ibn Sīnā's modal syllogisms should be alert to this.

Part II

Bukhārā (to c. 999)

Part II of this book gathers together the evidence for Ibn Sīnā's understanding of formal logic at the time when he left Bukhārā. In Section 4.2 we give evidence suggesting that Ibn Sīnā left Bukhārā in or soon after the year 999 in response to the fall of the Sāmānid regime. The section also gives evidence that the short essay *Short Epitome in Logic* was written while Ibn Sīnā was in Bukhārā, and very likely no later than 994 when he was in his teens. In his *Autobiography* Ibn Sīnā refers to three books that he wrote shortly before leaving Bukhārā (see the calculation on Ibn Sīnā's date of birth in Chapter 1 above), but unfortunately no syllogistic material survives from any of these three books. Another logical work of Ibn Sīnā that is probably from his time in Bukhārā is his *Epitome of the Principles of Logic*. Judging from the extract published by Silvia Di Vincenzo [42] I guess that this work is the earliest logical treatise that we have from Ibn Sīnā, but its logical content appears very thin. Gutas [74] p. 434f summarises the work and points to three manuscripts in Istanbul and one in Tehran.

At present this leaves us with one substantial logical work from Ibn Sīnā's time in Bukhārā, namely his *Short Epitome in Logic*. But in spite of its brevity (some 15 pages of Arabic), the work is one of the major innovative works in medieval logic. In it Ibn Sīnā shows how to adjust Aristotle's categorical logic so as to create a new hypothetical logic. In Part III we will find Ibn Sīnā developing and varying this hypothetical logic during the Gap period. *Short Epitome in Logic* also contains a new contribution to modal logic which is much harder to assess, because we don't know why Ibn Sīnā makes the modal claims that he does make. In his surviving works Ibn Sīnā begins to find answers to this question in *Twenty Questions*, probably in 1012 at the end of the Gap period.

Chapter 3

Text: *Short Epitome in Logic* (partial translation)

The page references are to the edition of Badawī [112].

/1.5/ Logic

/1.10/ Any expression that you use to signify many things under a single meaning is a universal. An example is ‘animal’. This is regardless of whether the many things are in your imagination or in the real world. /2.1/ Any expression that can’t signify with one and the same meaning several things that all have that meaning is [called] a particular. An example is ‘Zayd’.

/3.10/ Section: [De Interpretatione]

An expression that applies to many things either applies in a single meaning to equivalent things, in the same way as animal applies to human and horse, and [in this case] it is called univocal; or it applies in differing meanings in the way that *‘ayn* applies both to a dinar and to the eye, in which case it is called ambiguous; or else it is applied in a single meaning to things that are not equivalent—and in this case it is called doubtful—in the way that ‘existing’ is applied both to a substance and to an accident.

/3.15/ A name is an atomic expression that signifies a meaning without a salient time for it. A verb, also [known as] an action, is an atomic expression signifying both a meaning and the time of it, as when we say ‘was’. A sentence is any compound expression. A declarative sentence is one that allows of being assessed as true or false; it is also [called] a proposition.

/4.1/ A predicative¹ proposition is one used to judge about whether one thing, [called] the predicate, is true or not true of another thing which is [called] the subject, as when we say

(3.1) Zayd is a writer.

or²

(3.2) Zayd is not a writer.

The first [of these two sentences] is called an affirmation and the second is called a denial. A *muttaṣil*³ hypothetical proposition is one in which a judgment is made that a proposition, called the consequent, does or doesn't follow from /4.5/ another proposition, called the antecedent. The first [kind of meet-like hypothetical proposition] is an affirmation, as when you say

(3.3) If the sun is up then it is day.

and the second [kind] is a denial, as when you say:

(3.4) It is not the case that when the sun is up then it is night.

A *munfaṣil*⁴ hypothetical [proposition] is one in which a judgment is made [affirming] the exhaustiveness of the two propositions with reference to a conflict, or the denial of [such a judgment]. An example of the first is

(3.5) Either this number is even or it is odd.

/4.10/ An example of the second is⁵

(3.6) It is not the case that either this is even or it is two.

¹*ḥamlī*, from *ḥaml* 'predicate'. See Section 4.2 below for the relationship between 'categorical' and 'predicative'.

²'Writer' is *kātib*. The word is an active participle, ambiguous between 'writer' (i.e. literate) and '(is) writing'. Either way it expresses a separable accident, i.e. a property that the same thing can have and not have at different times of its existence.

³*muttaṣil* is from *waṣl* 'junction'. 'Hypothetical' is *sharṭī*, literally 'conditional'. The history that led to these names is chaotic, and in *Shorter Summary in Logic* Ibn Sīnā adds to the chaos; see Section 6.2 below.

⁴*munfaṣil* is from *faṣl* 'separation'. Again Ibn Sīnā's usage adds to the chaos of the terminology; see Chapter ?? below.

⁵Badawī reads *farḍan* 'odd'. A better reading is *ithnayni* 'two' in two manuscripts.

There are eight [kinds of] predicative⁶ proposition: [(i)] singular affirmative, as when you say

(3.7) Zayd is a writer.

[(ii)] singular negative, as when you say

(3.8) Zayd is not a writer.

and the subject in both of these together is a particular⁷ expression; [(iii)] unquantified affirmative, as when you say

(3.9) The person has suffered a loss.

[(iv)] unquantified negative, as when you say

(3.10) The person has not suffered a loss.

the subject in both of these is a universal and its extent is given by the judgment about it /4.15/ as unquantified; [(v)] quantified universal affirmative, as when you say

(3.11) Every human is an animal.

[(vi)] quantified universal negative, as when you say

(3.12) No person is a stone.

[(vii)] existential⁸ affirmative, as when you say

(3.13) Some person is a writer.

[(viii)] existential negative, as when you say:

(3.14) Not every human is a writer.

and

(3.15) Some person is not a writer.

⁶These sentence forms are in fact the categorical ones, cf. Note 1 above. This illustrates how the context can shrink the class of predicative propositions.

⁷*juz'ī* is literally 'particular', though one would have expected *shakhṣī* 'singular' here.

⁸*juz'ī* again, but here the contrast is with *kullī* 'universally quantified', so 'existential(ly quantified)' is a better fit with modern logical terminology.

both of these are negative about 'some', and this allows that the [corresponding] affirmation is true of 'some' [of the subject].

/5.1/ A contradictory pair in the singular [propositions] consists of two propositions which differ as affirmative and negative, where the subjects agree in meaning and so do the predicates, and [where the two propositions agree in] condition⁹ and relation and part and whole—if there are a part and a whole—and act and potential and time and place. In the quantified propositions [it is required] that these conditions are satisfied, and also that one of the two propositions is universal and the other is existential.

/5.5/ The modes¹⁰ of propositions are three: necessary, possible and impossible. The necessary mode is as when you say

(3.16) The human is an animal.

The impossible is as when you say

(3.17) The human is a stone.

The possible is as when you say

(3.18) The human is a writer.

Conversion makes the subject a predicate and the predicate a subject, leaving affirmation and denial and truth-value the same as before. The universal negative [proposition] converts symmetrically.¹¹ Thus when no X is a Y, then no Y is an X; so when no person is /5.10/ a stone, no stone is a person. As for the universal affirmative and the existential affirmative

⁹*shart* is 'condition' as in 'meeting a condition', not as in 'in good condition'. From this text alone it is not clear what 'condition' means here, but see Section 4.3 below.

¹⁰*jila* here seems to mean 'mode', though the word has a range of meanings including 'aspect', 'point of view', 'interpretation'. The triple 'necessary', 'possible' and 'impossible' appear together in al-Fārābī *Syllogism* [59] 16.8–17.8 as three kinds of 'matter' (*mādda*). But the author of *Shorter Summary in Logic* seems to have another source; like Ammonius he regards the three as the three matters rather than as kinds of matter, and his word for 'necessary' is *wājib* rather than al-Fārābī's *darūrī*. See Chapter 7 below for further discussion.

¹¹A predicative sentence converts 'symmetrically' (Arabic *mithla nafsīhi*, literally 'like itself') if it is equivalent to the sentence got from it by transposing the subject and the predicate. A predicative sentence form is said to convert symmetrically if every sentence of that form converts symmetrically. For example the categorical sentence form 'No B is an A' converts symmetrically, since for every choice of nouns A and B, 'No B is an A' is equivalent to 'No A is a B'.

propositions, they don't have to convert to universal propositions; in fact when every human is an animal or some moving thing is black, that doesn't imply that every animal is a human or every black thing is moving. But they do have to convert to an existential proposition, so when every X or some X is a Y then some of what is a Y is an X. But the existential negative [proposition] doesn't convert, since it is not the case that when /5.15/ not every animal is human, it has to be that not every human is an animal.

Syllogism

A syllogism is composed from sentences, such that when they are assumed, another sentence follows from them by themselves. An example of this is when you assume that every body is composite and that everything composite is created; it follows from this that every body is created.

/5.20/ Some syllogisms are recombinant¹² and some are exceptive.¹³ Recombinant syllogisms consisting of predicative propositions are [classified into] three figures. /6.1/ When the [part] that is repeated in the two premises, like 'composite' in the example above, is the predicate in the first of the two propositions and the subject in the second, [the syllogism] is said to [belong to] the first figure. When this repeated [part] is the predicate in both the premises, [the syllogism] is said to be [in] the second figure. Or else [the repeated part] is the subject in both premises and [the syllogism] is said to be [in] the third figure. This middle [term] is of a nature to combine the two extreme [terms] /6.5/ into a conclusion and to be excluded from what is explicit¹⁴ [in the conclusion], so that one of the two extremes, called the minor term, forms the subject of the conclusion, and the premise [containing] it is called the minor premise; while the other extreme, called the major term, forms the predicate of the conclusion, and the premise [containing] it is called the major premise.

¹²This seems to be the first recorded occurrence of *iqtirānī* as a term of logic. See Chapter 5 below on its probable origin.

¹³*istithnā'ī*. The word is taken from al-Fārābī, who borrowed it from the linguists; cf. [36] pp. 56f. The use of this linguistic term is ingenious but not of much help for understanding the logic. Later logicians, and perhaps already Ibn Sīnā, may have read the word etymologically as 'involving a duplication'.

¹⁴Badawī thinks *min al-bayyini* 'from what is explicit' is really *min al-bayna* 'from the between', i.e. from between the extremes. But Badawī's reading makes sense only for syllogisms in the first figure.

Section: [Categorical syllogisms]

The first figure yields a conclusion only when the minor premise is affirmative and the major premise is universal. /6.10/ The dominance¹⁵ in quality (i.e. affirmative and negative) and in mode (i.e. necessity and non-necessity) belongs to the major premise. An example of the first [mood of the first figure]¹⁶ is¹⁷

- Every *C* is a *B*;
 (3.19) and every *B* is an *A*, however it is.
 So every *C* is an *A*.

[The mode of the conclusion is] as above except when the minor premise is possible and the major premise absolute,¹⁸ in which case the conclusion

¹⁵The Arabic word for 'conclusion', *natīja*, is a metaphor: literally a *natīja* is a brood of sheep or camels. The underlying idea is that a conclusion is generated by the marriage of the two premises. The origin of the metaphor is unknown—it has not been found in Greek or Syriac sources. But several Arabic logicians (including the Brethren of Purity, Ibn Sīnā and Ibn Rushd) made use of the metaphor. Ibn Sīnā's contribution was that each property of the conclusion can be assumed to be inherited from one of the premises, so we can determine the conclusion by determining which premise is responsible for each property. Ibn Sīnā describes the premise in question as having the *'ibra* for the given property; I translate *'ibra* as 'dominance'. See further Hodges [93].

¹⁶Later the Latin Scholastics introduced names for these moods. The names contained coded information about the mood; for example *a* stands for universal affirmative, *e* for universal negative, *i* for existential affirmative and *o* for existential negative. (The three vowels in order describe the major premise, the minor premise and the conclusion, reflecting the usual order in which the Latins put the premises.) The present mood was called *Barbara*.

¹⁷*kayfa kāna*, 'however it is'. This and the similar phrase *kayfa ittafaqa* 'however it happens to be' occur quite often in Ibn Sīnā's logical expositions. Here the implication is that the sentence 'Every *B* is an *A*' can be varied in some way. Of course it can be varied by putting different words for *B* and for *A*; but this applies to all formal sentences and doesn't call for special mention here. More likely the variation that Ibn Sīnā has in mind here is putting modes on the sentence. Then the phrase is picked up by the sentence below which says that the conclusion is 'as above' (*ka-dhālika*) except in a special case; i.e. the conclusion has the same mode as the second premise (apart from the special case). So this is a restatement of the rule for dominance in mode.

¹⁸*muṭlaq*. The word is used in the Baghdad Standard translation of *Prior Analytics* for sentences that are categorical, i.e. not modalised, for example in translating *Prior Analytics* i.2, 25a1. It doesn't appear in al-Fārābī *Syllogism* [59]. There are just two occurrences in *Short Epitome in Logic*, namely here ([112] 6/12) and at the statement of dominance for mode in third figure ([112] 8.2). These occurrences are not explained, but they copy the usage in the Baghdad Standard translation; there is not yet any hint of the central place that various kinds of absolute sentence will take in Ibn Sīnā's logic from *Middle Summary in Logic* onwards.

is possible. The second [mood of this figure]¹⁹ is

- (3.20) Every *C* is a *B*;
and no *B* is an *A*, however it is.
So no *C* is an *A*, [with mode] as above.

The third [mood of this figure]²⁰ is

- (3.21) Some *C* is a *B*;
and every *B* is an *A*, however it is.
So some *C* is an *A*, [with mode] as above.

The fourth [mood of this figure]²¹ is

- (3.22) /6.15/ Some *C* is a *B*;
and no *B* is an *A*.
So some *C* is not an *A*.

The remaining [premise-pairs in first figure] have no conclusion following from them.²²

The second figure [premise-pairs yield a conclusion when they obey] the condition that the major premise is universal and the two [premises] differ as affirmative and negative. The first mood of [this figure]²³ is when you say:

- (3.23) Every *C* is a *B*;
and no *A* is a *B*.
We claim that it follows from [the premise-pair] that no *C* is an *A*.

The demonstration of this [mood] is that we convert the major premise so that it becomes 'No /6.20/ *B* is an *A*', and we reduce²⁴ [the premise-pair] to the first figure and we infer [the conclusion] above. The second mood [of

¹⁹Scholastic *Celarent*.

²⁰Scholastic *Darii*.

²¹Scholastic *Ferio*.

²²From *Middle Summary in Logic* onwards, Ibn Sīnā describes premise-pairs from which no conclusion follows as *‘aqīm* 'sterile', exploiting the metaphor discussed in Note 15 above. The absence of this word here is an indication that *Short Epitome in Logic* is an early work.

²³Scholastic *Cesare*.

²⁴*nurjī‘u*. Reduction to first figure moods is Aristotle's way of justifying second- and third-figure moods. In *Middle Summary* Ibn Sīnā will defy Aristotle by giving justifications that don't involve reduction to first figure; see Section 14.3 below.

this figure]²⁵ is:

- No /7.1/ C is a B ;
(3.24) and every A is a B .
It yields [a conclusion] as above.

It is proved by converting the minor premise so that it yields 'No A is a C ', and then it is converted to 'No C is an A '. The third mood [of this figure]²⁶ is as when you say

- Some C is a B ;
(3.25) and no A is a B .
It yields [the conclusion] that some C is not an A .

It is proved by converting the major premise. The fourth mood [of this figure]²⁷ is as when you say:

- Not every C is a B ;
(3.26) and every A is a B .
It yields [the conclusion] that /7.5/ not every C is an A .

It is proved not by conversion but by ecthesis²⁸ [as follows]. Let the some which is a C but not a B be D . Then we have

- No D is a B ;
(3.27) and every A is B .
This yields that no D is an A .

But

- (3.28) D is some C ;²⁹
so not³⁰ every C is an A .

The dominance for the mode [in the second figure] belongs to the negative premise, because the negative premise reduces to the major premise in the first figure by conversion or ecthesis, and the dominance for mode in the first figure belongs to the major premise. But the truth³¹ is that when the premises are a mixture, one necessary and the other not necessary, then the conclusion is necessary.

²⁵Scholastic *Camestres*.

²⁶Scholastic *Festino*.

²⁷Scholastic *Baroco*.

²⁸*iftirād*; see BELOW for more on the method of ecthesis, which derives from Aristotle.

³¹This is an example of the Aristotle-versus-truth format discussed in Section 2.2 above.

/7.10/ The third figure [premise-pairs yield a conclusion when they obey] the condition that the minor premise is affirmative and there has to be a universal [premise]. The first mood of [this figure]³²:

- Every *B* is a *C*;
 (3.29) and every *B* is an *A*.
 It yields [the conclusion] that some *C* is an *A*.

It reduces to the first [figure] by converting the minor premise. The second mood [of this figure]³³ is

- Every *B* is a *C*;
 (3.30) and no *B* is an *A*.
 So not every *C* is an *A*.

It reduces to the first [figure] by converting the minor premise. The third mood [of this figure]³⁴ is

- Some *B* is a *C*;
 (3.31) and every *B* is an *A*.
 It yields [the conclusion] that some *C* is an *A*.

It is proved by converting the minor premise. The fourth mood [of this figure]³⁵:

- Every /7.15/ *B* is a *C*;
 (3.32) and some *B* is an *A*.
 It yields [the conclusion] that some *C* is an *A*.

It is proved by converting the major premise, and then one converts the conclusion. Or [one proves it] by ecthesis, by specifying the thing that is some *B* [and] an *A* to be *D*, so that every *D* is an *A*. Then

- When we say that every *D* is a *B*;
 (3.33) and that every *B* is a *C*;
 it yields [the conclusion] that every *D* is a *C*.

Then

- When we say that every *D* is a *C*;
 (3.34) and that every *D* is an *A*;
 it yields [the conclusion] that some *C* is an *A*.

³²Scholastic *Darapti*.

³³Scholastic *Felapton*.

³⁴Scholastic *Datissi*.

³⁵Scholastic *Disamis*

The fifth mood [of this figure]:³⁶

- Every B is a C ;
(3.35) and not every B is an A .
It yields [the conclusion] that not every C is an A .

This is proved not by conversion but by ecthesis. The /7.20/ sixth mood [of this figure]:³⁷

- Some B is a C ;
(3.36) and no B is an A .
So some C is not an A .

This is proved by conversion /8.1/ of the minor premise.

The dominance in mode [in the third figure] belongs to the major premise, which is conveyed to the major premise in the first figure by conversion or ecthesis. That is, unless the minor premise is possible and the major premise is absolute.

[Section: Recombinant hypothetical syllogisms]³⁸

Note that the meet-like hypothetical sentences form premise-pairs that are described in the same way as the figures above, if we put their antecedent in place of the subject and their consequent in place of the predicate. Thus if the antecedent in /8.5/ one of the premises is the consequent in the other, it is [a mood in] the first figure. If [a clause is] consequent in both premises, then it is [a mood in] the second figure. If it is antecedent in both premises, then it is [a mood in] the third figure. The hypothetical proposition that is composed with its antecedent and its consequent being the two extremes [of the premises] is the conclusion. The conditions [for having a conclusion] are the same as the previous ones. The universal affirmative meet-like [proposition] is as when we say

- (3.37) Whenever A is B , C is D .

The universal negative [meet-like proposition] is as when we say

- (3.38) It is never the case when A is B that C is D .

³⁶Scholastic *Bocardo*.

³⁷Scholastic *Ferison*.

³⁸The recombinant hypothetical syllogisms form Ibn Sīnā's first new logic, which we call wholly *muttaṣil* logic. We examine it in detail in Chapter 7 below.

The existential affirmative [meet-like proposition] is as when you say

(3.39) /8.10/ Sometimes when *A* is *B*, *C* is *D*.

The existential negative [meet-like proposition] is as when you say

(3.40) Sometimes it is not the case, when *A* is *B*, that *C* is *D*.

Or:

(3.41) It is not the case that whenever *A* is *B*, *C* is *D*.

An example of the first mood of the first figure is

Whenever *A* is *B*, *C* is *D*;
(3.42) and whenever *C* is *D*, *H* is *Z*.
It yields [the conclusion]: Whenever *A* is *B*, *H* is *Z*.

An example of the first mood of the second figure is

Whenever *A* is *B*, *C* is *D*;
(3.43) and it is never the case when *H* is *Z* that *C* is *D*.
It yields [the conclusion] that it is never the case when *A* is *B* that
H is *Z*.

This is proved as above, /8.15/ by conversion. An example of the first mood of the third figure is

Whenever *C* is *D*, *A* is *B*;
(3.44) and whenever *C* is *D*, *H* is *Z*.
It yields [the conclusion]: Sometimes when *A* is *B*, *H* is *Z*.

This is proved by conversion. Now it is left to you to formulate and test the remaining compounds of [meet-like premises].³⁹ Where they use ecthesis, it is like when you say

It is not the case that whenever *C* is *D* then *H* is *Z*;
(3.45) and whenever *A* is a *B*, *H* is *Z*.
We say that it yields [the conclusion]: It is not the case that whenever *C* is *D* then *A* is *B*'

³⁹This sentence is characteristic of Ibn Sīnā from two points of view: the reader is set some homework (see Section 2.1 above), and Ibn Sīnā invokes *imtihān* 'testing' as a method of research in logic (see BELOW).

The demonstration of this is that we specify⁴⁰ the posit in which C is D and in which H is not Z , namely just while /8.20/ G is T .⁴¹ So

- (3.46) It is never the case when G is T that H is Z ;
and whenever A is B , H is Z .
So it is never the case when G is T that A is B .

Then we say:

- (3.47) It is sometimes the case, when C is D , that G is T ;
and it is never the case, when G is T , that A is B .
This yields [the conclusion]: It is not the case that whenever C is D then A is B .

/9.1/ Section: [Exceptive syllogisms]⁴²

An exceptive syllogism has either a meet-like premise or a difference-like one. If it has a meet-like premise then [there are two cases.] Either it excepts the antecedent [of this premise] unaltered, and its conclusion is the consequent unaltered, as when you say:

- (3.48) If this is a human then it is an animal;
but it is a human.
So it is an animal.

Excepting the contradictory negation /9.5/ of the antecedent doesn't yield a conclusion. Thus [if you except]

- (3.49) But he is not a human.

it doesn't follow from this that he is an animal or that he is not an animal. If the exception is from the consequent [of the meet-like premise], so that the

⁴⁰Where Badawī [112] 8.19 reads *immā nafsa al-waḍʿi*, read *anna nuʿayyina al-waḍʿa* following Ülken [113] 8.11. Neither edition gives any critical apparatus at this point.

⁴¹Here Ibn Sīnā introduces two new letters T and G with the intention that the sentence ' T is G ' expresses the posit (or assumption) that C is D and H is not Z . But there is not the slightest reason to think that this assumption has the form ' T is G '. This is evidence that the expressions ' C is D ', ' T is G ' and so on are essentially propositional variables. Aristotle almost never uses single letters to represent propositions, and Ibn Sīnā is dutifully following Aristotle's notation here.

⁴²The exceptive syllogisms are taken from al-Fārābī *Syllogism* [59] 31.7–33.17 (see [36] 53–58 and 133–136), except that Ibn Sīnā replaces al-Fārābī's exclusive disjunctions by inclusive disjunctions. The resulting logic is HL1 in our notation. It is examined in Section 6.1 below.

contradictory negation of the consequent is excepted, it yields [as conclusion] the contradictory negation of the antecedent, as when you say:

- (3.50) But he is not an animal.
Then it yields [the conclusion] that he is not a human.

But when the consequent is excepted unaltered, it doesn't follow that it yields any conclusion. If you said

- (3.51) But it is an animal.

it wouldn't follow that it is a human or that it is not a human.

/9.10/ In [a premise-pair with] a difference-like [hypothetical premise], when one excepts one of [the clauses of the difference-like premise] unaltered, it yields [as conclusion] the contradictory negation of the remainder as they were, forming a difference-like [conclusion] when there are many [clauses remaining], or the contradictory negation of the sole remainder as it was. An example of the first case is:⁴³

- (3.52) This number is either abundant or deficient or equal.
So if one excepts that it is deficient,
it yields [the conclusion] that it is not abundant and not equal;
or that it is neither abundant nor equal.

An example of the second is:

- (3.53) This number is either even or odd;
but it is odd.
So it is not even.

When one /9.15/ excepts the contradictory negation of one of [the clauses], it yields the remainder, unaltered and as it was, or the one [remaining], unaltered and as it was. An example of this is:

- (3.54) But it is not abundant;
so it is either deficient or equal.

And also:

- (3.55) But it is not odd;
so it is even.

⁴³This classification of natural numbers (i.e. positive integers) was for some reason of great interest to Neoplatonist mathematicians. A proper factor of n is a number that divides n and is less than n . For each number n write $S(n)$ for the sum of the proper factors of n . One says that n is abundant if $S(n)$ is greater than n , deficient if $S(n)$ is less than n , and equal if $S(n)$ equals n . Today the numbers n with $S(n) = n$ are called perfect; examples are 6 and 28.

When the difference-like premise is not paradigm—namely when it has both affirmative and negative clauses, or when both clauses are negative⁴⁴—then it yields no conclusion unless one excepts a contradictory negation [of a clause]. An example of this is

- (3.56) Either Abdullah /10.1/ is in the sea, or he is not drowning;
 but he is drowning.
 So he is in the sea.

[Or:]

- (3.57) But he is not in the sea;
 so he is not drowning,

And when you say 'But he is in the sea', or 'He is not drowning', nothing follows from [either of these]. And similarly [you can argue]

- (3.58) Either Zayd is not an animal or Zayd is not a plant;
 but he is an animal;
 so he is not a plant.

[Or:]

- (3.59) But he is a plant.
 So he is not an animal.

But nothing follows from your saying 'He is not /10.5/ an animal' or 'He is not a plant'.

A paradigm *munfaṣil* proposition is one which includes the expression 'without exception'.⁴⁵

Section: [Syllogism of absurdity (*khalf*)]

⁴⁴This is most easily read not as an explanation of the meaning of paradigm *munfaṣil* sentences, but as a syntactic criterion for distinguishing between paradigm and non-paradigm *munfaṣil* sentences. Curiously Ibn Sīnā seems to be still using this criterion in his later logic HL3 RENAME (cf. Chapter ??), though this sentence in *Short Epitome in Logic* may be the last place where he states the criterion explicitly. This is a striking illustration of the continuity of his logical assumptions throughout his career.

⁴⁵By contrast with Note 44, here he seems to be describing the difference in meaning between paradigm and non-paradigm *munfaṣil* sentences. But has he stated it correctly? See BELOW.

The syllogism of absurdity is that the contradictory negation of the goal⁴⁶ is taken and there is added to it [as further premise] a true premise forming a productive syllogism which yields a thing that is clearly impossible⁴⁷, and it is known that the cause of that /10.10/ impossibility is not the composition of the syllogism and not the true premise, but rather its cause is the impossibility of the contradictory negation of the goal. So therefore [the contradictory negation of the goal] is impossible, so the contradictory negation [of this contradictory negation] is true. If you prefer, you [can] take the contradictory negation of the impossible [conclusion], and add it to the true [premise], and then it yields the goal directly.

[Section: Induction, likening, enthymeme]

Induction is [a kind of argument] that yields a content applying to a universal because of its truth in all or some of the individuals [of the universal], as when it is judged that every animal moves its lower jaw when it chews. But this is not a reliable [conclusion], /10.15/ since sometimes animals are different from what you think, like the crocodile.

Likening⁴⁸ is where a content is applied to something that is absent, by means of something present, using something observed as an example. But sometimes [the present and the absent] differ. The most reliable cases of [likening] are where the thing represented or overlapped is a thing that causes the content to apply to what is observed. But it is not reliable, because sometimes what causes the content to apply to the observed case is because of what it is that is observed. And sometimes the overlap consists

⁴⁶*maṭlūb* 'goal'. The name refers to a fiction, perpetuated by Alexander of Aphrodisias and al-Fārābī among others, that whenever we consider a premise-pair we have in mind a proposed conclusion that we am to prove or refute from the premise-pair. This proposed conclusion is called the goal. The standard criterion for distinguishing between major and minor terms in a syllogism depended on the goal: the major term was the predicate of the goal. (Cf. BELOW.) In *Short Epitome in Logic* Ibn Sīnā speaks of goals only in two cases where it is plausible that we have a proposed conclusion before making the syllogism. The first is here at [112] 10.8, where we prove a conclusion by forming a syllogism with the contradictory negation of the goal as one of its premises. The second case is where we are systematically building up a deductive science; cf. Badawī [112] 11.16.

⁴⁷This is a logical error, copied from al-Fārābī *Syllogism* but corrected in al-Fārābī's later work *Shorter Summary in Logic*. What is needed is that the syllogism yields a thing that is 'clearly false', so that the contradictory negation of the goal is false rather than impossible. See the discussion in Chatti and Hodges [36] pp. 60f.

⁴⁸Likening (*tamthīl*) is Aristotle's *parádeigma* in *Prior Analytics* ii.24, but seen through the lens of al-Fārābī *Syllogism* [59] 36.1–37.6 (cf. [36] 67–69)

of a universal meaning that divides into two parts, in such a way that the cause is one of the two parts and the classification is not included /10.20/ in the subdivision that is referred to the cause. But if [the likening] is not obstructed by a two-way division of this kind, and if it is true that the content holds because of a cause, then the likening can be rearranged into a demonstration.

/11.1/ The enthymeme⁴⁹ is a syllogism in which only the minor premise is stated, as when people say

(3.60) He wanders about at night, so therefore he is confused.

so that the major premise is suppressed, or else [it is suppressed] through redundancy or sophistry.

⁴⁹*ḍamīr*, 'implicit'. The word appears to be one of al-Fārābī's many coinages of logical terminology; cf. [36] p. 35. In his *Syllogism* [59] 14.16, 15.8–12 he uses it for discourse where some things are not said because it can be assumed that everybody is thinking them. Elsewhere (though not in *Syllogism*) he uses it more specifically for syllogisms where one premise is left unspoken because it says something that is generally believed. This kind of syllogism is one of the things that Aristotle means when he speaks of *enthūmēma*, particularly in connection with rhetoric; in *Prior Analytics* ii.27, 70a10–25 he speaks of *enthūmēma* and says that one kind is a syllogism with a premise suppressed. Now *Short Epitome in Logic* [112] 11.1f mentions *ḍamīr* at a point which corresponds to Aristotle's mention of *enthūmēma*, between likening and the end of *Prior Analytics*. But al-Fārābī's *Syllogism* says nothing to connect *ḍamīr* with this part of *Prior Analytics*, so *Short Epitome in Logic* is presumably relying on a translation of *Prior Analytics* at this point. But the Baghdad Standard translation here spells out *anthūmīmā* and uses the root *kbt* rather than *ḍmr* to express suppression of a part of the syllogism. So *Short Epitome in Logic* may have been relying on a further source for this material. We might draw the same conclusion from the example syllogism at *Short Epitome in Logic* [112] 11.1, which is an unusual form of a standard enthymeme; the standard version appears later at *Short Epitome in Logic* [112] 13.8f in the section on rhetorical syllogisms.

Chapter 4

The structure, date and previous sources of *Short Epitome in Logic*

Chapter 3 translated the parts of the text of *Short Epitome in Logic* most relevant to formal logic. In Chapter 4 we regard *Short Epitome in Logic* as Ibn Sīnā's own account of the logic that was already classical before his time. But much of *Short Epitome in Logic* is, as far as we know, Ibn Sīnā's own original creation. These original parts can be grouped into three: the inference engine and related ideas, wholly *muttaṣil* logic, and Ibn Sīnā's system of alethic modal logic. Chapters 5, 6 and 7 will take up these three topics in turn.

4.1 Structure

Short Epitome in Logic follows the structure of Aristotle's *Organon*. In Badawī's edition [112] the work is fifteen pages long, and divides up as shown in the chart below.

Part	Pages of [112]	corresponding to
1.	1–2, 14–15	<i>Eisagōgē</i>
2.	2–3	<i>Categories</i>
3.	3–5	<i>De Interpretatione</i>
4.	5–11	<i>Prior Analytics</i>
	5–6	preliminary definitions
	6–8	categorical syllogisms
	8	wholly <i>muttaṣil</i> syllogisms
	9–10	exceptive syllogisms
	10–11	absurdum, induction etc.
5.	11–12	<i>Posterior Analytics</i>
6.	12	<i>Topics</i>
7.	12–13	<i>Sophistical Refutations</i>
8.	13	<i>Rhetoric</i>
9.	13–14	<i>Poetics</i>

As the chart shows, the author of *Short Epitome in Logic* was particularly interested in the syllogisms that occur in the part corresponding to *Prior Analytics*.

The translation in Chapter 3 above is of the two parts corresponding to *De Interpretatione* and *Prior Analytics*, from pages 3–11 in Badawī [112]. Closer examination of these two parts reveals that three descriptions account for all of the content in these parts:

- (4.1) (i) material taken from an Arabic translation or precis of *Prior Analytics*;
(ii) material taken from al-Fārābī *Syllogism* [59];
(iii) new material characteristic of Ibn Sīnā.

Let me run quickly through these pages 3–11 and indicate how they divide up between (i), (ii) and (iii). I also give references to places where we will discuss the material in more detail.

3.10–4.18 describes the parts of speech and some sentences that can be built up from them. This is standard Aristotelian material found also in al-Fārābī *Syllogism*, except that the definition and classification of hypothetical sentences at *Short Epitome in Logic* 4.4–10 are in al-Fārābī and not in Aristotle. We will discuss the categorical sentence forms in Section 4.2 below and the hypothetical sentence forms in Chapter 6.

5.1–6 introduces conditions of contradiction. This is followed by a section claiming to be about modes ‘necessary’, ‘possible’ and ‘impossible’.

This material on ‘modes’ is not in Aristotle and very probably comes from the Roman period. Contrary to first impressions, it has nothing to do with modality. We will discuss it briefly in Section 7.4 ‘Modes in al-Fārābī and Ammonius’. Nearly everything that *Short Epitome in Logic* says about conditions of contradiction or Ammonius matters (though neither al-Fārābī nor Ibn Sīnā mentions Ammonius in this context) is taken from the fuller discussion in al-Fārābī.¹ We discuss conditions of contradiction and other kinds of logical condition in Section 5.5.

At 5.7–15 *Short Epitome in Logic* introduces conversion of categorical sentences. This is uncontroversial material that is discussed in *Prior Analytics* i.2, and also in al-Fārābī [59] 17.9–18.14, [36] pp. 122f. From 1012 onwards Ibn Sīnā will be working with a new kind of predicative sentence that obeys different conversion rules. This is a major change from *Short Epitome in Logic*, and we will consider it in detail in Part IV below.

5.16–6.7 gives initial definitions on categorical syllogisms. This is the first place where ideas distinctive of Ibn Sīnā appear. They include his semi-mechanical procedure for deriving the conclusion from a pair of premises, which we refer to below as the inference engine. They also include the notions of a recombinant (*iqtirānī*)² syllogism and a premise with dominance (*‘ibra*). We discuss all this in Chapter 5.

6.8–8.2 runs through the standard fourteen valid moods of the categorical syllogism. The basic information is all in both *Prior Analytics* i.4–6 and al-Fārābī [59] 21.12–31.6, [36] 126–133, but there are some variations between Aristotle and al-Fārābī which we discuss in Section 4.5. In most of these variations *Short Epitome in Logic* follows al-Fārābī rather than Aristotle. Ibn Sīnā also gives some views on when modes added to the premises allow us to deduce that the conclusion is necessary; these views reject some strongly held views of Aristotle, and as far as we know they are Ibn Sīnā’s innovations. We will examine these modal views in Section 7.3 below, and they continue to make appearances in later chapters too. Thus 6.8–8.2 combines two lists of syllogistic moods: the fourteen categorical moods and a list of modal moods got by adding modes in appropriate ways to the categorical moods. We will refer to these two lists of moods together as the ‘*Short Epitome in Logic* list of syllogisms’.

At 8.3–22 we meet another innovation of Ibn Sīnā, in this case a new hypothetical logic which we discuss in Section 6.1 under the name of wholly

¹Contradiction [59] 14.11–16.12, [36] pp. 119–121, Ammonius matters [59] 16.12–17.8, [36] pp. 121f.

²At *Middle Summary in Logic* 119.21–120.1 Ibn Sīnā claims it as his own discovery that there are recombinant syllogisms that are not predicative.

muttaṣil logic.

9.1–10.6 introduces the exceptive syllogisms. These are not in Aristotle's published work, and *Short Epitome in Logic* takes them and their name 'exceptive' from al-Fārābī, see [36] pp. 56f. But there is a novelty: for *Short Epitome in Logic* the non-paradigm *munfaṣil* sentences ' p or q ' express that at least one of p and q is true; this reading seems to be new with Ibn Sīnā, and he continues to use it throughout his career. (For al-Fārābī the incomplete *munfaṣil* sentence expresses that at most one of p and q is true.) We discuss the form of exceptive syllogisms found in *Short Epitome in Logic* in Section 6.2 below.

10.7–12 describes *reductio ad absurdum* in terms that closely match al-Fārābī *Syllogism* but not al-Fārābī's later version *Short Syllogism* (cf. Note 47 to Chapter 3 above). 10.13–21 describes induction and likening³ (the latter corresponding to Aristotle's *parádeigma*). The description of likening is particularly close to that in al-Fārābī *Syllogism*. 11.1f describes enthymeme⁴ under the name *ḍamīr*, in this sense probably a coinage of al-Fārābī.

This analysis shows that some of the material in *Short Epitome in Logic* comes from al-Fārābī and some is original with Ibn Sīnā. We can safely infer that the traditional attribution of *Short Epitome in Logic* to Ibn Sīnā is correct.

There is less to show that Ibn Sīnā had had access to an Arabic version of

³Likening (*tamthīl*) is a form of argument where we observe that D is an A and we infer that some other thing C which is not visible to us is also an A , because C is similar to D in being a B . Al-Fārābī explains that to deploy this argument we must find B so that C is a B , and so that D being a B is the cause of D being an A . This appears to be al-Fārābī's own argument in 36.1–37.6 of his *Syllogism*; you have to be rather generous to Aristotle to find it in Aristotle's discussion of *parádeigma*. Ibn Sīnā's account of likening in *Short Epitome in Logic* is packed down tight, but it does identify the crucial role of B as cause. He must have taken it from al-Fārābī *Syllogism*, since likening is not considered in al-Fārābī's *Short Syllogism*, and the account of likening in al-Fārābī's *Long Commentary on Prior Analytics* is quite different.

⁴The mention of enthymeme is significant, because it answers to *Prior Analytics* ii.27, 70a10, but there is nothing in al-Fārābī *Syllogism* to indicate this. So presumably Ibn Sīnā read it himself in *Prior Analytics*; this is our clearest evidence that he had access to *Prior Analytics* when he wrote *Short Epitome in Logic*. However, *Short Epitome in Logic* [112] 11.1 refers to enthymeme as *ḍamīr*, as Ibn Sīnā will do in his later writings, while the Baghdad Standard translation of *Prior Analytics* writes *anthūmīmā* at this point. Possibly Ibn Sīnā is reading another translation of *Prior Analytics*, now lost. More likely Ibn Sīnā wanted an Arabic word for this concept, and he knew (for example from al-Fārābī's *Syllogism* 14.16 and 15.8, or from al-Fārābī's *Rhetoric* [62] 63.5–7) that al-Fārābī used the word *ḍamīr* in this context. See [36] pp. 34f for indications that al-Fārābī introduced the word *ḍamīr* in this meaning to logic.

Prior Analytics when he wrote *Short Epitome in Logic*. Probably the strongest evidence that he did is the fact that *Short Epitome in Logic* consistently gives the categorical syllogisms with formal sentences, i.e. using alphabetical letters rather than material (i.e. natural language) nouns for terms. This is a format used by Aristotle throughout *Prior Analytics* i.4–6, but it is rare in al-Fārābī; see Section 4.5 below. Also the mention of enthymemes at 11.1f indicates a knowledge of *Prior Analytics*, which has them at the corresponding place; al-Fārābī *Syllogism* doesn't mention them.

I am not aware of anything in *Short Epitome in Logic* that is not included under (i), (ii) or (iii) of (4.1) above. So Ockham's razor would suggest that al-Fārābī *Syllogism* and Arabic translations of Aristotle were the only logical sources that Ibn Sīnā used when he wrote *Short Epitome in Logic*.⁵

4.2 Date

There are no early records describing when *Short Epitome in Logic* was written.⁶ So any dating of it must use its contents. I give three arguments. In my view they are progressively stronger in terms of their cogency, but progressively weaker in the limits that they put on the dating of *Short Epitome in Logic*.⁷

The first argument rests on our estimate of Ibn Sīnā's birth date, together with a comparison of (4.1) with what Ibn Sīnā himself tells us in his *Autobiography* about his sources of logical information during his youth. In Section 1.1 we estimated that Ibn Sīnā was born no later than 976, so that he reached the age of eighteen no later than 994. Set this alongside the following passage from *Autobiography*, from the point where the young Ibn Sīnā

⁵What logical or philosophical texts are likely to have been available in the Bukhārā bookshops around the year 990? In his *Autobiography* Ibn Sīnā tells us: 'One afternoon I was at the booksellers' quarter when a crier came up holding a volume which he was hawking for sale. ... I bought it and it turned out to be Abū-Naṣr al-Fārābī's book *On the Purposes of the Metaphysics*.' ([117] pp. 32–35, trans. Gutas [74] p. 17). The Sāmānid rulers of Bukhārā prided themselves on making Bukhārā a centre of Persian-Islamic culture, and probably any book available in Baghdad could be expected to turn up sometimes in Bukhārā.

⁶There is some early information about when *Short Epitome in Logic* was bound up together with the other two parts of *ʿUyūn al-ḥikma*, but I don't see that this helps us. See Gutas [74] pp. 417–419.

⁷The three arguments below are an edited version of those in Section 3 of Hodges [95].

was enrolled in the service of the Sultan Nūḥ ibn Maṣṣūr:

(4.2) One day I asked his permission to enter their library, look through it, and read its contents. He gave me permission and I was admitted to a building with many rooms; in each room there were chests of books piled one on top of the other. In one of the rooms were books on the Arabic language and poetry, in another jurisprudence and so on in each room a separate science, I looked through the catalogue of books by the ancients and requested those which I needed. I saw books whose very names are unknown to many and which I had never seen before nor have I seen since, I read those books, mastered their teachings, and realized how far each man had advanced in his science. So by the time I reached my eighteenth year I had completed my study in all these Philosophical Sciences, ([117] pp. 34–37, trans. Gutas [74] p. 18)

Before this library visit, Ibn Sīnā had 'mastered' (*aḥkamtu*) the sciences of logic, physics and mathematics (*Autobiography* [117] pp. 30f) in a period of intense self-education.

Evidently Ibn Sīnā considered that at age eighteen⁸ he passed a significant threshold in his relation to the science of logic. By that date he had 'mastered' the science (but he gives no criteria for this assessment). Soon after it he was working within the tradition of logical scholars, including people he had previously been unaware of. In today's language we might say he had passed from being a pupil of logic to being a researcher in logic.

When in relation to this threshold did Ibn Sīnā write *Short Epitome in Logic*? One thing that emerged from our analysis of its contents is that the work shows no awareness of any figures in the logical tradition apart from Aristotle and al-Fārābī. There is no sign that when he wrote the book, Ibn Sīnā had even heard of Theophrastus, Galen, Alexander of Aphrodisias or Themistius for example. So the book reports only pre-threshold logic.

On the other hand *Short Epitome in Logic* seems to take full advantage of Ibn Sīnā's period of self-education. Some features of wholly *muttaṣil* logic, as Ibn Sīnā reports it in *Short Epitome in Logic*, are as advanced as anything known to be available in hypothetical logic before Ibn Sīnā himself. This includes the notion of a recombinant syllogism. See Chapter 5 below for more

⁸*balaghta thamānī ʿasharata sinatan min ʿamrī*; Gohlman translates 'I had reached the age of eighteen' [117] p. 37. On my understanding Gutas's translation of this phrase means a year younger than Gohlman's, but I won't adjudicate between them. In any case 'eighteen' here might be more symbolic than factual.

on this point, and for likely connections between recombinant syllogism and the study that Ibn Sīnā reports that he made of the *tartīb* ('ordering') of syllogisms, before the age of eighteen according to his *Autobiography*. It's significant that some of the most advanced features of the logic of *Short Epitome in Logic* correlate well with the kind of logical experiment that Ibn Sīnā reports he was carrying out before the age of eighteen; again Chapter 5 will consider some evidence of this.

Also, noting the significance that Ibn Sīnā gives to this threshold, it would seem entirely natural for Ibn Sīnā to mark the threshold by writing a book summing up the main points that he reckoned he had learned in logic while reaching the threshold. *Short Epitome in Logic* is an excellent fit for such a book.⁹

The second argument rests on the dating of the Hypothetical Breakthrough, in which Ibn Sīnā collected up a number of logics whose syllogisms have conclusions that are hypothetical sentences. In Section 9.1 below we assemble evidence that the Hypothetical Breakthrough was complete by around 1004 at latest. In Chapter 9 we show that all the logics of the Hypothetical Breakthrough are in one way or another developments from the wholly *muttaṣil* logic introduced in *Short Epitome in Logic*. But the reader can check from the text of *Short Epitome in Logic* in Chapter 3 above that the author of *Short Epitome in Logic* shows no awareness at all that wholly *muttaṣil* logic can be generalised into a range of new logics, or that it would be worthwhile to make this generalisation. This shows that when *Short Epitome in Logic* was written, the Hypothetical Breakthrough had not yet begun. Assuming that the Hypothetical Breakthrough took Ibn Sīnā at least four years to complete, this places the date of *Short Epitome in Logic* to no later than 1000. This is some eight years later than the date suggested by the first argument, but the dates involved are a little more secure.

Our third argument can be read as a commentary on Gutas's remark ([74] p. 419) that *Short Epitome in Logic* (i.e. the Logic part of the collection '*Uyūn al-hikma*') 'appears relatively early'. Gutas doesn't document this remark. There are different kinds of feature that could point to a work of Ibn Sīnā being 'relatively early'. For example the work might contain theories or concepts that we know Ibn Sīnā didn't subscribe to in his later years. Or it might have signs of immaturity of thinking. We can find examples of both these kinds in the part of *Short Epitome in Logic* translated in Chapter

⁹Silvia Di Vincenzo [42] pp. 73–75 points to some evidence implying that Ibn Sīnā was already proficient in writing logical books while he was living in Bukhārā.

3 above.

We begin with two examples of the former kind: logical claims that we can show were part of Ibn Sīnā's thinking from some date onwards, though they don't fit with his beliefs when he wrote *Short Epitome in Logic*. There is a danger of circularity here: how can we know that Ibn Sīnā believed something from date D onwards, but not when he wrote *Short Epitome in Logic*, without already knowing that he wrote *Short Epitome in Logic* before date D? To escape from this circularity we will rely on the fact that it may be possible to pin down Ibn Sīnā's relevant beliefs at enough dates after D, and in enough detail, to make it highly unlikely that he quietly backtracked during this period. It's a great help to us that the dates of Ibn Sīnā's logical writings from 1013 onwards are mostly secure (though we will need to correct the date generally given for *Twenty Questions*).

Here follow two logical theses that Ibn Sīnā had not adopted when he wrote *Short Epitome in Logic*, but had come to adopt with full conviction by 1013 at the latest. The first is in hypothetical logic and the second is in predicate logic.

The first of these theses will be discussed in more detail in Section 9.3 below, so here I confine myself to a sketch. In *Short Epitome in Logic* Ibn Sīnā advises the reader to avoid using *muhmal* sentences since they are liable to lead to syllogisms that are only superficially valid. Also in *Short Epitome in Logic* Ibn Sīnā shows how categorical sentences and syllogisms can be adjusted to form hypothetical sentences and syllogisms, by putting sentences for terms; these new hypothetical syllogisms are valid for reasons that are close analogues of those for categorical logic. In *Short Epitome in Logic* Ibn Sīnā makes these adjustments only for quantified categorical syllogisms, never for syllogisms containing *muhmal* sentences. This is hardly surprising given his advice in *Short Epitome in Logic* to avoid *muhmal* sentences. But by the time when he writes *Middle Summary in Logic*, we find that Ibn Sīnā has completely changed his mind and now also recommends adjusting *muhmal* categorical sentences to form hypothetical sentences. This change of attitude is clear in all of Ibn Sīnā's logical works from *Middle Summary in Logic* onwards. It is also clear in al-Barakāt's *Al-kitāb al-mu'tabar*, which dates it to maybe 1004 at the latest. (This comment relies on our dating of al-Barakāt's sources; see Section 8.3.)

The second thesis is in predicative logic: it consists of a notion that is not present in *Short Epitome in Logic*, but we will see that Ibn Sīnā adopts it in his Jurjān period and never abandons it. In *Short Epitome in Logic* Ibn Sīnā twice describes premises of some syllogisms as 'absolute' (*mutlaq*). In

Badawī's edition the occurrences are at [112] 8.2:

- (4.3) The dominance in mode [in the third figure] belongs to the major premise, which is conveyed to the major premise in the first figure by conversion or ecthesis. That is, unless the minor premise is possible and the major premise is absolute.

and [112] 6.12:

- (4.4) [In a first figure syllogism in first mood the conclusion] is as above except when the minor premise is possible and the major premise ['Every B is an A , however it is'] is absolute.

In Section 4.5 below we consider what Ibn Sīnā means by 'absolute' in *Short Epitome in Logic*, and we give evidence that he intends the word in the way that was established by Theodorus in his Arabic translation of *Prior Analytics*. Namely, an 'absolute' sentence is a quantified categorical sentence, and 'absolute' means 'not qualified by a modal word such as necessary or possible or contingent'. But when we consider what Ibn Sīnā means by 'absolute' in *Twenty Questions* and later works, it's clear that he means something different. He introduces two new kinds of absolute sentence, which he calls respectively 'Themistian absolute' and 'Alexandrine absolute'. For example when 'No B is an A ' is read as a Themistian absolute sentence, it means 'Everything that is at some time a B is at some time not an A '. As evidence that this means something different from the Theodoran absolute 'No B is an A ', Ibn Sīnā points out that the Theodoran absolute satisfies *e*-conversion:

- (4.5) 'No B is an A ' entails 'No A is a B '.

whereas negative universal Themistian absolute doesn't satisfy *e*-conversion, because

- (4.6) 'Every sometimes- B is sometimes not an A ' doesn't entail 'Every sometimes- A is sometimes not a B '.

(To see the falsehood of (4.6), consider 'Every sometimes-human is sometimes not laughing', given Ibn Sīnā's belief that everything that sometimes laughs is throughout its existence a human. Section 4.5 below has more to say on this.)

We will see in Part IV below how Ibn Sīnā continues to develop and refine the notion of 'Themistian absolute'. By the time he comes to write *Syllogism*, the name has changed to 'broad absolute', still not obeying *e*-conversion. The change from (Theodoran) absolute to broad absolute is a

central theme in the development of Ibn Sīnā's mature modal logic, and it's inconceivable that he would have casually flipped back to (Theodoran) absolute in the middle of this development.¹⁰

Finally we turn to examples of immaturity in *Short Epitome in Logic*. Some of these are immaturities in al-Fārābī *Syllogism*, which the author of *Short Epitome in Logic* has copied almost verbatim. These include logical errors in al-Fārābī's descriptions of the ecthetic proof of *Baroco*¹¹ and of proof by absurdity.¹²

Also *Short Epitome in Logic* [112] 5.5f claims to introduce modes, but in fact the supposed 'modes' are really Ammonius matters, which have no direct connection with modes. (See Section 7.4 below.) I wish we knew what Ibn Sīnā was thinking when he made this confusion. Did he realise that at this stage he didn't understand what a mode is? We come back to this question in Section 7.4 below.

4.3 Categorical and predicative sentences

In this and the following two sections¹³ we summarise some standard notions of Aristotelian logic that Ibn Sīnā makes use of in *Short Epitome in Logic*.

4.3.1 Predicative sentences

Short Epitome in Logic describes two kinds of sentence, namely 'predicative' (*ḥamlī*) sentences and 'hypothetical' (*shartī*) sentences. Not every sentence (*qawl*) is predicative or hypothetical. Predicative and hypothetical sentences are 'propositions', i.e. they can be true or false;¹⁴ presumably this excludes commands, requests and questions from being predicative or hypothetical. I presume too that Ibn Sīnā intends that no sentence is both predicative and hypothetical; but this depends on how one defines 'hypothetical', and we will leave that topic to Section 8.3 below. CONFIRM THIS

¹⁰This also rules out the speculation of Gutas [74] p. 419, that *Short Epitome in Logic* was written in 1013–1014, a time at which the introduction of Themistian absolute was well under way.

¹¹See Footnote 27 to Chapter 3 above, with Chatti and Hodges [36] p. 52.

¹²See Footnote 47 to Chapter 3 above.

¹³See Chatti [34] Chapter 3 for a review of categorical logic in al-Fārābī and Ibn Sīnā.

¹⁴A sentence that can be true or false is called 'declarative' (*jāzīm*) and a 'proposition' (*qaḍīya*), according to *Short Epitome in Logic* [112] 3.19.

REF.

Ibn Sīnā defines ‘predicative’ in a way that explains the name *ḥamlī*:

- (4.7) The predicative proposition is one in which it is judged whether or not a thing—the predicate (*maḥmūl*)—is true of a thing—the subject (*mawḍūʿ*)—as when we say ‘Zayd is a writer’ or ‘Zayd is not a writer’.
(*Short Epitome in Logic* [112] 4.1f)

The expressions naming the subject and the predicate of a predicative sentence—‘Zayd’ and ‘writer’ in the example above—are called its *terms* (*ḥadd*, plural *ḥudūd*).

The Arabic verb *ḥamala*, literally ‘to carry’, was used by Theodorus to translate Aristotle’s Greek *katēgoreîn* ‘to affirm, of a thing, that it has a property’. Thus ‘Zayd is a writer’ affirms that Zayd has the property of being a writer. The choice of *hamala* to translate this Greek word was not very accurate, because *hamala* is used both for affirming and for denying. The passive participle *maḥmūl* means ‘what is affirmed or denied’, i.e. the property. The adjective *ḥamlī* is used for sentences that affirm or deny, of a thing, that it has a property. The Greek *katēgoreîn* translates into Latin as *praedicare*, hence ‘predicate’. But *katēgoreîn* also gives us the adjective ‘categorical’, which we will come back to in Subsection 4.3.4.

4.3.2 Quantified and *muhmal*

Besides the general definition (4.7) above, *Short Epitome in Logic* also gives a more specific list of types of predicative sentence, telling us there are eight types as follows. There are four quantified (*maḥṣūr*) types:

- (4.8) 1 Every human is an animal. (universal affirmative)
2 No person is a stone. (universal negative)
3 Some person is a writer. (existential affirmative)
4 Not every human is a writer. (existential negative)
or: Some person is not a writer. (" ")
(*Short Epitome in Logic* [112] 4.15–18.)

Here ‘universal’ is *kullī*, ‘existential’¹⁵ is *juzʿī*, ‘affirmative’ is *mūjib* and ‘negative’ is *sālib*. The properties ‘universal’ and ‘existential’ are said to give the ‘quantity’ (*kam*) of a sentence, while the properties ‘affirmative’ and ‘negative’ give its ‘quality’ (*kayfa*). The remaining four types of predicative

¹⁵More usually called ‘particular’ in writing about Aristotelian logic.

sentence have no quantifiers. They are:

- | | | | |
|-------|---|------------------------------------|---------------------------------|
| | 5 | The human has suffered a loss. | (<i>muhmal affirmative</i>) |
| | 6 | The human has not suffered a loss. | (<i>muhmal negative</i>) |
| (4.9) | 7 | Zayd is a writer. | (<i>singular affirmative</i>) |
| | 8 | Zayd is not a writer. | (<i>singular negative</i>) |
- (*Short Epitome in Logic* [112] 4.11–14.)

'Singular' is *shakhṣī*. Aristotle used 'indeterminate' (Greek *adióristos*)¹⁶ for sentences that have no quantifier. But the rules of Arabic grammar require every sentence to carry some indication of definiteness or indefiniteness, and this has the effect that Arabic has no exact equivalent to a Greek indeterminate sentence. At some early date Arabic logicians agreed to translate 'indeterminate' as *muhmal*, a word that refers to sheep or camels that are allowed to wander freely. As we will see in Section 7.1, al-Fārābī suggested that singular sentences beginning 'the' could count as *muhmal* in Arabic. The listing (4.9) shows that *Short Epitome in Logic* accepted this recommendation.

Al-Fārābī thought he had found a use in formal logic for the resulting *muhmal* sentences with 'the', as a support for his theory of 'tolerance' (*sumāḥa*); Section 7.1 below will give details of this. But in *Short Epitome in Logic* Ibn Sīnā regarded this use of sentences with 'the' as a recipe for 'syllogisms of resemblance' (*al-qiyāsāt al-mashbiha*) which look plausible but fail to obey logical laws.¹⁷

- (4.10) The way to avoid [syllogisms of resemblance] is that . . . and that one beware of *muhmal* sentences and makes no use at all of them.
(*Short Epitome in Logic* [112] 12.19–13.3 in section on sophistical arguments)

We will see in Section 9.3 below that Ibn Sīnā changed his mind during the Gap period and found a different and entirely defensible way of using *muhmal* sentences in hypothetical logic.

4.3.3 Material and formal, matter and form

We will refer to the examples of predicative sentences in (4.8) and (4.9) as 'material' sentences, because they are natural language sentences with no

¹⁶Also translated as 'indefinite'.

¹⁷Maybe thirty years later Ibn Sīnā in [124] iv.4, 210.13 claims to have shown tolerance towards a false move by an unnamed logician (who is almost certainly al-Fārābī, cf. Street [205]); this looks like mild mockery. Cf. also [124] ii.2, 91.1

logical symbolism in them. Their subject and predicate are ‘material terms’. But Ibn Sīnā, following Aristotle in *Prior Analytics*, often points to the same sentence forms by using alphabetical letters, one in place of the subject and a different letter in place of the predicate. In this style the material sentences of (4.8) and (4.9) might be written as follows:

- | | | | | |
|--------|---|------|---------------------------------------|---------------------------|
| | 1 | (a) | Every <i>B</i> is an <i>A</i> . | (universal affirmative) |
| | 2 | (e) | No <i>C</i> is an <i>A</i> . | (universal negative) |
| | 3 | (i) | Some <i>C</i> is a <i>B</i> . | (existential affirmative) |
| | 4 | (o) | Not every <i>B</i> is a <i>C</i> . | (existential negative) |
| (4.11) | | (o) | or: Some <i>B</i> is not a <i>C</i> . | (" "). |
| | 5 | (ma) | The <i>C</i> is an <i>E</i> . | (muhmal affirmative) |
| | 6 | (mn) | The <i>F</i> is not a <i>G</i> . | (muhmal negative) |
| | 7 | (sa) | <i>D</i> is a <i>B</i> . | (singular affirmative) |
| | 8 | (sn) | <i>D</i> is not an <i>A</i> . | (singular negative) |

The alphabetical letters that are used to replace the material terms are called ‘term letters’, and the sentences with term letters instead of material terms are said to be ‘formal sentences’.¹⁸

Two formal sentences are said to be ‘of the same form’ if they are the same except for the choice of letters.¹⁹

Alexander of Aphrodisias found it useful to think of a material sentence as being made up of a form (i.e. a corresponding formal sentence) and a ‘matter’ (*mādda*) which tells us what material terms to put in place of the term letters in the form. For example the sentence ‘The human is an animal’ has form ‘The *S* is a *P*’, and a matter that assigns ‘human’ to *S* and ‘animal’ to *P*. Alexander’s definitions are rather loose; today we would probably express the matter as an ordered pair (human, animal) where the first term goes to the subject letter and the second term goes to the predicate letter. I will think of Alexander’s ‘matters’ in this way.²⁰

¹⁸One weakness of Ibn Sīnā’s logical terminology throughout his career is that he has no words for the concepts ‘material’ and ‘formal’, though his use of term letters makes clear that he has these two concepts.

¹⁹A ‘sentence form’ is an equivalence class of formal sentences under the relation ‘having the same form’. The ‘form’ of a material sentence is the sentence form of any sentence got by replacing the material terms by distinct letters. Aristotle normally names a sentence form by writing a formal sentence that has that form, such as ‘Not every *B* is an *A*’; or else by describing its quantity and quality, in this case existential negative. The symbols between parentheses in the second column of (4.11) are names for the sentence forms shown.

²⁰Alexander’s notion of matter fed into the theory of modes, and this will become important for us in Chapter 7 below.

4.3.4 Categorical and predicative

We return to the words 'categorical' and 'predicative'. They both translate Greek *katēgorikós*, the first directly and the second via Latin. We have seen in Subsections 4.3.1 and 4.3.2 that in *Short Epitome in Logic* Ibn Sīnā uses 'predicative' for a broad class of sentences as in (4.7), and also for a narrower class of sentences with restricted forms as in (4.8) and (4.9). There are a number of situations where it would be helpful to have a vocabulary for distinguishing the broad class from the narrow. For example the syllogisms that Aristotle discusses in *Prior Analytics* i.4–6 take all their premises and conclusions from the narrower class of predicative sentences. Today these syllogisms are often described as 'categorical syllogisms', and accordingly we will define 'categorical sentences' to be sentences of the forms described in (4.8) or (4.9), or their formal counterparts as in (4.11).²¹ This has the consequence that a syllogism is categorical if and only if its premises and conclusion are all of them categorical sentences. A 'categorical form' is the form of a categorical sentence, as in (4.11).

The symbols in the second column of (4.11) are a convenient shorthand for the categorical forms. The first four (*a*, *e*, *i*, *o*) were first introduced in the Latin West at least a century after Ibn Sīnā's death. They can be used in several ways. For example 'Every *B* is an *A*' can be written as

$$(4.12) \quad (a)(B, A)$$

and 'Every horse is an animal' can be written as

$$(4.13) \quad (a)(\text{horse}, \text{animal}).$$

When we come to Ibn Sīnā's Themistian or broad absolutes, we will change *a*, *e* etc. to (*a-t*), (*e-t*) etc..

There are some Arabic words that might be offered as descriptions which pick out the categorical sentences. One is the adjective *muṭlaq*,²² which

²¹Some modern writers refer to a kind of sentence called 'assertoric', though usually without defining this word. My impression is that at least when discussing Latin Scholastic logic they usually mean something close to 'categorical'. But in Arabic logic from Ibn Sīnā onwards and outside Spain, much of the interest lies in predicative sentences that have temporal non-permanent modes, and in this context there is nothing achieved by saying 'assertoric' without saying what one means.

²²The word *muṭlaq* is a passive participle of the common Arabic verb *ṭaluqa* which has a range of meanings around the central idea of removing restrictions or conditions. Hence the English translation of *muṭlaq* as 'absolute', which comes from a passive participle of the Latin verb *absolvo* and has a similar range of meanings.

means ‘absolute’ in the sense of being free from restrictions. The word doesn’t appear in al-Fārābī’s *Syllogism*, but Theodorus in his translation of *Prior Analytics* uses it frequently to express that a sentence (usually a premise or conclusion of a syllogism) carries no mode of necessity, possibility or contingency. So we can read ‘absolute’ as meaning ‘categorical’ in Theodorus’s text. In Section 4.2 above we noted that the word appears twice in *Short Epitome in Logic*, both times in contexts where the natural reading is the same as that of Theodorus.

However, we will see in Section 8.3 below that at least from the time of *Twenty Questions* in around 1012, Ibn Sīnā begins to introduce new kinds of predicative sentence which he calls ‘absolute’. Among others they include the sentences that he calls ‘Themistian absolute’ (or ‘Theophrastan absolute’), and those that he calls ‘Alexandrine absolute’. By the time he comes to write *Syllogism*, Ibn Sīnā will no longer refer to ‘absolute’ sentences in Theodorus’s sense. Rather he describes the Themistian absolute as ‘broad absolute’ (*muṭlaq āmm*); he describes the Alexandrine absolute as ‘narrow absolute’ (*muṭlaq khāṣṣ*) or ‘paradigm absolute’ (*muṭlaq ḥaqīqī*). Chapter 11 below will examine the new ‘absolute’ sentences that Ibn Sīnā introduces, most of them by the end of the Jurjān period.

There are a very few occasions in *Syllogism* when Ibn Sīnā wants to pick out the categorical sentences among the predicative sentences. For example at *Syllogism* [124] ii.4, 113.9 and 114.1 he mentions that some second-figure syllogisms are not productive if their sentences are understood to be broad or narrow absolute, but they will be productive under certain conditions, one of them being that the sentences are categorical. For ‘categorical’ he uses *mashhūr*²³ which normally means ‘generally accepted as being true’ and in this sense is an important notion in Aristotelian theory of debate. But as Ibn Sīnā is using it, the truth or falsehood of the sentences is irrelevant, and perhaps *masshūr* should be read here as ‘generally accepted among logicians as a suitable sentence for use in syllogisms’, which we can abbreviate to ‘standard’. We find *mashhūr* already used at *Middle Summary in Logic* [114] 123.8 to describe the ‘standard’ account of the second figure in categorical syllogisms. Ibn Sīnā never uses *masshūr* to describe anything that he himself invented, such as broad absolute or narrow absolute sentences.²⁴

²³This is *mash-hūr*, not to be confused with *mas-hūr*, which means bewitched (as with the evil eye). I thank a mathematics student of mine called Mashur who was eager I should not confuse these two words.

²⁴Ibn Sīnā has a different word for sentences got from another sentence by removing a restriction. He calls them *mujarrad*, literally ‘stripped’ but normally translated as ‘ab-

4.4 Categorical and predicative syllogisms

The Peripatetic tradition tends to use the word 'syllogism'²⁵ rather loosely—it varies according to the context and to the stage of development of the theory. For purposes of the present section it will be enough to say that a syllogism is an inference in which a conclusion is derived from two premises.

We call a syllogism a 'categorical syllogism' if its premises and conclusion are all of them categorical sentences. Likewise a 'predicative syllogism' is a syllogism in which the premises and the conclusion are predicative sentences. If the premises and conclusion of a predicative syllogism are material sentences, we say that the syllogism is a 'material syllogism'. In practice the sentences in a categorical or predicative syllogism are assumed to be quantified; singular and *muhmal* sentences do occasionally appear but on an ad hoc basis.²⁶ Hence we can generally assume that the premises and conclusion of a categorical syllogism belong to any of the four classes described in Section 4.3: universal affirmative, universal negative, existential affirmative, existential negative.

Just as we describe the form of a predicative sentence by putting distinct term letters for the distinct material terms, we do the same for predicative syllogisms. The resulting syllogism with term letters is a 'formal syllogism'. For example in *Short Epitome in Logic* Ibn Sīnā gives the following material syllogism:

- (4.14) Every body is composite;
and everything composite is created.
It follows that every body is created.
(*Short Epitome in Logic* [112] 5.18f)

The form of this syllogism can be described by putting distinct term letters

strat(ed)'. The restriction could be a mode but it could also be for example the restriction of a formal sentence to one of its material instances. Ibn Sīnā uses this word sometimes in logic, but more often in psychology, where his use of abstraction influenced the Scholastic philosophy of Thomas Aquinas [37].

²⁵Arabic *qiyās*, literally 'analogy'. This translation goes back to a period of Arabic logic from which almost no texts survive, and it is not known why a word meaning 'analogy' was thought to be a suitable translation for 'syllogism'.

²⁶'*Muhmal* propositions behave like existential propositions . . . , singular propositions behave like universal propositions.' (Ibn Sīnā *Syllogism* ii.4, 109.11–13).

for the material terms:

- (4.15) Every C is a B ; $(a)(C, B)$;
 and every B is an A . $(a)(B, A)$.
 It follows that every C is an A . $or : So (a)(C, A)$.
 (*Syllogism* [112] 5.18f)

We can use this example to introduce some terminology that comes from Aristotle. The term B is in both premises and not in the conclusion, and so it is called the ‘middle term’ (*awsat*). The term A is predicate of the conclusion; we call it the ‘major term’ (*akbar*), and the premise containing it is the ‘major premise’ (*kubrā*). The term C is subject of the conclusion; we call it the ‘minor term’ (*aṣghar*), and the premise containing it is the ‘minor premise’ (*ṣuḡhrā*). The major and minor terms are called the ‘extremes’ (*ṭaraf*, plural *aṭrāf*) of the premise-pair.

Ibn Sīnā normally uses the letters A, B, C (or rather their Arabic counterparts a, b, j) for the major term, the middle term and the minor term respectively.

Two formal syllogisms are said to have ‘the same form’ if each of them can be transformed into the other by making a one-to-one replacement of the term letters (so that a term letter occurring twice gets replaced by the same term letter at each occurrence).²⁷ A form of a syllogism is called a ‘mood’.²⁸

Aristotle in *Prior Analytics* i.4–6 recognised fourteen categorical syllogistic moods, and classified them into three ‘figures’. Or so it seems at first, though the truth is more complicated. In fact he recognised forty-eight forms of premise-pair, where a ‘premise-pair’ (Arabic *qarīna* or *iqtirān*)²⁹ is an ordered pair of quantified formal categorical sentences with one term in common; the two sentences are distinguished as ‘major’ and ‘minor’. For each of these premise-pairs, Aristotle knew what quantified formal categorical sentences he would consider as possible candidates to be conclusions entailed by the two premises. To be precise, we say that a quantified formal categorical sentence χ is a ‘candidate conclusion’ for a premise-pair (ϕ, ψ)

²⁷As in Footnote 20 above, ‘having the same form’ is an equivalence relation, and its equivalence classes are the forms of formal syllogisms. The ‘form’ of a material syllogism is the syllogism form of any formal syllogism got by replacing the material terms by term letters (again replacing the same material term by the same letter at each occurrence).

²⁸*darb*, following al-Fārābī’s *Syllogism*.

²⁹*qarīna* is at *Middle Summary in Logic* [114] 120.10; *iqtirān* is at *Guidance* [121] 92.9).

(where ϕ is the minor premise and ψ is the major premise) if

- (4.16) the subject term of χ is the term letter that occurs only in the minor premise ϕ , and the predicate term of χ is the term letter that occurs only in the major premise ψ .

Given a premise-pair (ϕ, ψ) , Aristotle would consider its four candidate conclusions, and determine whether any of them are entailed by ϕ and ψ together. If none are, then the premise-pair is not 'syllogistic'.³⁰ Aristotle established that thirty-four of the forms of premise-pair are in this sense not syllogistic. In the remaining fourteen cases the premise-pair is syllogistic and entails at least one of its candidate conclusions. Aristotle followed (but never explained) the following rule for choosing one of these entailed candidates to be the 'conclusion' of the premise-pair.

Rule for Identifying Conclusions in categorical syllogisms. If the premise-pair entails exactly one of the candidates, this one is counted as the 'conclusion' (Greek *sumpérasma*, Arabic *natīja*) of the premise-pair. If it entails more than one, then there is always a unique entailed candidate which is strongest, where 'Every B is an A ' counts as stronger than 'Some B is an A ', and 'No B is an A ' counts as stronger than 'Not every B is an A '. This strongest candidate is counted as the conclusion of the premise-pair.³¹

Adding to the complications, Aristotle at least once described a sentence χ as the 'conclusion' (*sumpérasma*) of a premise-pair when the premise-pair doesn't even entail χ . Ibn al-Muqaffa^c, the author or translator of the earliest known Arabic logical text, distinguished 'sound conclusions' which are conclusions as in the Rule for Identifying Conclusions above, from 'broken conclusions' which are conclusions that are not entailed by given premises. Sound conclusions and broken conclusions in Ibn al-Muqaffa's sense can respectively be called 'logical conclusions' and 'pseudo-conclusions'. In the ninth or tenth century al-Fārābī and al-Dimashqī, probably acting together, reverted to Aristotle's usage and used *natīja* both for

³⁰More precisely Aristotle says that in the case of this premise-pair 'there is not a syllogism'.

³¹Neither Aristotle nor Ibn Sīnā lays out these rules for identifying the conclusion of a premise-pair. I am simply stating the rules that they follow in practice. At *Twenty Questions* [115] xix, 103.7f Ibn Sīnā says 'It was a fault in Galen that he drew many conclusions from the same syllogism, because he reckoned that what follows from the conclusion of the syllogism, and the syllogism, and the conclusion of the syllogism, are all the same thing.'

logical conclusions and for pseudoconclusions. Ibn Sīnā followed them in this, misguidedly in my view.³²

There is a lot here that needs to be clarified and justified. One key issue that was picked up by Aristotle’s successors was the question of deciding which of the premises in a premise-pair is the major and which is the minor. At least from the time of Alexander of Aphrodisias to that of Ibn Sīnā, a fiction was adopted in order to answer this question. The fiction was that whenever a premise-pair is considered, we are also given a categorical sentence containing the two extremes as its terms, together with the task of determining whether the given sentence can be proved or refuted from the premises. Alexander called the given sentence a *problēma*; the Arabic logicians described it as the ‘question’ (*mas’ala*) or the ‘goal’³³ (*maṭlūb*).

The minor term and the major term were defined to be respectively the subject term and predicate term of the goal. The minor premise and the major premise of the premise-pair were defined to be respectively the premise containing the minor term and the premise containing the major term. In Section 5.2 we will see how Ibn Sīnā quietly removed the assumption that there is always a goal.

Tracing back through the definitions, and following Ibn Sīnā’s convention of using A, B, C respectively for the major term, the middle term and the minor term, we find that the conclusion (if there is one) will have subject C and predicate A —say for short that it has the form CA —and that the first premise has the form either CB or BC , and the second premise has the form either AB or BA . This allows four possibilities as in the following diagram. Each of Aristotle’s valid categorical syllogisms has the format shown in one of the first three ‘figures’ (*shakl*, plural *ashkāl*) of the diagram.

		1st figure	2nd figure	3rd figure	4th figure
(4.17)	1st premise	$C B$	$C B$	$B C$	$B C$
	2nd premise	$B A$	$A B$	$B A$	$A B$
	conclusion	$C A$	$C A$	$C A$	$C A$

³²I hope to provide some more of the history in Hodges [99] and [101]. Pseudoconclusions are Aristotle’s main method for proving non-entailment; he refers to this method as ‘terms’ (Greek *hóroi*). Meanwhile an example of pseudoconclusions being called *natiḥja* by Ibn Sīnā is at *Syllogism* [124] iv.2, 197.12f.

³³Sometimes translated as ‘objective’.

The fourth figure is included in the diagram only for mathematical completeness. In Section 5.2 I will suggest that Ibn Sīnā invokes the operation of the inference engine to rule out the fourth figure as 'unnatural'.³⁴

A premise-pair that has a conclusion is said to be 'productive' (*muntij*, a word introduced by al-Fārābī and al-Dimashqī in this sense, cf. [36] p. 25, and used in *Short Epitome in Logic* at 10.9 and 12.19).³⁵ If it has no conclusion—i.e. if none of the four candidate conclusions follows from the premises—then in later works from *Middle Summary in Logic* onwards Ibn Sīnā says that the premise-pair is 'sterile' (*ʿaqīm*); *Short Epitome in Logic* has no special expression for this case.

4.5 Aristotle's methods for categorical syllogisms

For each of the sterile categorical premise-pairs, Aristotle gives in *Prior Analytics* i.4–6 a proof that it is sterile, using the method of terms (i.e. pseudo-conclusions).³⁶ Al-Fārābī, who certainly didn't understand these sterility proofs, simply ignored them. Ibn Sīnā also ignored them in *Short Epitome in Logic*. Much later in *Syllogism* vi.2 he tried to construct sterility proofs for sterile premise-pairs in one of his new hypothetical logics; see Hodges [99] for a review. For the rest of this book we consider only productive premise-pairs.

Following Aristotle's *Prior Analytics* i.1, 24b22–24 and i.4, 26b28–30, the translator Theodorus [139] defines a syllogism as 'perfect' (*kāmil*) if it doesn't need anything other than the premises for showing that the conclusion follows from the premises. He adds Aristotle's remark that all first-figure categorical syllogisms are perfect. In *Short Epitome in Logic* Ibn Sīnā spells out the first-figure moods (*Barbara*, *Celarent*, *Darii*, *Ferio*) without justifications and without any mention of their being perfect. The lack of any mention of perfection probably doesn't indicate any disagreement with Aristotle, since Ibn Sīnā repeats Aristotle's definition of 'perfect' in *Middle Summary in Logic* [114] 100.4 and at *Pointers and Reminders* [135] i.7.4, 141.15.

In *Short Epitome in Logic* [112] 6.17–8.1 Ibn Sīnā lists the remaining ten

³⁴In *Prior Analytics* i.7 Aristotle takes the number of valid categorical syllogisms up to nineteen by dropping the condition (4.16). Since I'm not aware that Ibn Sīnā ever makes any use of this device, I say no more about it.

³⁵Incidentally the form of this adjective is one of several indications that the finite form of the verb is *yuntiju* and *antaja*, not *yantiju* and *nataja* as one sometimes sees suggested.

³⁶There are a few small and easily repaired mistakes in his proofs. These mistakes fooled al-Barakāt but not Ibn Sīnā.

productive categorical premise-pairs together with their conclusions, and in each case he says how the mood is justified. He uses two forms of justification, namely ‘reduction to the first figure’ (*rujū^c ‘ilā l-shakli l-awwal*, [112] 6.20) and ‘ecthesis’ (*iftirāḍ*, [112] 7.5). Both these kinds of justification add some steps of argument between the premises of the syllogism and its conclusion, mostly using one-premise inferences from the following diagram:

$$\begin{array}{ccc}
 (a)(B, A) & (e)(B, A) \xleftrightarrow{\gamma} & (e)(A, B) \\
 \downarrow \beta & & \\
 (i)(A, B) \xleftrightarrow{\alpha} (i)(B, A) & (o)(B, A) \xrightarrow{\delta} & (a)(D, B), (e)(D, A)
 \end{array}$$

(4.18)

Aristotle did little more than give names to these inferences. When we come to the question of ‘foundations’ discussed in *Twenty Questions*, it will become clear that many of the ‘foundations’ that Ibn Sīnā is trying to justify or replace in *Twenty Questions* are one-premise inferences as in (4.18).

The inferences α , β and γ in (4.18) above are examples of ‘conversions’ (*‘ukūs*, singular *‘aks*). Ibn Sīnā’s terminology for talking about conversions is seriously inadequate. Let me say what terminology I think he needed.

Consider the formal categorical sentence $(i)(B, A)$. Its ‘converse’³⁷ is the categorical sentence $(i)(A, B)$ got by transposing the two terms B and A ; the ‘original’ of which it is the converse is called *aṣl*. Similarly the converse of $(e)(B, A)$ is $(e)(A, B)$, and the converse of $(a)(B, A)$ is $(a)(A, B)$.

Converses are classified by their logical relationship to their original as follows. If a formal sentence ϕ entails its converse ψ , then we say that ϕ ‘logically converts to’ ψ . If ϕ and its converse ψ each entail the other, then we say that ϕ ‘converts symmetrically’; Ibn Sīnā says in this case that ϕ ‘converts to itself’, *yan‘akisu mithla nafsih*. Inspecting the examples above, we see that $(i)(B, A)$ and $(e)(B, A)$ each convert symmetrically, but that $(a)(B, A)$ doesn’t convert logically.

We describe either direction of the symmetric inference α as ‘*i*-conversion’, and either direction of the symmetric inference γ as ‘*e*-conversion’.

³⁷ Also called *‘aks* by Ibn Sīnā, thus confusing the syntactic output of a conversion with the entailment that gives this output.. To avoid confusion between senses of *‘aks* I suggest following *Syllogism* [124] ii.3, 105.7 where Ibn Sīnā uses *in‘ikās* ‘reflection’ for converse.

This leaves β , which is strictly not a conversion at all, out in the cold. But Aristotle felt that the inference β was similar enough to α and γ to deserve being described as a conversion too. Accordingly it has become customary to describe β as '*a*-conversion'. Note that β is needed for justifying the two categorical moods *Darapti* and *Felapton*.

The two-conclusion entailment δ in (4.18) is called '*o*-ecthesis'. Note that it introduces a new term, the same in both conclusions. *ékthesis* is a Greek word sometimes translated as 'setting out', and in Arabic it is normally translated as *iftirād*.

Some other logical relationships between pairs of sentences that are in the diagram (4.18) vary with different interpretations of the sentences; for further details see the discussion of replicas in [98] Section 4.3.³⁸ But one further relationship that is stable, at least within categorical logic, is that $(e)(B, A)$ and $(i)(B, A)$ are contradictory negations of each other. In other words, however we choose material terms to put for A and B , one of the two sentences $(e)(B, A)$ and $(i)(B, A)$ will be true and the other will be false.

We illustrate reduction to first figure by justifying the second-figure mood *Camestres*. The method is to start from the premises of *Camestres* and then use a combination of first-figure *Celarent* and two *e*-conversions to reach the conclusion of *Camestres*. As in Chapter 3 above, the premises and conclusion of *Camestres* can be written as

- (4.19) No C is a B ;
 and every A is a B .
 Therefore no C is an A .

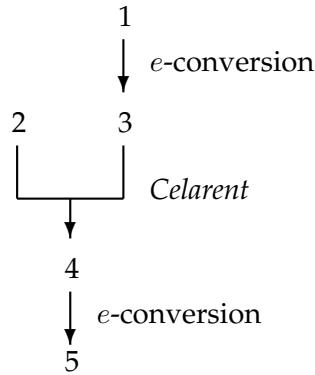
To show how *Celarent* and two conversions can be fitted into this array, logicians today often use the three-column format:

- | | | | |
|--------|---|----------------------|------------------------|
| | 1 | No C is a B . | minor premise |
| | 2 | Every A is a B . | major premise |
| (4.20) | 3 | No B is a C . | 1 <i>e</i> -conversion |
| | 4 | No A is a C . | 2,3 <i>Celarent</i> |
| | 5 | No C is an A . | 4 <i>e</i> -conversion |

The first column numbers the sentences used, and the third column gives the reason why each sentence is used. (For example sentence 4 is the con-

³⁸Different replicas use different first-order translations for justifying *a*-conversion. At *De Interpretatione* 79.11–80.5 Ibn Sīnā interprets $(a)(B, A)$ as if it read $(\forall x(Bx \rightarrow Ax) \wedge \exists x Bx)$. The added clause $\exists x Bx$ is called the 'existential augment', and its use is said to give 'existential import' to $(a)(B, A)$.

clusion of *Celarent* with minor premise 2 and major premise 3.) It can be helpful to draw a tree diagram with the premises at the top and the conclusion at the bottom:



(4.21)

One of Aristotle's basic assumptions can be expressed in terms of diagrams like (4.21): if the arrows express entailments and the entailments are strung together as in the diagram, then the two topmost sentences together entail the bottom sentence.³⁹

The argument sketched above shows that the premises of *Camestres* entail the sentence 'No *C* is an *A*'. But by themselves they don't show that it is the (logical) conclusion of *Camestres*. For that we need to apply the Rule for Identifying Conclusions in the previous section, as follows. The candidate conclusions are 'Every *C* is an *A*', 'No *C* is an *A*', 'Some *C* is an *A*' and 'Not every *C* is an *A*'. Using entailments from (4.18), the premises entail two of these: 'No *C* is an *A*' and 'Some *C* is not an *A*'. The stronger of these, in the sense of the Rule, is 'No *C* is an *A*', so this is the conclusion. But how do we know that the premises don't also entail one or both of 'Every *C* is an *A*' and 'Some *C* is an *A*'? Both Aristotle and Ibn Sīnā often omitted this step in justifying *Camestres* and other categorical syllogisms. But the missing justification can always be given using Aristotle's method of pseudoconclusions.

We turn to an example of ecthesis. Ecthesis involves introducing a new letter, say *D*. Reading early logicians, there is room for debate about

³⁹This assumption is sometimes referred to as 'transitivity', because it can be seen as generalising the transitivity property of entailment: if *p* entails *q* and *q* entails *r* then *p* entails *r*. Smiley [202] Definition 1(ii) states a more abstract and general form of the assumption, which includes shapes of transitivity used to justify other second- or third-figure moods.

whether D is supposed to stand for an individual or a class; Aristotle's comments on ecthesis can be read either way. But for both al-Fārābī and *Short Epitome in Logic* we can resolve the question: in justifications of syllogisms they mean D to be a class. Al-Fārābī [59] 25.17–19 writes 'There was assumed " B was absent from some C ", and this shows that B is denied of the whole of that some. Let us take out that some on its own and call it D .' This surely makes D a class. Ibn Sīnā's language at [112] 7.5f is similar.

One mood that *Short Epitome in Logic* justifies by ecthesis is *Baroco* in second figure. Neither of the premises of *Baroco* has either the form i or the form e , so reduction to first figure by use of a conversion is not available for it. Instead *Short Epitome in Logic* justifies the mood as follows (except that we have to repair the nonsensical ' D is some C ' copied from al-Fārābī and discussed by Chatti and Hodges [21] p. 52).

- | | | | |
|--------|---|---------------------------|--------------------|
| | 1 | Not every C is a B . | minor premise |
| | 2 | Every A is a B . | major premise |
| | 3 | Every D is a C . | 1 ecthesis |
| (4.22) | 4 | No D is a B . | 1 ecthesis |
| | 5 | No D is an A . | 4, 2 <i>Cesare</i> |
| | 6 | Some D is a C . | 3 a -conversion |
| | 7 | Some C is a D . | 6 i -conversion |
| | 8 | Not every C is an A . | 7, 5 <i>Ferio</i> |

At line 5 the proof uses *Cesare*, a second figure mood. But *Short Epitome in Logic* shows how to justify *Cesare* by reduction to first figure, so there is no circularity here. However, the proof that the conclusion is 'Not every C is an A ' is incomplete; by the Rule for Identifying Conclusions we still need to show that the premises don't entail the stronger candidate 'No C is an A '.

Aristotle also has a third method for justifying second- or third-figure syllogisms, namely reduction to absurdity. Ibn Sīnā knows this method as 'syllogism of absurdity' (*qiyāsu l-khalf*), and he describes it at *Short Epitome in Logic* [112] 10.8–12. In fact this description is set in the context of proving a syllogism. But in *Short Epitome in Logic* none of the categorical syllogisms are justified by this method. Ibn Sīnā's reason for avoiding this method in *Short Epitome in Logic* is not clear. Perhaps he is simply following al-Fārābī's *Syllogism*, which also avoids this method. In Section 16.4 we will come to Ibn Sīnā's novel justification of syllogisms of absurdity in his own *Syllogism*, and there will be more to say in the light of his new method.

Besides giving justifications for all the fourteen moods of formal categorical syllogisms, Aristotle in *Prior Analytics* i.4–6 also describes and illustrates a method for showing that a formal categorical premise-pair is sterile. This method will be discussed in detail in [100]. In *Twenty Questions* and *Middle Summary in Logic* Ibn Sīnā will use the method in a very indirect and unexpected way to justify introducing temporal modes. Section 10.1 will explain how Ibn Sīnā does this. In the present section we briefly examine how Aristotle (at *Prior Analytics* [124] i.4, 26a36–39) proves the sterility of the premise-pair in first figure

- (4.23) No C is a B .
Some B is an A .

This will give us some useful technical vocabulary.

Write ϕ for the first formal sentence in (4.23), and ψ for the second. Write Φ for the premise-pair (4.23); we can think of Φ as being the ordered pair (ϕ, ψ) . Write I for the function that has domain $\{A, B, C\}$, and takes A to the material term ‘white thing’, B to the material term ‘horse’ and C to the material term ‘swan’; we can express this function I by writing

$$(4.24) \quad I = \begin{pmatrix} A & : & \text{white thing} \\ B & : & \text{horse} \\ C & : & \text{swan} \end{pmatrix}$$

We say that I is an ‘instantiation’ of the term letters A, B, C in its domain. Also if the function I is applied to the term letters in (4.23), the results are

$$(4.25) \quad I(A) = \text{white thing}, \quad I(B) = \text{horse}, \quad I(C) = \text{swan}.$$

We ‘apply’ I to ϕ by putting $I(C)$ in place of the letter C and $I(B)$ in place of the letter B , so that ϕ is changed to

$$(4.26) \quad \text{No swan is a horse.}$$

So (4.26) is a material sentence; we call it an ‘instance’ of the formal sentence ϕ , and we refer to it as $\phi[I]$. Similarly $\psi[I]$ is the material sentence

$$(4.27) \quad \text{Some horse is a white thing.}$$

which is an ‘instance’ of ψ .

In the same way we can apply I to the whole premise-pair Φ , and the result is the following instance of Φ :

$$(4.28) \quad \begin{array}{l} \text{No swan is a horse.} \\ \text{Some horse is a white thing.} \end{array}$$

We write this instance as $\Phi[I]$, and we also refer to it as ' Φ under I '; so for example C under I is 'swan'. Thus an instantiation I turns formal sentences into instances of them.⁴⁰

We will show that the premise-pair (4.23) has no logical conclusion. The method involves showing that it does have what we will call a 'pseudoconclusion', which we find as follows. Since the premise-pair is in first figure, its candidate conclusions have subject C and predicate A . If θ is one of these candidates, we can write the material sentence $\theta[I]$. If all of $\phi[I]$, $\psi[I]$ and $\theta[I]$ are true sentences, then we call θ a 'formal pseudoconclusion' of Φ , and $\theta[I]$ a 'material pseudoconclusion' of $\Phi[I]$. Alexander of Aphrodisias proved:

(4.29) **Alexander's Theorem.** If a formal categorical premise-pair has both an affirmative universal formal pseudoconclusion and a negative universal formal pseudoconclusion, then it is a sterile premise-pair.⁴¹

This theorem captures Aristotle's method for proving sterility of categorical premise-pairs. In the case where Φ is (4.23) and I is as in (4.24), Aristotle applied I so as to find the material pseudoconclusion 'Every swan is a white thing', giving the formal pseudoconclusion 'Every C is an A '. Then he changed 'swan' to 'crow' and found the material pseudoconclusion 'No crow is a white thing' and hence the formal pseudoconclusion 'No C is an A '. By Alexander's Theorem this proved that Φ is sterile.

In this section we drew on the listing of categorical syllogisms in *Short Epitome in Logic* [112] 6.8–8.1, ignoring the fact that this listing includes a number of comments that have no obvious relevance to categorical syllogisms. These include statements of the *'ibra* for necessity, and some references to 'how it is' (*kayfa kāna*). In Section 7.2 we will see that these extra items refer to a different reading of *Short Epitome in Logic* [112] 6.8–8.1, not as a list of categorical syllogisms but as a list of modal predicative syllogisms. This is one illustration of a typical trick of Ibn Sīnā's exposition: if

⁴⁰I used to call these functions 'interpretations', a name which goes back to the nineteenth-century logicians Peacock and Boole. But now I think we have too many other things called 'interpretations'.

⁴¹In fact Alexander proved more than this: he showed that a formal categorical premise-pair is sterile if and only if it has both an affirmative universal formal pseudoconclusion and a negative universal formal pseudoconclusion, and that this is the only pair of forms for which this equivalence holds. For example the premise-pair can have two pseudoconclusions which are contradictory negations of each other, and still be productive. See [7] 89.29–91.33. On my reckoning this is the best theorem in Peripatetic logic that was proved after Aristotle and before Ibn Sīnā.

two distinct theories are similar enough, he tries to expound both of them simultaneously, with occasional remarks to explain where the two theories diverge. Another example, from much later in his career, is his double version of the proof search algorithm in *Syllogism* [124] ix.6; see Hodges [84] Section 5.2.

Chapter 5

The inference engine

5.1 Defining syllogism

The definitions of ‘syllogism’ in the Arabic *Prior Analytics*, al-Fārābī’s *Syllogism* and Ibn Sīnā’s *Short Epitome in Logic* are as follows.

- (5.1) As for the syllogism: it is a discourse such that when more than one thing is posited in it, something else follows, of necessity through the truth of those posited things by themselves.
(*Prior Analytics* [139] 184.3f)

- (5.2) A syllogism is a discourse in which more than one thing is posited, such that when [these things] are composed (*ullifat*), something other than them follows from them, by themselves, and not by accident but necessarily.
(Al-Fārābī *Syllogism* [59] 19.8f)

- (5.3) A syllogism is composed (*mu'allaf*) from sentences, such that when they are assumed, another sentence follows from them by themselves.
(*Short Epitome in Logic* [112] 5.17)

There are obvious family resemblances between these definitions. But one point in which al-Fārābī and Ibn Sīnā differ from Aristotle (both the Arabic Aristotle and the Greek) is that the two medieval authors make it explicit that in order to draw the conclusion the premises must be ‘composed’, i.e. brought together in some suitable way.

This is not an accidental point. Both al-Fārābī and Ibn Sīnā elsewhere spell out further details of how the premises must be combined in the mind in order to generate the conclusion in a semi-mechanical way. Thus al-Fārābī:

- (5.4) Syllogisms in general are things that are arranged in a certain order (*turattabu tartīban mā*) in the mind such that when the things have been put in this order, the mind as a result finds itself unavoidably looking down at something else of which it was ignorant before, so that it knows it now, and thus the mind is equipped to be bound by the thing it looked down at, just as if it [already] knew that thing.
(*Expressions* [55] 100.3–5)

Ibn Sīnā gives similar accounts in various books throughout his career. Two of these accounts are in *Short Epitome in Logic*. Here is the first; I added numbers to mark the stages of the process:

- (5.5) The middle [term] is of a nature to (i) combine the two extreme [terms] into a conclusion, and to (ii) be excluded from what is explicit [in the conclusion], so that (iii) one (*aḥad*) of the two extremes, called the minor term, forms the subject of the conclusion, and the premise [containing it] is called the minor premise; while (iv) the other extreme, called the major term, forms the predicate of the conclusion, and the premise [containing it] is called the major premise. (v) The first figure yields a conclusion only when the minor premise is affirmative and the major premise is universal. (vi) The dominance (*‘ibra*) in quality, i.e. in affirmative and negative, and in mode, i.e. in necessity and non-necessity, belongs to the major premise. ...
(*Short Epitome in Logic* [112] 6.4–11)

And here is the second, again with numbers for the steps. This passage is taken from the part of *Short Epitome in Logic* corresponding to *Sophistical*

Refutations:

- Syllogisms of resemblance are those which fall short of the aforementioned conditions of entailment. [The way to] avoid [these syllogisms] is to ensure that (vii) the terms of the syllogism enter stepwise and singly (*yakhṭīru martabatan mufradatan*) as the meanings of the expressions, and to strive to ensure that (viii)
- (5.6) the middle term is used in one of the two premises in the same sense as in the other, and that the major and minor terms appear in the premises in just the same sense as in the conclusion, with no distinction either in the sense of the terms [themselves] or in any [implied] conditions (*sharā'it*) or interpretations.
(*Short Epitome in Logic* [112] 12.19–13.3)

The process described in these two passages from *Short Epitome in Logic* finds the conclusion of a syllogism by (viii) identifying a term that occurs in both premises with the same meaning, (ii) removing this term, (i) taking the two remaining terms as the terms of the conclusion, (iii)+(iv) determining which of these two terms is the subject of the conclusion and which is the predicate, and (vi) determining from the forms of the premises whether the conclusion is affirmative or negative. There is also a check (v) whether any conclusion can be drawn.

The procedure is semi-mechanical.¹ Some phrases in the first passage read as if the parts of the syllogism make the construction themselves—as if the repeated term removes itself, for example. But these can only be a quixotic metaphor; clearly the mind is what performs the process.

Systems that carry out particular forms of reasoning by mechanical manipulations of the syntax are in common use in informatics today, particularly in expert systems where they go by the name of ‘inference engines’.² Ibn Sīnā’s description does involve meanings as well as syntax, but otherwise it fits the bill. We will refer to it henceforth as the ‘inference engine’.

Among several words that Ibn Sīnā associates with the inference engine, there is a phrase ‘it comes into the mind’ (*yakhṭīru bi-l-bāl*) which Ibn Sīnā

¹But presumably not fully mechanical, since step (viii) requires checking that a word carries the same sense at two occurrences.

²See Hayes-Roth et al. [80]. An abstract device of a similar kind is the Warren Abstract Machine [6], which we can recognise as a wide-reaching generalisation of Ibn Sīnā’s inference engine, including the identification of the repeated term and a procedure for removing it. The identification is a special case of the operation known to modern logicians as ‘unification’; Ibn Sīnā has his own technical name for it, *idghām* (*Syllogism* viii.3, 410.2).

uses to express that the inference engine is at work.³ The phrase appears in this sense at:

	<i>Middle Summary in Logic</i> [114]	6.5, 86.13f, 230.19–231.14
	<i>Shorter Summary in Logic</i> [131]	11.13, 105.9, 120.10–121.1
	<i>Madkhal</i> [118]	i.6, 34.21–36.2; i.7, 41.8
(5.7)	<i>Syllogism</i> [124]	i.2, 15.15–16.7; iv.1, 187.1–4; v.4, 270.14; viii.1, 393.14f, 395.1–5; viii.2, 402.14; ix.1, 418–421; ix.19, 543–547; ix.22, 565.15
	<i>Debate</i> [126]	28.10, 82.2
	<i>Easterners</i> [116]	30.20f
	<i>Pointers and Reminders</i> [135]	i.1.11, 48.1; i.6.1, 126.10f

These occurrences run right through the second half of Ibn Sīnā's career. The phrase is not found in *Short Epitome in Logic*, though *Short Epitome in Logic* does contain the key passages (5.5) and (5.6), and the second of these speaks of the terms of a syllogism 'entering' (presumably the mind).

To the best of my knowledge, Ibn Sīnā never uses the phrase *yakhtiru bi-l-bāl* in his writings on psychology, for example in his *Soul* [130]. This suggests that he regards the inference engine as a fact about syllogistic reasoning, not about the nature of conscious thinking in general.

While we are thinking about the shapes of proofs, there is another puzzle that we should mention, though I don't know of any place where it is directly discussed before modern times. Consider a formal syllogism, for example

- (5.8) Some *B* is a *C*.
 Every *B* is an *A*.
 Therefore some *C* is an *A*.

According to (5.3) above, this syllogism occurs when we assume 'Some *B* is a *C*' and 'Every *B* is an *A*'. But these are not meaningful sentences, because of the meaningless alphabetical terms *A*, *B*, *C* that they contain. So how can they be assumed? To borrow a modern expression, the problem is that this

³In classical Arabic the phrase just means 'it comes to mind'. The idea of specialising it to the processing of syllogisms seems to be due to al-Fārābī in his *Long Commentary on Prior Analytics* [57] 450–494. For example at 474.10 he says 'If we are not aware of a thing and it never comes to mind (*yakhtiru bi-bālin*), then it is impossible for us to make it the goal'.

talk of assuming the formal premises doesn't correctly describe the 'logical form' of the inference from the premises to the conclusion.

There are several ways to handle the problem. The following is not a definitive answer but it will serve us adequately in this book. The cause of the problem is the meaningless formal terms. We can give them the appropriate kind of meaning by interpreting them as the variables for implicit universal quantifiers over things named by material terms, as follows:

- (5.9) For all classes A , B and C nameable by material terms,
 if something in B is in C ,
 and everything in B is in A ,
 then something in C is in A .

So the logical form of the syllogism is a universally quantified 'if ... then ...' sentence. In BELOW we will find it helpful to extend this analysis to modal syllogisms.

In the following four sections we look at four other notions closely linked with the inference engine.

5.2 The syllogistic order

At (5.4) above we quoted al-Fārābī as saying that the contents of the premises of a syllogism are arranged in an order (*tartīb*) that compels the mind to move to the conclusion of the syllogism. For Ibn Sīnā this order is the order in which the terms enter the inference engine if the engine is to do its job properly. At *Syllogism* [124] ix.18, 539.14 he calls this order the 'syllogistic arrangement' (*naẓm qiyāsī*); at *Syllogism* [124] ix.6, 460.6 it is the 'natural order' (*tartīb ṭabīʿī*), and at *Debate* [126] 305.12f it is the 'syllogistic order' (*tartīb qiyāsī*).

Both al-Fārābī and Ibn Sīnā are developing some passages where Aristotle analyses what goes wrong when we make a mistake of reasoning. Ibn Sīnā translates Aristotle's explanations into failures of the inference engine.

For example:

- (5.10) ... like when someone knows that this is a mule, and knows that every mule is barren, but when he doesn't put these two facts together as they come into his mind, it is possible that nevertheless he thinks that there is a foetus in the belly of this mule. This is because these two premises are only potentially a cause of the conclusion; they become an actual cause of the conclusion when they come into the mind together in the order (*tartīb*) which naturally makes a conclusion and leads towards the conclusion. (*Middle Summary in Logic* [114] 230.16–19, cf. *Syllogism* [124] ix.19, 544.13–17)

Here Ibn Sīnā is following Aristotle *Prior Analytics* ii.21, 67a34ff.

In a debate between a questioner and a responder, one of the tasks of the questioner is to invite the responder to accept or reject various sentences in turn, in order to get the responder to agree to the conclusion of a syllogism whose premises he has accepted. (In this context the syllogism will usually be a compound syllogism with three or more premises.) Aristotle likes to point out that the questioner can get an advantage in a debate of this kind by hiding his intentions from the responder. One way to do this is to mix up the order in which he puts his questions. For example:

- (5.11) If [the questioner puts the questions] explicitly as 'Every *B* is an *A*; every *Z* is a *B*', this is not in the syllogistic arrangement. In fact when the term in common to these [premises] is not posited in both of the premises [with its two occurrences] next to each other, the conclusion won't be intuited—how could it be when [the syllogistic order] is not made explicit? (*Syllogism* [124] ix,18, 539.14–16)

(Cf. Aristotle *Topics* [12] 156a23–26, trans. Barnes [20] vol. 1 p. 262: 'It is a useful rule, too, not to secure the axioms on which the deductions are based in their proper order, but alternately those that conduce to one conclusion and those that conduce to another; for if the appropriate ones are set side by side, the conclusion that will result from them is more obvious in advance.')

The example of a failure of syllogistic order in (5.11) may help to ac-

count for Ibn Sīnā's rejection of the syllogistic fourth figure:

- The best of doctors mentions this [fourth class of syllogistic moods], but he doesn't take the view that we do. Here we re-
- (5.12) ject it because it is unnatural, unacceptable and inappropriate for the conduct of enquiry and reflection.
(*Syllogism* [124] ii.4, 107.11f)

(The doctor is presumably Galen.) It's a pity that Ibn Sīnā doesn't explain why he thinks the fourth figure is unnatural. One possibility is that he puts into the fourth figure arguments like

- Every B is an A .
(5.13) Every Z is a B .
Therefore every Z is an A .

This is not in the 'natural order' for the reason mentioned in (5.11), namely that the two occurrences of B are unnecessarily far apart. On the other hand at *Syllogism* [124] i.6, 62.5f Ibn Sīnā seems to be claiming that an example of fourth figure is

- No C is a B ;
(5.14) and some B is an A .
Therefore some A is not a C .

which doesn't separate the two occurrences of the middle term. My guess is that Ibn Sīnā is still rejecting (5.14) on the basis of the inference engine, but in this case relying on the requirement that in the syllogistic order the premises 'lead towards the conclusion' (as in (5.10) or *Syllogism* ix.1, 419.2).

To expand this a little: we saw in (5.5), (iii) and (iv), that the inference engine determines which of the premises is the major and which is the minor. This determination is needed for deciding which figure we are in. Ibn Sīnā sometimes pays lip service to Alexander's rule for finding the major and the minor premise; this rule assumes that the premises have a goal (*maṭlūb*). But here is a problem: already in *Prior Analytics* used the three figures to classify not only syllogisms but also sterile premise-pairs. In *Syllogism* [124] vii.2 Ibn Sīnā himself will do the same for premise-pairs in a form of hypothetical logic. A sterile premise-pair normally doesn't have a goal. So the inference engine should have a way of distinguishing the major premise from the minor in any premise-pair, using just properties of the premise-pair. Goals should be irrelevant.⁴

⁴'The basic question is whether to take account of premiss-order.' (Thom [215] p. 27.)

Ibn Sīnā's practice shows how to do it, though he never explains it openly.⁵ The premises in a premise-pair are always given in some order; we take the first premise to be the minor premise and the second to be the major premise. Since Ibn Sīnā uses this rule in practice, we can call it 'Ibn Sīnā's rule for major and minor'. We can then define the major term to be the term occurring only in the major premise, and the minor term to be the term occurring only in the minor premise, running the definitions of Section 4.4 backwards. The four candidate conclusions of the premise-pair are the four categorical sentences that have the minor term as subject and the major term as predicate.

There is one case where Ibn Sīnā doesn't need his rule for major and minor. This is the case where the middle term in a premise-pair is predicate in one premise and subject in the other. It follows from Ibn Sīnā's rejection of the fourth figure that this premise-pair can only be put in the first figure, and hence the minor premise is the one with the middle term as predicate, and the major premise is the one with the middle term as subject.

There are also cases where a premise-pair is productive or sterile depending on which of the premises we take as the major. One example is at *Middle Summary in Logic* [114] 138.11, where Ibn Sīnā gives the conclusion, and from this we can infer that he is treating the first premise as major and the second as minor, breaking his rule. But examples like this, where Ibn Sīnā must have broken his rule, are vanishingly few.

Ibn Sīnā's rule for major and minor makes it automatic that the syllogistic order of a premise-pair puts the minor premise first. This is the opposite order to that in the Latin West.

5.3 Dominance

Step (vi) of the inference engine procedure relies on a rule called an *'ibra* for determining the quality and mode of the conclusion. To the best of my knowledge this word appears, as a term of logic, for the first time in *Short Epitome in Logic*, and Ibn Sīnā continues to use it regularly in later works,

⁵In (5.5) the word *aḥad* 'one' for the minor term could also be read as 'first'. So the reader is at least primed to take the minor term as the one appearing only in the first premise to enter the inference engine, and the major term as the term only in the second premise to enter.

as follows:

	<i>Short Epitome in Logic</i> [112]	6.10, 7.7f, 8.1
	<i>Middle Summary in Logic</i> [114]	130.16, 131.15, 133.14, 134.1
	<i>Shorter Summary in Logic</i> [131]	66.4, 67.3, 69.5, 69.8
(5.15)	<i>Guidance</i> [121]	98.9, 101.8, 102.1, 102.6
	<i>Syllogism</i> [124]	iii.3: 157.11, 159.16; iv.2, 198.11; iv.3, 204.15; vi.1, 304.4
	<i>Pointers and Reminders</i> [135]	i.7.4: 147.12, 148.1, 148.10, 151.11

(At *Pointers and Reminders* [135] i.7.4, 147.8 Ibn Sīnā replaces *‘ibra* by *ḥukm* in the same meaning. The reason for this variation is unknown.) The rule relies on the metaphor, known in earlier Arabic logic, that the conclusion of a syllogism is a child resulting from the marriage of the two parents/premises, so that it inherits its characteristics from one or other of the parents. Ibn Sīnā uses *‘ibra* for the factor that determines which parent passes on a particular characteristic; so ‘dominance’ seems an appropriate translation.⁶ Of course Ibn Sīnā could have introduced the term *‘ibra* without reference to an inference engine, but the fact is that it does first appear as part of the machinery of the inference engine.

When Ibn Sīnā first introduces an *‘ibra*, in *Short Epitome in Logic* [112] 6.9–11 for categorical syllogisms in the first figure, he states it for two properties of the sentences: (1) quality, i.e., being affirmative or negative, and (2) being either necessary (*ḍarūrī*) or not necessary (*ghayr ḍarūrīn*). Quality is a syntactic property that can be read off directly from the sentence. So when Ibn Sīnā says that the *‘ibra* for quality in the first figure lies with the major premise, he probably means that for any predicative first-figure syllogism, the conclusion is affirmative if and only if the major premise is affirmative.

Since *Short Epitome in Logic* doesn’t make clear what kind of property Ibn Sīnā intends necessity or non-necessity to be, our best bet is to assume that he intends these modal properties to be syntactic too, in a way that makes cases (1) and (2) parallel. Thus modal sentences can be classified as ‘necessary’ or ‘non-necessary’, and the classification can be read off from the syntax. The *‘ibra* for necessity in the first figure then says that for a pred-

⁶See Hodges [93] for more on this metaphor, including the fact that Ibn Sīnā uses *‘ibra* also in biology, for the factor that determines the gender of a child. Another of his innovations that comes from the same metaphor is *‘aqīm* ‘sterile’ for a premise-pair with no syllogistic conclusion; though we first find this usage in *Middle Summary in Logic*, not in *Short Epitome in Logic*. The paper [93] also illustrates how the metaphor that premise = parent is used by the Brethren of Purity and by Ibn Rushd.

icative syllogism in first figure, the conclusion will fall under the classification 'necessary' if and only if the major premise falls under the classification 'necessary'. *Short Epitome in Logic* allows a sentence that is not necessary to be either absolute or possible/contingent; so if the major premise is not necessary, what should follow about the classification of the conclusion? Since Ibn Sīnā doesn't answer this question in *Short Epitome in Logic*, we can follow our hunches. My hunch is that where the conclusion inherits its properties from its parents, a conclusion from a necessary minor premise and an absolute major premise is not necessary, so it should be absolute. The same goes with 'possible/contingent' for 'absolute'. In BELOW Ibn Sīnā will begin to replace unscientific appeals to an *'ibra* by reasoned arguments, and these reasoned arguments will turn out to support my hunch. (Maybe Ibn Sīnā and I were both lucky.)

For the moment we leave open the question what syntactic features show that a sentence is classified as necessary. A first indication is that the sentence contains a word or phrase meaning 'necessary'. But in Chapter 7 below we will see that this is not a necessary and sufficient condition.

Quantity (universal or existential) is syntactic, just as quality is. So we might have expected that Ibn Sīnā would state an *'ibra* for quantity too.⁷ Perhaps he was discouraged from stating an *'ibra* for quantity by the fact that there is no such *'ibra* in the third figure. In the later work *Syllogism* he seems to consider only *'ibar*⁸ for necessity; though because he never mentions this restriction, the result is likely to mislead the reader.

In Chapter 7 we will begin to examine Ibn Sīnā's understanding of modal logic. The first step will be to reconstruct as far as we can, from his remarks in *Short Epitome in Logic* on dominance with respect to necessary and non-necessary, which modal syllogisms he claims to accept and which he claims to reject. In this way we will identify Ibn Sīnā's Modal Challenge. Much of the rest of this book will uncover the steps that he took in order to meet this challenge.

⁷It's clear what the *'ibra* for quantity should say. In first and second figures this *'ibra* lies with the minor premise. In third figure there is no *'ibra* for quantity, because in third figure the conclusion is never universal even when both premises are universal; and in fourth figure the requirement for the conclusion to be universal is that the minor premise is negative. See Hodges [98] Theorem 9.4.3.

⁸Plural of *'ibra*.

5.4 Recombinant syllogisms

At *Short Epitome in Logic* [112] 5.20 Ibn Sīnā tells us:

- (5.16) Some syllogisms are recombinant (*iqtirānī*) and some are exceptive (*istithnā'ī*). The recombinant syllogisms that are predicative fall into three figures.

These are the only two occurrences of the word *iqtirānī* in *Short Epitome in Logic*, and it is not known as a property of syllogisms in any earlier work. In this passage it is not defined, but we are given examples of *iqtirānī* and *istithnā'ī* syllogisms. Immediately after (5.16) is a section [112] 6.1–8.2 which defines the three figures and gives examples of all the valid fourteen categorical moods; presumably these are examples of the recombinant moods that are predicative. Later, a section [112] 9.1–10.6 explains the exceptive moods. Between these two sections, in [112] 8.3–22, a new kind of syllogism is introduced, for example

- (5.17) Sometimes when CD , GT ;
and it is never the case, when GT , that AB .
It yields that it is not the case that whenever CD , AB .

Presumably these syllogisms of the new kind are examples of syllogisms that are recombinant but not predicative.

The adjective *iqtirānī* is an adjective from the nouns *iqtirān* and *qarīna*, both of which in logic mean ‘premise-pair’; the underlying idea is of two or more things being connected or composed into a single unit. This and the inference engine tell us how to understand what a recombinant syllogism is. It’s a syllogism in which parts of the two premises are brought together to form the conclusion. As (5.5) says, at step (i), when the middle term has been identified and removed, the remaining extreme terms from the two premises are ‘combined into a conclusion’. Categorical syllogisms are recombinant in this sense, but exceptive syllogisms are not. Examining the sample syllogism (5.17) of the third and new kind, we see that the conclusion can be reached by copying the action of the inference engine: the ‘middle term’ is GT , leaving ‘extremes’ CD from the first premise and AB from the second. The conclusion is formed by combining CD with AB .

In Ibn Sīnā’s later logical works *iqtirānī* (‘recombinant’) appears quite

often, particularly in the later articles of *Syllogism*:

	<i>Short Epitome in Logic</i> [112]	5.20
	<i>Middle Summary in Logic</i> [114]	119.17–120.2, 150.6f, 161.12, 162.6, 163.16, 169.8, 213.22, 214.9, 237.11
	<i>Shorter Summary in Logic</i> [131]	52.4–6, 83.4, 92.8
	<i>Guidance</i> [121]	91.8, 92.1, 105.1, 105.9, 109.1
(5.18)	<i>Syllogism</i> [124]	ii.4, 106.1–12; v.2, 249.6; vi.6, 357.2; viii.1, 389.6–9, 397.6; ix.1, 415.5–418.11, 423.4–425.19; ix.2, 432.1f; ix.3, 433.5, 445.2, 456.10; ix.6, 462.7; ix.7, 471.9; ix.9, 491.4–7; ix.14, 518.5, 522.10; ix.20, 551.9, 552.9; ix.21, 555.4, 558.2
	<i>Pointers and Reminders</i> [135]	i.5.2, 139.11–13; 140.4–9; i.7.2, 140.14f, 142.6; i.8.4, 161.3

In *Middle Summary in Logic* Ibn Sīnā begins his discussion of recombinant syllogisms by defining ‘recombinant’—something he omitted to do in *Short Epitome in Logic*. He defines:

- (5.19) The difference between [recombinant and exceptive syllogisms] is that in the recombinant [syllogism] neither the goal nor its contradictory negation is mentioned explicitly [in either premise] . . . while in the exceptive [syllogism] either the goal or its contradictory negation is mentioned explicitly [in a premise].
(*Middle Summary in Logic* [114] 119.17–20)

To make sense of this definition we have to read ‘goal’ (*maṭlūb*) as meaning conclusion.⁹

5.5 Taking care of conditions

The Arabic word *sharṭ*¹⁰ means a proviso or stipulation in a contract, or more generally a condition. Together with its adjective *sharṭī* ‘conditional’,

⁹By the time of *Syllogism*, Ibn Sīnā quite often uses *maṭlūb* to mean conclusion. For example *Syllogism* [124] i.2, 14.12f: ‘In the case of other sciences, one of them helps another in the sense that a goal in the one that helps forms a premise and matter for the one that is helped.’

In other words, the same sentence is a conclusion in the helper science and a premise in the helped science.

¹⁰Plural *shurūṭ* or *sharāʾiṭ*.

it appears frequently in Arabic logic with a range of meanings. These meanings in logic fall into two classes.

In the first class of meanings, a sentence was originally described as *sharṭī* if it had the form ‘If ϕ then ψ ’, and the clause ϕ was called the *sharṭ*. But as logic developed in the hands of Ibn Sīnā among others, the class of sentences described as *sharṭī* expanded greatly. For example in Ibn Sīnā’s usage a sentence ‘If ϕ then ψ ’ was a special case of a class of *sharṭī* sentences called *muttaṣil* (literally ‘continuous’), as described in Chapter 6 below. In the sixth century the Latin author Boethius studied a class of sentences which included more than the *muttaṣil* but less than what Ibn Sīnā came to count as *sharṭī*, and Boethius’s name for sentences in this class was *hypotheticus*. Many modern studies, including the present book, borrow this word from Boethius and translate *sharṭī* as ‘hypothetical’.

The second class of meanings of *sharṭ* in Arabic logic can be summed up as: necessary and sufficient condition for a piece of text to have a certain property of logical interest. In Ibn Sīnā the two chief examples of conditions in this sense are

- conditions of contradictory negation (*sharā’iṭu l-naqīḍ*), e.g. *Short Epitome in Logic* 12.16.
- conditions of productivity (*sharā’iṭu l-intāḥ*), e.g. *Short Epitome in Logic* 12.19.

We begin with the conditions of contradictory negation. Aristotle gave examples of a sentence *S* which is either true or false, where simple syntactic adjustments will find a sentence *T* that is true if *S* is false and false if *S* is true. Each of the sentences *S* and *T* is said to be the ‘contradictory negation’ of the other, in Arabic *naqīḍ*.¹¹ In Aristotle’s logic these contradictory negations are used in proofs by *reductio ad absurdum*.

Aristotle also observed that no purely syntactic rules are enough to guarantee that *T* has the opposite truth value to *S*. This is because people often mean more than they say explicitly; so to be sure that *S* and *T* have opposite truth values, we need to bring to the surface any implied

¹¹In Arabic such pairs of sentences are easy to find, since most sentences can be negated by putting *laysa* ‘it is not the case that’ at the beginning of them. But Greek has no such device. Cf. the discussion of double negation in Ibn Sīnā in paragraph 7.7 of Hodges [88]. OR SECTION BELOW IN THIS BOOK.

parts of the meanings of *S* and *T*. As the Arabic Aristotle expresses it:

- (5.20) Let contradiction be this: I mean affirmation and denial that are opposite to each other. And by 'opposite' I mean that the corresponding [terms] in the opposed sentences are identical in meaning and not just agreeing in their wording, together with the other requirements of similarity that we make because of trouble-making by the sophists. (Hunayn's translation [139] 117.9–12, via Syriac from Aristotle's *De Interpretatione* 17a33–37)

These 'requirements of similarity' are what Ibn Sīnā refers to as the 'conditions of contradictory negation'. Ibn Sīnā also adopted a phrase for the action of bringing to the surface the relevant implied meanings. He called it 'taking care of' or 'protecting' (*murāʿa*) the conditions.¹²

Early commentators on Aristotle's *Sophistical Refutations* scoured the text of the *Refutations* for indications of what these 'other requirements of similarity' might be. Ebbesen [45] p. 172 compares versions of a scholion on the Greek original of (5.20) in a number of sources and reduces them to a standard list of requirements, which he attributes to Porphyry. There are several Arabic versions of this list, including the following:

- al-Fārābī, *Syllogism* [59] 14.11–15.13, [36] pp. 119f.
 Ibn Sīnā, *Short Epitome in Logic* [112] 5.1–4, partly repeated at 12.15;
 Ibn Sīnā, *Guidance* [121] 84.5–85.3.
 Ibn Sīnā, *De Interpretatione* [123] 43.8–44.7.
 Ibn Sīnā, *Pointers and Reminders* [135] i.3.10, 83.15–84.3.

The following table lines up the conditions mentioned by al-Fārābī and Ibn Sīnā against those in the Porphyry scholion:

Porphyry	part	relation	manner		time	
al-Fārābī	part		<i>jīha</i>		time	place
<i>Short Ep L</i>	part/whole	relation	<i>ḥāl</i>	act/pot	time	place
<i>Guidance</i>	part/whole	relation		act/pot	time	place
<i>De Int</i>	part	relation	<i>jīha</i>	act/pot	time	place
<i>Pointers</i>	part/whole	relation	<i>sharṭ</i>	act/pot	time	place

The four lists from Ibn Sīnā show that he had his version of the list in a stable form from the early *Short Epitome in Logic* to the late *Pointers and*

¹²This is another example of a term of Arabic logic that is borrowed from the language of animal husbandry; the participle *rāʿin* means shepherd or herdsman. Cf. Footnote 15 in Chapter 3 above..

Reminders. He always added ‘place’, following al-Fārābī, and he ignored al-Fārābī’s presumably accidental omission of ‘relation’. The column under ‘manner’ seems to show that he was unclear what al-Fārābī meant by *jiha* (mode? aspect?). Porphyry probably intended ‘manner’ to stand for act versus potential here, but Ibn Sīnā missed this and always added act versus potential as a further condition. All these forms of the list mention a condition on ‘time’.

Ibn Sīnā uses the full phrase ‘taking care of the conditions’ (and some close variants) in the following places:

	<i>Twenty Questions</i> [115]	ii, 83.3f
	<i>Middle Summary in Logic</i> [114]	109.6, 117.12f
	<i>Shorter Summary in Logic</i> [131]	70.6
	<i>De Interpretatione</i> [123]	45.9, 102.10–12
(5.21)	<i>Syllogism</i> [124]	i.5, 38.6,10; 39.10;
		ii.4, 113.11; iii.1, 138.16;
		vii.2, 384.11f;
	<i>Sophistics</i> [127]	22.4, 54.15, 101.13–102.1
	<i>Easterners</i> [116]	48.9, 75.1
	<i>Pointers and Reminders</i> [135]	i.3.10, 83.15–84.3; i.5.2, 111.11–13
	<i>Autobiography</i> [117]	pp. 28f

My strong impression is that as we move from the beginning of Ibn Sīnā’s career to the end, Ibn Sīnā became increasingly concerned about the importance of *murāʿa*, but his concern about this developed in two directions. In the first direction he broadened the scope of *murāʿa* away from Aristotle’s list of requirements of similarity; the primary aim was no longer to find contradictory negations, but to pin down the meaning of any sentence. He even came to regard this as the primary aim of logic. The opening sentence of *Pointers and Reminders* tells us:

- (5.22) The aim of logic is to be for the human a lawlike tool, attention to which will protect him (*murāʿa*) against error in thinking.
(*Pointers and Reminders* [135] i.1.1, 39.2)

So the logician is a shepherd, guarding the meanings of our thoughts. In Section 1.3 above, quotation (1.5), an anonymous student of Ibn Sīnā from around the same time as *Pointers and Reminders* makes a similar point.

The second direction is almost the opposite of the first. In it Ibn Sīnā concentrates the ‘taking care’ on time and the conditions for finding the contradictory negation of a sentence that talks about time. Around the end

of the Gap period, both *Twenty Questions* and *Middle Summary in Logic* argued for introducing new sentence forms for logic. These new sentence forms contained 'Sometimes' and other quantifiers over times, something not foreseen at all in *Short Epitome in Logic*. If these new sentence forms are to be introduced into formal syllogisms, we need to know how to find their contradictory negations. But Aristotle has told us that if a sentence refers to a time, its contradictory negation must refer to the same time. So, as Ibn Sīnā comments, to be sure that our syllogisms using these new sentences are valid, we need to be able to ensure that

- (5.23) the two propositions have a property that is difficult to take care of (*murāʿa*), namely that the time is one and the same in both of them if possible, and under the same condition if possible.
(*Syllogism* [124] ii.4, 113.11f)

The property is difficult to take care of because the reference may be to an indefinite time, for example in a quantification over times. I am not sure that Ibn Sīnā ever sorted out this conundrum.

Ibn Sīnā and many of his successors followed Aristotle's exhortation to bring tacit assumptions to the surface, as a way of generating new sentence forms for logical analysis. This is such a basic procedure that it deserves a name. In Section 8.2 below I will call it 'Adding Careful Conditions' and say some more about it. See also the further discussion in [100].

Both Aristotle and Ibn Sīnā seem to have believed that Adding Careful Conditions made their reasoning more rigorous. When used appropriately it could have this effect. But in practice it could have the opposite effect, by introducing an ambiguity about which statement was being discussed. It could be (1) the underlying categorical sentence, (2) the categorical sentence plus an intended but unspoken interpretation of it, (3) the categorical sentence expanded to make the intended interpretation explicit, (4) an approximation to the expanded sentence, got by ignoring subtler features, ...

There is less to be said here about conditions of productivity. These are necessary and sufficient conditions for a premise-pair to be productive. Ibn Sīnā did a disservice to his successors by encouraging them to use conditions of productivity without finding a systematic way to check their correctness. Also it would have crowned his research into compound syllogisms if he had found a justifiable way of extending conditions of productivity to sets of more than two premises.

Chapter 6

Hypothetical syllogisms and their semantics

In Section 4.3 above we met al-Fārābī's distinction between predicative (*ḥamlī*) and hypothetical (*sharṭī*) sentences. We examined what al-Fārābī had to say about predicative sentences, and how Ibn Sīnā described these sentences in *Short Epitome in Logic*. Then in Section 4.4 we moved on from predicative sentences to predicative syllogisms. In his accounts of predicative sentences and syllogisms in *Short Epitome in Logic*, Ibn Sīnā follows al-Fārābī rather faithfully.

The present chapter will compare al-Fārābī's treatment of hypothetical sentences and syllogisms with Ibn Sīnā's treatment of them in *Short Epitome in Logic*. The situation here is quite different from what we saw with predicative sentences and syllogisms. In *Short Epitome in Logic* Ibn Sīnā describes two different hypothetical logics. The first (in order of exposition in *Short Epitome in Logic*) is what we will call 'wholly *muttaṣil* logic'. This is a new logic, based entirely on Aristotle's categorical logic but switched around so that the terms are sentences instead of nouns. In *Short Epitome in Logic* Ibn Sīnā makes no connection between it and al-Fārābī's exceptive hypothetical logic, except for borrowing a few technical terms. (In Section 11.2 we will see that by the Gap period Ibn Sīnā has started to ask whether there is in some sense a more general hypothetical logic that includes both wholly *muttaṣil* logic and exceptive logic. His enquiries in this direction continue at least to *Syllogism v* in the 1020s, though I think not always helpfully.)

The second hypothetical logic in *Short Epitome in Logic* is a variant of al-Fārābī's exceptive hypothetical logic. We explore it in Section 6.5. We

will see there that in *Short Epitome in Logic* and *Middle Summary in Logic* at opposite ends of the Gap period, the versions of wholly *muttaṣil* logic and of exceptive logic that Ibn Sīnā presents have no sentence forms in common. Ibn Sīnā never confused the two logics, even when he was discussing them together.

The name *sharṭī* is an adjective from the noun *sharṭ* 'condition'. By expanding the class of *sharṭī* sentences as in *Short Epitome in Logic*, Ibn Sīnā makes it harder to sustain the claim that 'conditional' is an appropriate name for *sharṭī* sentences. For instance the new existential *sharṭī* sentences (affirmative and negative) are introduced in *Short Epitome in Logic* as variants of categorical 'Some *B* is an *A*' and 'Some *B* is not an *A*', neither of which has any hint of a condition.¹ So it's not surprising that Ibn Sīnā in a number of places suggests a new way of understanding the word *sharṭī*.

For example 'sharṭī sentence' is (on a definition suggested by Ibn Sīnā in *Easterners* [116] 61.8–17) a sentence with two or more clauses, where the truth of the sentence doesn't imply either the truth or the falsehood of either of the clauses on its own.² This removes the suggestion of a condition, but it also overlooks the quantifier that wholly *muttaṣil* logic inherits from categorical logic. It also overlooks single-clause sentences that result from coordination, like 'This number is either even or odd', which Ibn Sīnā persisted in counting as *sharṭī*. This was almost guaranteed to lead to logical errors; see BELOW.

In practice *sharṭī* in logic is usually translated today as 'hypothetical', in line with the Latin description used by Boethius.³

¹Compare the translations of these two categorical sentences into first-order logic as $\exists x(Bx \wedge Ax)$ and $\exists x(Bx \wedge \neg Ax)$; Ibn Sīnā anticipates the existential negative case at *Middle Summary in Logic* [114] 90.5 and *Syllogism* [124] v.3, 253.11f. What may be the same new existential *sharṭī* sentences appear also in *Epitome of the Principles of Logic* [42], though the published partial text of *Epitome of the Principles of Logic* makes it hard to be sure what meaning Ibn Sīnā attached to these sentences in this work.

²In *De Interpretatione* [123] 41.6f Ibn Sīnā says 'In [a hypothetical proposition] one sentence is made out of two sentences combined with each other, in a way that disqualifies either of them from being a declarative sentence.' He argues that in 'If ϕ then ψ ' the addition of 'If' prevents ϕ from being true. Here he is committing the wardrobe fallacy, see Frege [65] p. 157 and REF. I translate the quotation from *Easterners* in a way that downplays the fallacy.

³An alternative translation to 'hypothetical' is 'propositional', since al-Fārābī's examples are mostly formed by putting a propositional operator between the clauses. But like Ibn Sīnā's suggested definition of *sharṭī*, this overlooks the quantifiers in Ibn Sīnā's new sentences. (So my use of 'propositional' to translate *sharṭī* in some of my early papers on Ibn Sīnā's logic was a mistake, for which I apologise.)

6.1 Semantical structures and overloading

Ibn Sīnā's passage from categorical logic to wholly *muttaṣil* logic depends on his understanding of the semantics⁴ of categorical logic.

In *Middle Summary in Logic* Ibn Sīnā gives a brief but illuminating semantics for categorical sentences.

- (6.1) The quantifier (*ḥaṣr*) in predicative propositions determines how far the content (*ḥukm*) extends, i.e. whether it applies to each of the subject [individuals] (*kull wāḥid min al-mawḍūʿ*) or to some of them.

(*Middle Summary in Logic* [114] 88.15)

This sentence breaks down every material categorical sentence into three main components: the subject term, the content and the quantifier. The subject term describes a set or class, which we will call the 'subject class': the members of this class are what Ibn Sīnā describes as the 'subject individuals' (literally 'individual[s] of the subject'). Ibn Sīnā doesn't say how the subject term describes this class. But we can assume that they are the individuals that the subject word or phrase is true of; for example horses are what the subject word 'horse' is true of.

The content of the categorical sentence is what the sentence asserts of all or some of the subject individuals. For example the sentence 'Every horse is a quadruped' asserts 'is a quadruped' of every horse. The sentence 'No plant flies' asserts 'doesn't fly' of every plant. 'Some birds don't sing' asserts 'doesn't sing' of some birds. The content can be read off from the predicate and the presence or absence of negation.⁵

The main function of the quantifier is to tell us what individuals in the subject class the content is asserted of. The answer will be either 'each' or 'some'.

Ibn Sīnā states (6.1) for predicative sentences, not categorical ones. So (6.1) should also make sense for modalised categorical sentences; but in *Middle Summary in Logic* Ibn Sīnā omits to tell us how the modalised version of (6.1) goes. Sometime during the Gap period he became aware that he needed a systematic way of explaining the meanings of predicative sentences that are categorical but with alethic modes and other kinds of decoration added. So he began to make frequent use of a format devised by

⁴Semantics studies how meanings are assigned to text.

⁵It seems that the notion of content was introduced to logic by al-Fārābī, very possibly from the language of Islamic jurisprudence. See Chatti and Hodges [36] pp. 26, 28.

al-Fārābī for this purpose. It will be helpful for us to give a first taste of this format here. Further details can wait until Section 10.4 below.

Following al-Fārābī, Ibn Sīnā applies the format to universally quantified sentences. The examples that we know from quotations all conform to the following sketch.

Everything said to	be not be	actually necessarily possibly	a <i>B</i>
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is said to	be not be	actually necessarily possibly	an <i>A</i>
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For a reason to be explained in Section 10.4 below, we refer to this diagram as al-Fārābī's *dictum*. (Ignore the phrase 'said to be', which is a byproduct of Peripatetic understanding of meanings; Section 10.3 will explain this.)

Note that al-Fārābī's *dictum* allows a modal predicative sentence to have two modes, one for the subject and one for the predicate. Also it allows for two negations, one for the subject and one for the content.

Also the modes (both for the subject and for the predicate) are immediately next to the subject or predicate.⁶ This creates some dissonance in negative sentences. For example the sentence 'Necessarily no *B* is an *A*' expresses a necessity. But if we put 'necessarily' as a mode on the predicate in al-Fārābī's *dictum*, it falls within the scope of the content negation, so that the effect is to deny a necessity. The moral is that if we want to express a negative necessary sentence using al-Fārābī's *dictum*, we have to

⁶This is one definition of *de re* modality. However, Ibn Sīnā's version of al-Fārābī's *dictum* is more complicated, so I am not sure how helpful it is to speak of Ibn Sīnā's new modes as *de re*.

write the mode as ‘possibly’, using the fact that ‘not possibly’ is equivalent to ‘necessarily not’.⁷

In Chapter 10 we will describe some of the tools that Ibn Sīnā himself assembled during the Gap period in order to guide his new inventions in logic.⁸

It will help us later if we note here an extension of categorical logic which was introduced by Alexander of Aphrodisias in his *Commentary on Aristotle’s Prior Analytics Book 1*. He claimed ([7] 344.20–27, trans. based on Mueller [10] 28.21–27) that the following argument is syllogistic:

- A* is greater than *B*, which is greater than *C*.
 (6.2) Everything which is greater than what is greater than something
 is also greater than the something.
 Therefore *A* is also greater than *C*.

As they stand, the premises and conclusion of this syllogism are not predicative sentences, because they each have three subjects. But later scholars have read Alexander as suggesting that the three subjects should be read as a single ordered triple, so that the syllogism reads as follows:

- ABC* is a triple such that *A* is greater than *B* and *B* is greater than *C*.
 (6.3) Every triple of things where the first is greater than the second
 and the second is greater than the third is such that the first is
 greater than the third.
 Therefore *ABC* is a triple such that *A* is greater than *C*.

This forms a categorical syllogism in mood *Barbara* with first premise and conclusion singular. Note in particular that in the second premise Alexander has managed to pack three universal quantifiers ‘Every first thing ... every second thing ... every third thing’ into a single universal quantifier over triples. After Alexander this became a standard move for simplifying arguments with several quantifiers (all universal or all existential). Ibn Sīnā refers to it with other examples at *Middle Summary in Logic* [114] 92.16–18 and *Syllogism* [124] v.3, 256.11–15.

⁷In practice Aristotle understood this. For example at *De Interpretatione* i.10, 30b9f: ‘If the negative [premise] is necessary, so that no *B* is possibly an *A* ...’.

⁸The rest of this section contains things that Ibn Sīnā was apparently the first to discover, though he seems never to have understood the implied logical issues. (They require an understanding of free versus bound variables, which he lacked.) Since they confused him, they may very well confuse you too, at least at first reading. If they do, my advice is to come back to this material when it is needed, for example when reading Section 10.4.

The predicate of the second premise and conclusion of the syllogism (6.3) is true or false of triples. Today we would normally write this term using variables, say ℓ , m and n , for example as

(6.4) triple (ℓ, m, n) such that ℓ is greater than n .

The variables ℓ , m and n are called 'parameters' (or 'arguments', but in our context that might be confusing). The predicate (6.4) has three parameters, so it is described as 'three-ary' (or '3-ary') and said to have arity 3. A standard categorical predicate is true or false of single individuals, and hence is 'one-ary' or of arity 1.

Ibn Sīnā will consider terms that are true or false of an individual x at a time τ , for example

(6.5) person x who is alive at time τ .

This term has two parameters, x and τ , so it is 'two-ary' (or '2-ary' or 'binary') and said to have arity 2. A logic in which all the terms are 2-ary is said to be 'two-dimensional'.⁹

In both natural and formal languages it sometimes happens that one and the same symbol or word is used in two places, once as a 1-ary term and once as a 2-ary term. Both linguists and computer scientists sometimes describe this situation by saying that the term is 'overloaded'. There are several examples of overloading in *Middle Summary in Logic*. The first is the following hypothetical syllogism at [114] 151.7f:

Whenever AB , every C is a D .
 (6.6) Every D is an H .
 So whenever AB , every C is an H .

The second premise is (at least at face value) a straightforward categorical sentence, so that the term D in it is 1-ary. But the first premise has a quantification over times; it says that if τ is any time at which A is a B , then it is true at τ that every C is a D . This last clause unpacks to say that everything that is a C at τ is also a D at τ , so that both C and D are true or false of pairs where the second individual in the pair is a time. So we need to be able to

⁹This name was suggested by C. S. Peirce's student Oscar Mitchell [166] (1883), who designed a two-dimensional logic in which the second parameter of each term ranges over times. He used this logic to analyse inferences involving sentences such as 'Some of the Browns were at the sea-shore during the summer'. See [87] Sections 3 and 6 for references and context.

read C and D in the first premise as 2-ary. In this reading D is overloaded in this syllogism.

Logical text that *prima facie* contains overloaded terms needs careful handling if we want to do justice both to the laws of logic and to the author's intentions. For example one approach is to stipulate that occurrences of an overloaded term with different arities should be treated as occurrences of distinct terms with no implied logical relationship between them. But this will almost certainly not be fair to Ibn Sīnā's intentions with the syllogism (6.6), since the premises are certainly sterile if D in the first premise is read as a different term symbol from D in the second premise.¹⁰

An alternative approach is to recall Aristotle's advice at (5.20) to bring to the surface any implied requirements of similarity. In other words, one should assume that D in the second premise is in fact 2-dimensional, and add text to show how this occurrence of D depends on time. For example one could replace D in the second premise by 'thing that is always a D ' or 'thing that is sometimes a D ' or 'thing that is a D now' or 'thing that is a D while it exists'. Of these additions, the first and fourth would produce valid syllogisms, though the syllogism produced by the first would be useful only for discussing eternal individuals.

6.2 Wholly hypothetical syllogisms

In *Short Epitome in Logic* [112] 8.3–22 and [113] 7.17–8.14 (translated in Chapter 3 above) Ibn Sīnā introduces a new logic. Immediately after defining categorical logic, he continues:

Put an antecedent in place of the subject and a consequent in
(6.7) place of the predicate [in each sentence].
(*Short Epitome in Logic* [112] 8.4)

This is a little too brief. We will see below in (6.15) that the sentences of wholly *muttaṣil* logic each have two clauses; as in the second paragraph of Section 1.2 above, the first clause is called the 'antecedent' and the second is called the 'consequent'. So what Ibn Sīnā is telling us in (6.7) is that we should put sentences in place of the two terms in a categorical sentence, and as a result the first introduced sentence becomes an antecedent and the second becomes a consequent. If we are constructing a formal wholly

¹⁰[To avoid invalidity] one strives to ensure that the middle term doesn't apply in one of the two premises except in the same way that it applies to the other.
(*Short Epitome in Logic* [112] 12.20–13.1).

muttaṣil sentence rather than a material one, then Ibn Sīnā writes each of the sentences using two term letters, as in '*A* is a *B*'.

Here we meet a problem. In categorical logic a term letter can be interpreted by any noun or noun phrase. So if wholly *muttaṣil* logic is to be analogous to categorical logic, it should be possible to interpret '*A* is a *B*' by any sentence. But if we interpret *A* and *B* by nouns, we get an affirmative categorical sentence, not an arbitrary sentence.

In his account of wholly *muttaṣil* logic in *Syllogism* vi.1, Ibn Sīnā tells us

- (6.8) We say '*A* is a *B*', '*C* is a *D*' [etc.] to represent a predicative sentence in any of the eight forms.
(*Syllogism* [124] vi.1, 296.2)

(See (4.11) above for the eight forms of categorical sentence, which are presumably what Ibn Sīnā has in mind here.) Unfortunately this fails to resolve the problem, because (6.8) could mean either that the sentences that can be represented by '*A* is a *B*' are *exactly* the categorical sentences, or that they are *at least* the categorical sentences.

A passage in *Short Epitome in Logic*, if taken literally, settles the matter in favour of 'at least'. Here Ibn Sīnā gives an argument to derive the conclusion in the case of the wholly *muttaṣil* analogue of the mood *Baroco*. This syllogism is

- It is not the case that whenever *C* is an *I*, *H* is a *Z*;
and whenever *A* is a *B*, *H* is a *Z*.
(6.9) We say that this yields: It is not the case that whenever *C* is an *I*,
A is a *B*.
(*Short Epitome in Logic* [112] 8.17f)

After a plausible reconstruction of the text calling on both Badawī's edition

and Ülken's, Ibn Sīnā's demonstration of this syllogism runs as follows:

We stipulate that the situation in which C is an I and H is not a Z holds when G is a T , so we have:

It is never the case, when G is a T , that H is a Z ;
and whenever A is a B , H is a Z .
So it is never the case, when G is a T , that A is a B .

(6.10) Then we say:

Sometimes when C is an I , G is a T .
And it is never the case, when G is a T , that A is a B .
It yields that it is not the case that whenever C is an I
then A is a B .

(*Short Epitome in Logic*, Badawī [112] 8.19–22, Ülken [113] 53.4–8)

Here the sentence ' G is a T ' is required to be true just when C is an I and H is not a Z .²² But there is no reason to believe that this condition can be written as a categorical sentence. So we have to suppose that Ibn Sīnā allows ' G is a T ' to be a placeholder for whatever form of sentence the context requires.²³

Since these pairs of letters, as in ' A is a B ', are understood to be interpretable by sentences of any form, we will call them 'term letters of sentence type'. Though Ibn Sīnā himself doesn't contract these pairs to single letters, it would be harmless for us to replace the pairs by single letters, say P , Q etc., as sentence-type terms. For example, reading Ibn Sīnā literally, a material instance of

(6.11) Every P is a Q .

could be

(6.12) Every [^cAbdallah is speaking] is an [^cAmr is running].

In (6.12) we enclose the sentence matters in square brackets. The sentence '^cAbdallah is speaking' is the antecedent and the sentence '^cAmr is running' is the consequent.

6.3 'When'

Our immediate problem now is that the material sentence (6.12) makes little sense. Without further explanation we have no idea how to evaluate it

as true or false. But fortunately we have already seen where to look for an explanation, in Ibn Sīnā's understanding of what makes a material categorical sentence true or false, in Section 6.1 above. In the case where the subject term letter is of noun-type, the letter will be interpreted by a noun which describes the class of subject individuals, i.e. individuals which fit the description given by the noun. Analogously, when the subject term is of sentence-type, the matter which interprets it must characterise a class of individuals that fit the description given by the sentence.

In *Short Epitome in Logic* Ibn Sīnā doesn't tell us what these individuals could be. But he is more forthcoming in later writings, as in *Middle Summary in Logic*:

- (6.13) The quantifier (*ḥaṣr*) in predicative propositions indicates how far the content (*ḥukm*) extends, i.e. whether it applies to each of the subject individuals or to some of them. In the same way the quantifier in the *muttaṣil* [proposition] expresses how far the content ... applies, i.e. whether it applies to every case (*waḍʿ*) of the antecedent, or to some of its cases.
(*Middle Summary in Logic* [114] 88.15–19)

Ibn Sīnā is relaxed about the exact meaning of 'case' (*waḍʿ*^c, literally 'posit'). It can mean a proposed or imagined situation in which the antecedent is true. It can also mean a point or period of time during which the antecedent is true, as for example:

- (6.14) Thus when you say 'Whenever *A* is a *B*, *C* is a *D*.' it could be that it is at every time (*waqt*) in which *A* is a *B* and that *C* is a *D* during that time, but not all that time but in some of it ...; but it is also possible that *C* is a *D* for so long as *A* continues to be a *B*
(*Middle Summary in Logic* [114] 147.14–17)

We will refer to the individuals of which the antecedent is true as the 'cases' or 'situations' of the antecedent, remembering that these 'cases' can be times or periods of time.

Immediately after instructing us to write sentences in place of nouns in categorical sentences (which taken literally produces strange new forms as in (6.12)), Ibn Sīnā tells us how he himself writes the resulting sentences.

Thus in English translation, quoting *Short Epitome in Logic* [112] 8.8–11:
(6.15)

- (From ‘Every A is a B ’)
Whenever A is a B , C is a D .
- (From ‘No A is a B ’)
It is never the case, when A is a B , that C is a D .
- (From ‘Some A is a B ’)
Sometimes when A is a B , C is a D .
- (From ‘Some A is not a B ’)
Sometimes it is not the case, when A is a B , that C is a D .
Or: It is not the case that whenever A is a B , C is a D .

Note the role of the phrase ‘when (*idhā*) A is a B ’ in three of the sentences of (6.15). It serves to name a class, viz. the class of cases or situations in which ‘ A is a B ’ is true. Immediately before the ‘when’ there is a quantifier that quantifies over this class. In Arabic as in English, this quantifier has to be an adverb (‘Never’, ‘Sometimes’ etc. rather than ‘No’, ‘Some’ etc.). Implicitly this pattern applies also to the two sentences with ‘Whenever’ if we note that (in Arabic as in English) ‘whenever X ’ can be paraphrased as ‘Always when X ’.

Ibn Sīnā himself has no name for the four sentence forms in (6.15). But we will need a name for them, so I propose ‘SEL *muttaṣil*’, where SEL is short for *Short Epitome in Logic*. The same name can be used both for variants of (6.15) where Ibn Sīnā uses other formal sentences for the terms (for example as in ‘Sometimes when C is a D , E is an F ’), and for material instances of the sentences of (6.15).

The syllogisms of this logic in *Short Epitome in Logic* also need a name. For this I propose ‘wholly SEL *muttaṣil*’ syllogisms, or for brevity ‘wholly *muttaṣil*’ syllogisms, so that the logic is ‘wholly *muttaṣil*’ logic, as in Section 6.2 above.

In 1963 Nicholas Rescher [187] proposed a translation of the sentences

in (6.15), using A and C as sentence-type term letters:

	Whenever A, C	$\mapsto (t)(A_t \supset C_t)$
	It is never the case, when A ,	
	that C	$\mapsto (t)(A_t \supset \sim C_t)$
(6.16)	Sometimes when A, C	$\mapsto (\exists t)(A_t \& C_t)$
	Sometimes when A , it is not	
	the case that C	$\mapsto (\exists t)(A_t \& \sim C_t)$

These translations together with a standard first-order semantics don't support all of the fourteen formal hypothetical syllogisms of this logic. DARAPT.I. This is a problem that goes back to the origins of wholly *muttaṣil* logic in categorical logic. In the categorical case there are two solutions, both known in a natural language form to al-Fārābī and to Ibn Sīnā. The first is to limit the structures to those where the interpretations of all term letters are nonempty classes. The second is to use standard structures but read 'Every A is a C ' as being false in structures where the interpretation of A is empty. The first of these solutions, carried over to wholly *muttaṣil* logic, is to use only sentence terms that are true in some situation; the second is to read 'Whenever A is B , C is D ' as false if there is no situation in which ' A is B ' is true.

In two works that we will date roughly twenty years later than *Short Epitome in Logic*, namely *Middle Summary in Logic* [114] and the closely related *Twenty Questions* [115], Ibn Sīnā tells us that his Peripatetic predecessors after Aristotle never gave syllogistic moods whose consequences are hypothetical sentences.¹¹ He prides himself on having found a range of new hypothetical syllogisms that fill this gap. At first it seems he must be deceiving himself or us, because we have seen that Alexander described exactly such syllogisms.¹² But Ibn Sīnā's claim is bold and specific, and he is unlikely to have made it if it could be refuted at once from available texts. It's relevant that Al-Fārābī's *Syllogism* and *Short Syllogism* never mention wholly hypothetical syllogisms. Also the catalogue (*Fihrist*, [172] 405.21f) of

¹¹*Middle Summary in Logic* 145.18–146.9, 147.7–10, *Twenty Questions* 101.21–103.14; see CHECK.

¹²In his *Autobiography* Ibn Sīnā tells us that when he first began to teach himself logic he read 'the texts and the commentaries' (*al-kutub, al-shurūkh*), though as far as I know there is nothing in the part of *Short Epitome in Logic* corresponding to *Prior Analytics* that indicates knowledge of any commentaries beyond the logical texts of al-Fārābī. Ibn Sīnā will surely have seen Alexander's commentary a few years later when he examined the contents of the library of Nūḥ ibn Maṣṣūr; see [117] 22.6, 34–36.

the Baghdad bookseller Ibn Nadīm in 988 tells us that he has two copies of commentaries by Alexander on *Prior Analytics*, both covering ‘the categorical figures’ but one more complete than the other. These translations will have run no further than Alexander’s commentary on *Prior Analytics* i.22, while Alexander’s first reference to wholly hypothetical syllogisms is in his commentary on *Prior Analytics* i.23. These facts together make it likely that at least when he wrote *Middle Summary in Logic* and *Twenty Questions*, Ibn Sīnā had never seen any reference to wholly hypothetical syllogisms, making the account of them in *Short Epitome in Logic* a genuine innovation.¹³

6.4 The inference rules of wholly *muttaṣil* logic

Ibn Sīnā remarks in several places that the wholly *muttaṣil* syllogisms obey rules or conditions which are analogous to those obeyed by the categorical syllogisms. FIND REFS. These remarks should be read in two parts: (a) the rules used to prove entailments in categorical logic, as in Section 4.5 above, carry over to analogous rules for proving entailments in wholly *muttaṣil* logic, and (b) the same but for rules proving nonentailment, again as in Section 4.5 above. Taken together, these rules tell us for example that wholly *muttaṣil* logic has four valid syllogistic moods in first figure, and that it has no more than fourteen valid moods in the first three figures, namely the hypothetical analogues of the fourteen valid categorical moods. To the best of my knowledge this was the first time that any logician read off the valid entailments of one logic as the analogues of the valid entailments of another logic.

Though Ibn Sīnā claimed (a), he never proved it or examined what kinds of argument would be needed to establish it. The same is true of (b) in his early work. But in *Syllogism* vi.6 he made a serious attempt to prove the sterility of some premise-pairs in an extension of wholly *muttaṣil* logic by adapting Aristotle’s method for proving sterility in categorical logic. The result was only partially successful; I plan to say more about it in [99].

However, we can give some examples of (a) using the analogues of categorical rules that Aristotle seems to have regarded as perfect, assuming that the perfection carries over to wholly *muttaṣil* logic. For example categorical logic obeys conversion of affirmative existential sentences:

(6.17) From ‘Some *A* is a *B*’ it follows that ‘Some *B* is an *A*’.

¹³It seems to follow that Ibn Sīnā was unaware of the Peripatetic syllogisms that Maróth REF sees as ancestors of his wholly hypothetical syllogisms.

From the construction of affirmative existential SEL *muttaṣil* sentences we can infer

- (6.18) From 'Sometimes when A is a B , C is a D ' it follows that 'Sometimes when C is a D , A is a B '.

Or for another example, in categorical logic the negative existential sentence

- (6.19) Some A is not a B .

can be paraphrased as

- (6.20) Something is both an A and not a B .

So we can infer from

- (6.21) Sometimes it is not the case, when A is a B , that C is a D .

that

- (6.22) Sometimes it is the case that A is a B and not the case that C is a D .

These and similar examples are useful to us for checking the accuracy of our readings of SEL *muttaṣil* sentences.

Ibn Sīnā was not the only logician who proposed to create a new logic by replacing noun-type terms in categorical logic by sentence-type terms, and then using inference rules that are the analogues of the categorical rules. George Boole in 1854, after describing categorical logic as concerned with 'primary propositions', continues:

- (6.23) The relations of *things* we express by primary propositions. But we are able to make Propositions themselves also the subject of thought, and to express our judgments concerning them. The expression of any such judgment constitutes a secondary proposition. . . . The discussion of the theory of Secondary Propositions is . . . interesting, from the close and remarkable analogy which it bears with the theory of Primary Propositions. It will appear, that the formal laws to which the operations of the mind are subject, are identical in expression in both cases. . . . Instead of classes of things, we shall have to substitute propositions, and for the relations of classes and individuals, we shall have to consider the connexions of propositions or of events.
(Boole, *The Laws of Thought* [28] pp. 162–4.)

There is also evidence suggesting that both Ibn Sīnā and Boole came to make this same advance as a result of realising that the semantics of categorical logic should be phrased in terms of classes and the relationships of inclusion and exclusion between classes.

In *Short Epitome in Logic* B8.3–22 Ibn Sīnā introduces a new form of syllogism. Its distinctive feature is said to be that both the premises are hypothetical *muttaṣil* sentences. The first example given is:

- Whenever A is a B then C is a D ;
 (6.24) whenever C is a D then H is an R .
 It yields that whenever A is a B then H is an R .²⁸

As we see in this example, the conclusion is a hypothetical sentence too. So we can borrow a term from Alexander of Aphrodisias and describe these new syllogisms as ‘wholly hypothetical’—this distinguishes them from the exceptive syllogisms where only one premise is hypothetical and the conclusion is generally not hypothetical. Wholly hypothetical syllogisms exactly like those in *Short Epitome in Logic* reappear in Ibn Sīnā’s *Guidance* 96.1–6, 101.1–3, 104.9–11 and *Syllogism* vi.1. They are also mentioned briefly in his *Middle Summary in Logic* 148.6–15, *Najāt* 83.4–9 and *Iṣārāt* 157.5–9 though without any examples. In some later works Ibn Sīnā introduced wholly hypothetical syllogisms using both *muttaṣil* and *munfaṣil* sentences; we will not consider these syllogisms in the present paper.

WHERE?: A case like (6.30) above counts as hypothetical because it can be paraphrased as a disjunction; cf. Note 2.

The wholly *muttaṣil* logic sentence forms are much more distinct from the *muttaṣil* sentences of exceptive logic than Ibn Sīnā’s descriptions in *Short Epitome in Logic* suggest. The chart below lists the *muttaṣil* sentence forms of exceptive logic and wholly *muttaṣil* logic in *Short Epitome in Logic* together with the versions of exceptive logic and wholly *muttaṣil* logic found in Ibn Sīnā’s later writings *Guidance* and *Syllogism*; the numbers on the right are the numbers of occurrences in each of the three works. We use p and q to

label the clauses.

		<i>Short Epit</i>		<i>Guidance</i>		<i>Syllogism</i>	
		Exc	WH	Exc	WH	Exc	WH
(6.25)	1. <i>in p fa-q</i>	1		1		14	
	2. <i>idhā p fa-q</i>					2	1
	3. <i>kullamā p fa-q</i>		9		8		18
	4. <i>laysa albatta idhā p fa-q</i>		6		4		18
	5. <i>qad yakūnu idhā p fa-q</i>		3		1		12
	6. <i>laysa kullamā p fa-q</i>		4		1		11
	7. <i>qad lā yakūnu idhā p fa-q</i>		1				

As the chart shows, sentence form 2 occurs both in exceptive logic and in wholly *muttaṣil* logic, but apart from this, the two logic have no sentence forms in common. Sentence form 2 is most easily read as a variant of sentence form 1 in exceptive logic and of sentence form 3 in wholly *muttaṣil* logic. We will return to it shortly.

There is also a systematic difference that separates the sentences of wholly *muttaṣil* logic from those of exceptive logic. In wholly *muttaṣil* logic every sentence is quantified with respect to times. The affirmative universal time quantifier is *kullamā* 'whenever', the negative universal time quantifier is *laysa albatta* 'never', and the existential quantifier is *qad yakūnu* 'sometimes'. We should add that in Arabic as in many languages, expressions that are literally about points of time are widely used for intervals, events, situations, cases etc. This is not visible in *Short Epitome in Logic* because of its shortage of material examples; but the reference at *Short Epitome in Logic* 8.19 to a 'posit' rather than a time might be a clue that Ibn Sīnā is happy with interpretations that go beyond times. By contrast the one *muttaṣil* hypothetical form in exceptive logic has no quantification, with respect to time or anything else.

The quantifiers in sentences of HL2 reflect where these sentences came from. According to *Short Epitome in Logic* B8.3ff we construct HL2 as follows. We take a (quantified) categorical sentence and replace its two terms by clauses, treating the first as antecedent and the second as consequent. Ibn Sīnā doesn't spell out what happens to the quantifier of the categorical sentence, but we can reconstruct it thus. Just as a term 'horse' stands for the class of horses, so the expression 'when *p*' (where *p* is a clause) stands for the class of times (or situations) where *p* holds. The quantifier of the categorical sentence is replaced by the corresponding quantification over this class of times. Just as in English the phrase 'Always when *p*' can be short-

ened to ‘Whenever p ’, so in Arabic the phrase *dā’iman idhā p* (or any phrase meaning this) can be shortened to *kullamā p*; and similarly with ‘Never’. In this way the sentence forms 3, 4, 5, 6 of the chart above are derived from the categorical forms ‘Every B is an A ’, ‘No B is an A ’, ‘Some B is an A ’ and ‘Not every B is an A ’.

What do these seven sentence forms mean? Form 1 translates straightforwardly as ‘If p then q ’, and form 3 translates as ‘Whenever p , q ’.¹⁴ As Ibn Sīnā indicates and we can deduce from the *kull-* ‘every’ of *kullamā*, form 3 contains a universal quantification over times. There is no such quantification in form 1.

There is a further difference between the meanings of these two forms, which comes to light in the analogy that Ibn Sīnā makes between wholly *muttaṣil* logic syllogisms and categorical syllogisms. The proof of the categorical mood *Darapti* involves reasoning from ‘Every C is a B ’ to ‘Some B is a C ’; the reasoning requires that if ‘Every C is a B ’ is true then there is at least one C . This property of the sentence is known as ‘existential import’. The wholly *muttaṣil* logic analogue of this step in reasoning goes from form 3 to form 5, where *qad yakūnu* means ‘sometimes’ (*Short Epitome in Logic* 8.15f). So in wholly *muttaṣil* logic the analogue of existential import holds and tells us that if a sentence of the form 3 is true then there is a time at which p is true. By contrast the form 1 carries no implication that p is ever true. In fact form 1 is standardly used (for example in mathematics) when one wants to show that p is false by showing that p entails something absurd. In brief, form 3 in wholly *muttaṣil* logic carries an existential import but form 1 in exceptive logic does not.

We return briefly to form 2. Already in the eighth century Arabic linguists were discussing whether this form carries an existential import. According to Sībawayhi, al-Khalīl contrasted *in* ‘if’ and *idhā* ‘when’ by comparing the sentences ‘I will visit you when the grapes ripen’ and ‘I will visit you if the grapes ripen’; the second is inappropriate because we know that the grapes will ripen. So it seems that al-Khalīl would have detected an existential import in form 2 but not in form 1. On the other hand *idhā* is distinguished from *kullamā* by the fact that while *kullamā* covers every occasion on which p holds (or in some contexts every such occasion in the future), *idhā* can point just to the first next occasion when p holds; this seems to be

¹⁴ At *Middle Summary in Logic* 88.6 Ibn Sīnā describes *fa-* as the ‘particle of response’ that introduces the consequent in a conditional sentence. This is a convenience for translating Aristotle’s logic into Arabic, but it is not the normal use of *fa-* in Classical Arabic; SEE [70] REF.

the point that Ibn Sīnā is making at REF. All these three forms are sensitive to context, but we have noted ways in which form 2 resembles 1 and other ways in which it resembles 3. The very few occurrences of this form in *Syllogism* probably expect to be understood in appropriate contexts for exceptive logic and for wholly *muttaṣil* logic.

Forms 6 and 7 are understood as synonymous. They are the HL2 analogues of the categorical 'Not every *B* is an *A*' and 'Some *B* is not an *A*' respectively.

The phrase *idhā p* in the forms 4, 5 and 7 deserves a mention, because it lays itself open to misunderstanding. I make the points first in English, but we are looking at linguistic universals that apply equally to Arabic. The phrase 'when *p*' refers to a class of times, namely those when *p* is true. We can quantify over this class by saying 'At all times when *p*'; this phrase can be paraphrased as 'Whenever *p*'. To quantify existentially over the same class we can say 'Sometimes when *p*'. Now note that 'Sometimes when I wake in the middle of the night I feel thirsty' says that there are times at which I wake in the middle of the night *and* I feel thirsty. It can not be read as 'There are times at which, *if* I wake in the middle of the night *then* I feel thirsty', which is logically illiterate. Ibn Sīnā himself says 'and' – WHERE? ANSWER *Middle Summary* 90.5 and 90.6.¹⁵

Now we interpret the forms 4, 5 and 7. We saw above that form 5 expresses 'There is a time at which *p* and *q*'. In the form 4 the *laysa* is (as often) moved forwards from its logical position. In fact it negates *q*, and *albatta* serves as a universal quantifier over times, so that the sentence expresses 'At every time when *p*, it is not the case that *q*'. (Contrast form 6 where *laysa* is in its logical position, so that the sentence means 'It is not the case that at every time when *p*, *q*'.) In form 7 the *lā* again negates *q*, so that we have 'Sometimes when *p*, not *q*', or equivalently 'There are times when *p* and it is not the case that *q*'.

¹⁵Rescher [187] added to the confusion by reading the existential affirmative case as 'Sometimes: when *A*, then (also) *C*'. In his first-order formalisation of this sentence he uses & and not $\rightarrow$, but readers are tempted to read the English rather than the symbols. The linguistic background, with reference to related views of medieval Arabic linguists, is discussed in [70].

6.5 Exceptive syllogisms

Turning to the exceptive hypothetical sentences, this class of sentences was defined by examples. Typical examples were

- (6.26) (i) If it is dark then it is night.
(ii) It is either greater than or equal to or smaller than.

Ibn Sīnā also learned from al-Fārābī that hypothetical sentences are classified as either ‘connected’ (*muttaṣil*) or ‘separated’ (*munfaṣil*); (i) above is connected and (ii) is separated.

If you find it hard to see from these examples what the defining characteristic of ‘hypothetical’ is, then you are in good company. It was only towards the end of his life, in *Easterners*, that Ibn Sīnā attempted to give a definition of ‘hypothetical sentence’ that fits the standard examples. We will examine it in Section REF BELOW. Even in his early *Short Epitome in Logic* Ibn Sīnā introduces examples of ‘hypothetical’ sentences that pay no regard to al-Fārābī’s hopeless attempts at definition of *muttaṣil* and *munfaṣil*. For this reason I think it is justified, when dealing with Ibn Sīnā’s versions of hypothetical logic, to abandon the literal but logically misleading translations ‘connected’ and ‘separated’. Instead I translate *muttaṣil* as ‘meet-like’ and *munfaṣil* as ‘difference-like’. (NOTE history of these terms.)

For al-Fārābī the syllogisms of hypothetical logic have a first premise that is hypothetical and a second premise that is predicative; the conclusion is usually predicative. A typical example with a *muttaṣil* first premise is modus ponens:

- If this is even then it is the sum of two equal numbers;
(6.27) this is even.
Therefore this is the sum of two equal numbers.

An example with a *munfaṣil* first premise is:

- This is either hot or cold or middling;
(6.28) This is hot.
Therefore this is not either cold or middling.

Al-Fārābī describes syllogisms of these forms as ‘exceptive’ (*istithnā’ī*). Ibn Sīnā shows little interest in exceptive syllogisms. Instead he prefers, throughout his career, to work with new sentence forms that he claims are hypothetical and support new and non-exceptive forms of syllogism. (NOTE In any case Ibn Sīnā is distinctly evasive about whether this second syllogism should be regarded as logically valid. REF.)

After listing the categorical syllogisms, Aristotle moves on in *Prior Analytics* i.8–22 to a systematic listing of modalised syllogisms. Neither Al-Fārābī in his *Syllogism* and *Short Syllogism* nor Ibn Sīnā in *Short Epitome in Logic* include any study of modal syllogisms corresponding to this—though we will see in Subsection 7.3 what *Short Epitome in Logic* does with modal syllogisms instead.

In *Prior Analytics* i.23 Aristotle turns to arguments ‘from a hypothesis’, among which he includes *reductio ad absurdum*. In the corresponding place al-Fārābī's *Syllogism* and *Short Syllogism* both have sections on hypothetical syllogisms and *reductio ad absurdum*. After this both works jump to induction, which is found (together with related topics including analogy and enthymeme) near the end of *Prior Analytics* in sections ii.23ff. *Short Epitome in Logic* follows Al-Fārābī by putting hypothetical syllogisms next after the listing of categorical syllogisms, and then moving on to *reductio ad absurdum*. But whereas Al-Fārābī studies only the hypothetical syllogisms that he calls ‘exceptive’, Ibn Sīnā moves first to another kind of hypothetical syllogism which he invented himself. As an innovation this kind belongs in Section 6.2 above. But now we review the treatment of exceptive syllogisms and *reductio ad absurdum* in *Short Epitome in Logic*.

The account of exceptive (*istithnā'ī*) syllogisms in *Short Epitome in Logic* follows one or both of Al-Fārābī's *Syllogism* [59] 31.7–33.17 and *Shorter Summary in Logic* [60] 82.7–86.1, which are quite similar.¹⁶ The general idea of an exceptive syllogism is that the first premise, said to be ‘hypothetical’ (*sharṭī*), lists two or more possible cases, and the second premise, which is a single-clause categorical sentence, has the effect of eliminating one of the listed cases, so that the conclusion is built out of the remaining case or cases, perhaps with an added negation.¹⁷

A common kind of exceptive syllogism, given by both al-Fārābī and *Short Epitome in Logic*, is *modus ponens*, which al-Fārābī illustrates with the example

- If this visible thing is a human, then it is an animal.
 (6.29) But it is a human.
 It is therefore an animal.

¹⁶Some modern writings refer to a class of sentences called ‘assertoric’, usually without explanation. This might be intended as a translation of *wujūdī*, which is used in several different senses, for example meaning what the Latins called *de inesse ut nunc*. The description *wujūdī* never appears in *Short Epitome in Logic*.

¹⁷Exceptive syllogisms can be traced back to Classical Greece, but the name ‘exceptive’ was almost certainly taken by Al-Fārābī from a formal similarity to the Arabic grammatical construction known as ‘exception’ (*istithnā'*); cf. [36] p. 56f.

The hypothetical premise here takes the form ‘If p then q ’, which a tradition going back to Classical Greece called ‘connected’ (Arabic *muttaṣil*); p is the antecedent and q the consequent. Al-Fārābī’s typical *muttaṣil* sentence had the form ‘If (or when) this is an X then it is a Y ’, where X and Y are replaced by Arabic nouns; *Short Epitome in Logic* follows suit in its description of exceptive syllogisms.

Both *Short Epitome in Logic* and al-Fārābī also discussed exceptive syllogisms where the hypothetical sentence takes the form

(6.30) Either this is an X_1 or it is an X_n or ... or an X_n .

where n is typically either 2 or 3 but could be greater. Traditionally this kind of sentence was called ‘separated’ (Arabic *munfaṣil*), though the name hides some very different interpretations of the sentence. Al-Fārābī recognised two interpretations, which he called ‘complete’ and ‘incomplete’. In the complete interpretation the sentence counts as true when exactly one of the X_i is true of ‘this’, in other words when both the following conditions hold:

(6.31) (Exclusive) At most one of the X_i holds.
(Exhaustive) At least one of the X_i holds.

The incomplete interpretation is exclusive but not exhaustive. As an example of exclusive but not exhaustive, al-Fārābī gives ‘This colour is either white or black’.

By contrast Ibn Sīnā in *Short Epitome in Logic* recognises the complete interpretation (renaming it *ḥaqīqī* which I translate as ‘paradigmatic’ for reasons given in Section 12.4 below), and ignores the incomplete. Instead he introduces a ‘non-paradigmatic’ interpretation, which is exhaustive but not exclusive. An example is

(6.32) Either Abdullah is in the sea, or he is not drowning.

This sentence is intended to tell us that at least one of ‘in the sea’ and ‘not drowning’ is true of Abdullah. So we can form an exceptive syllogism with second premise ‘He is drowning’ and conclusion ‘He is in the sea’. But the sentence doesn’t rule out that Abdullah might be in the sea and swimming safely, so it is not exclusive. Exhaustive but not necessarily exclusive disjunctions are not new with Ibn Sīnā; Galen in his *Institutio Logica* [68] v.1 had considered them under the name *paradiezeugménos*, though Ibn Sīnā may not have known this.

Why did Ibn Sīnā switch the two conditions ‘exhaustive’ and ‘exclusive’ in his choice of hypothetical *munfaṣil* premises in exceptive syllogisms? He

gives no reasons, but two suggest themselves. The first is that he is interested in paraphrasing *muttaṣil* sentences as *munfaṣil* and vice versa. The *muttaṣil* sentence 'If p then q ' paraphrases as the *munfaṣil* sentence 'Either not p or q ', but only if this latter sentence is read as exhaustive rather than exclusive.¹⁵ The second is that he may have come to the conclusion that requiring exhaustiveness rather than exclusiveness gives a reading more faithful to the normal use of 'either ... or' sentences in Arabic. (FIND THE TEXT ILLUSTRATING THIS.)

At the end of the section on exceptive syllogisms, *Short Epitome in Logic* has a sentence claiming that a paradigmatic *munfaṣil* sentence is one that contains the expression 'without exception' (*lā yakhlū*)—in other words it is exhaustive. This confuses exhaustive with exclusive, and it is a mistake that Ibn Sīnā is unlikely to have made after putting himself to the trouble of rewriting exceptive syllogisms so as to switch 'exhaustive' and 'exclusive'. Most likely the sentence is a marginal gloss by a reader who either rejected Ibn Sīnā's switch or was simply confused. Even experienced logicians can get confused by the difference between 'exhaustive' and 'exclusive'.

In fact Ibn Sīnā himself seems to make this confusion early in *Short Epitome in Logic* (at B4.11 CHECK) when he repeats the traditional statement that *munfaṣil* sentences make a claim about a 'conflict' (i.e. an exclusion) between two clauses. But this passage is one of many where Ibn Sīnā states a traditional position and then contradicts it later in the same work. The phenomenon occurs so often that it deserves a name. I will call it the Finite Injury Principle, viz. that you can say something false provided you leave yourself time to correct it before the end of the work.¹⁶

Al-Fārābī uses material terms, not term letters, throughout his exposition of exceptive syllogisms.¹⁷ Ibn Sīnā does likewise, not only in *Short Epitome in Logic* but also in his later expositions in *Middle Summary in Logic* and *Syllogism*. This is odd, given Ibn Sīnā's commitment to formal logic almost everywhere else. But Ibn Sīnā never really integrated exceptive syllogisms with his other logical systems, and never developed them beyond the account given in *Short Epitome in Logic*.

After listing exceptive syllogisms, *Short Epitome in Logic* B10.8–12 turns briefly to proof by reductio ad absurdum (*qiyās al-khālf*). Write $\bar{p}$ for the contradictory negation of the proposition p . Suppose we have a valid syllogism ' p, q (in some order); therefore r ', where q is true and r is 'clearly impossible'. Then we can deduce $\bar{p}$ from the true premises q and $\bar{r}$. This is clumsily phrased; for the deduction of $\bar{p}$ to be valid with true premises we need only that r is false, not that it is clearly impossible. By writing it

as he does, Ibn Sīnā commits the logical error that if p, q together entail a clear impossibility then one of p and q already entails a clear impossibility. Comparison with al-Fārābī's texts reveals that *Syllogism* commits exactly this logical error but *Short Syllogism* corrects it by requiring only that r is false. We infer that at this point Ibn Sīnā's source was *Syllogism* rather than *Short Syllogism*; in fact he presumably hadn't looked at the more correct account in *Short Syllogism*.

Ibn Sīnā's account does differ from al-Fārābī's *Syllogism* by leaving out al-Fārābī's dialectical setting which involves concessions made by a partner in a debate. This is a broad difference between Ibn Sīnā and al-Fārābī which continues into Ibn Sīnā's later writings on logic. Ibn Sīnā never bases a logical thesis on an appeal to dialogue.

SECTION ON 'the name hypothetical and the overcoat fallacy' HAS BEEN MOVED INTO THE HYPOTHETICAL STREAM.

Ibn Sīnā's discussion of exceptive moods contains nothing that is not either in the account of such syllogisms in al-Fārābī [59] 31.7–33.17, or a side-effect of Ibn Sīnā's own invention of the wholly hypothetical syllogisms. Let us begin with what he could have taken from al-Fārābī.

Exceptive moods have two premises, of which the first is a hypothetical sentence and the second is a categorical sentence. There are two kinds of hypothetical sentence, and both al-Fārābī and Ibn Sīnā refer to these two kinds as *muttaṣil* and *munfaṣil*. In the case of al-Fārābī, accepted translations of these two terms are *muttaṣil* 'connected', *munfaṣil* 'separated'. But Ibn Sīnā at *Short Epitome in Logic* implies something that al-Fārābī could hardly have accepted, namely that both the class of *muttaṣil* sentences and the class of *munfaṣil* sentences are closed under contradictory negation. If a sentence expresses that some things are connected, its contradictory negation will express that some things are *not* connected, and similarly with 'separated'. So Ibn Sīnā's generalisation makes it *prima facie* absurd to stay with the accepted translations. In working with Ibn Sīnā's hypothetical logic I translate *muttaṣil* as 'meet-like' and *munfaṣil* as 'difference-like'; see REF for some justification of these names.

For Al-Fārābī the typical *muttaṣil* sentence has the form 'If it is an A then it is a B ', where A and B are Arabic nouns, and 'it' can be replaced at both occurrences by 'this' or 'the number' or some other expression referring to an individual. The clause 'It is an A ' is called the antecedent and the clause 'it is a B ' is called the consequent. Both al-Fārābī and Ibn Sīnā recognise two kinds of exceptive syllogism, as illustrated by the following examples

which both authors give.

- If it is a human then it is an animal.
(6.33) It is a human.
Therefore it is an animal.

- If it is a human then it is an animal.
(6.34) It is not an animal.
Therefore it is not a human.

For al-Fārābī the typical *munfaṣil* sentence has the form 'It is either an *A* or ... or a *C*', where two or more incompatible alternatives are listed. The sentence is called complete if the listed alternatives include all possibilities (as with 'This water is either hot or cold or middling') and incomplete otherwise. For complete *munfaṣil* sentences we have the two exceptive moods, as in

- This water is either hot or cold or middling.
(6.35) It is hot.
Therefore it is not cold and not middling

- This water is either hot or cold or middling.
(6.36) It is not hot.
Therefore it is either cold or middling.

(The second premise could mention either 'cold' or 'middling' in place of 'hot', with corresponding changes in the conclusion.) When the *munfaṣil* premise is not complete then we have only the first of these two moods.

For all kinds of exceptive mood, the second premise is either one of the clauses of the first premise, or the contradictory negation of one of those clauses. The second premise is said to 'except' this clause; hence the name 'exceptive'.

Ibn Sīnā's account in *Short Epitome in Logic* disagrees with al-Fārābī on the incomplete *munfaṣil* sentences. At [112] 110.5f he defines the strict *munfaṣil* sentences to be those where the alternatives are listed exhaustively, i.e. those that al-Fārābī calls 'complete'. But his examples of non-paradigmatic at [112] 9.18–10.4 don't fit this definition. In both examples the alternatives in the first premise are exhaustive, but they are not incompatible. In other words they are examples of what today we call inclusive disjunctions. In his later accounts of exceptive moods Ibn Sīnā will smooth out the inconsistencies and make it clear that non-paradigmatic *munfaṣil* sentences are inclusive disjunctions; he will find it convenient that '*p* or *q*'

(inclusive disjunction) is equivalent to ‘if not- p then q ’. But there is no mention of this point in *Short Epitome in Logic*. It would be welcome if we found some evidence of how Ibn Sīnā came to write what he did about non-paradigmatic *munfaṣil* sentences in *Short Epitome in Logic*.

In his listing of the categorical syllogistic moods, Aristotle never gives examples with material (natural language) terms; instead he always uses term letters. Al-Fārābī mentions this point in his *Syllogism* [59], 22.4–23.4, adding that Aristotle did this ‘because he wanted to avoid people thinking that [the consequences] that followed from the [resulting] compositions [followed] because of the matters that the expressions signified’; in other words, Aristotle was making the point that his syllogistic logic was formal. So it is interesting that in *Syllogism* al-Fārābī gives several listings of categorical moods where he uses material examples and no term letters; in fact in his later revision *Short Syllogism* he uses no term letters at all. By contrast *Short Epitome in Logic* follows Aristotle and uses only term letters in his listing of categorical syllogisms. Can we infer that Ibn Sīnā, like Aristotle, wanted to emphasise the formality of formal logic, in contrast to al-Fārābī who had less interest in formality?

Chapter 7

Modes

Short Epitome in Logic offers us two passages on modes. The first is at [112] 5.5f and claims to explain what a mode is.

The second passage, at [112] 6.8–8.1, consists of comments on the categorical syllogisms, giving information about when modes can be attached to premises and/or conclusions of these syllogisms. The effect is to create a list of modal syllogisms that forms a variant of Aristotle’s *Prior Analytics* i. 9–22, with some rough edges. This list is clearly important for Ibn Sīnā, since it poses problems that he is still working on when he writes *Syllogism*. He makes one final advance on these problems in *Pointers and Reminders*, as we will see BELOW.

7.1 Modal sentences before Ibn Sīnā

We saw that in *Prior Analytics* i.4–6 Aristotle lists forty-eight formal categorical premise-pairs, and for each of these premises he tells us whether it is productive, and if it is productive, what its conclusion is. In *Prior Analytics* i.8–22 he does something similar for a different collection of premise-pairs, namely the fourteen productive categorical premise-pairs but with a label attached to at least one of the premises.

At *Prior Analytics* i.2, 25a1f and again at the beginning of *Prior Analytics* i.8 ((29b29f) Aristotle tells us that there are three kinds of premise in the syllogisms of *Prior Analytics* i.8–22. These three kinds are called ‘holding’ (Greek *hupárkhein*), ‘holding with necessity’ (Greek *ex anánkēs hupárkhein*) and ‘holding with possibility/contingency’ (Greek *endékhesthai hupárkhein*). The verb *hupárkhei* (with infinitive *hupárkhein*) means ‘holds’ as in ‘the predicate holds of the subject’. This verb appears in the descriptions of all

three kinds, but in the second and third kinds there are other labels referring to necessity and possibility (or contingency). Aristotle gives no name that covers both these labels. But by around AD200 we find Alexander of Aphrodisias describing these two labels as 'modes' (Greek *trópos*), and using Aristotle's word 'holds' (*hupárkhei*) as a third label that counts as being applied when neither of the other two labels is attached.

Alexander's language is reflected in Theodorus's translation of *Prior Analytics* in the Arabic Aristotle:

... the absolute (*muṭlaq*), the necessary (*iḍṭirārī*) and the possible/contingent (*mumkin*) sentences differ, one from another
(7.1) ... ([139] i.8, 29b29f)

Theodorus uses the Arabic *jīha* to translate 'mode', which is not a bad translation. He uses the Arabic *muṭlaq* for 'holds', and this is not really a translation at all; *muṭlaq* generally means 'unrestricted', so Theodorus is using it to express that the first kind of premise is not restricted to being necessary or possible (or contingent).

Aristotle uses one of his two labels in two different senses, namely 'possible' and 'contingent'. These are alternative translations of *endékhesthai*; Aristotle also uses *dunatós* for 'possible' and for 'contingent'. A sentence ϕ is contingent if both ϕ and not- ϕ are possible; so 'contingent' is sometimes described as 'two-sided possible'; 'one-sided possible' means simply 'possible'. The fact that Aristotle used the same word for 'possible' and for 'contingent' suggests that he himself was not always clear about which of the two notions he meant.

In spite of his ambiguities, it's convenient to think of Aristotle as having four modes, namely absolute, necessary, possible and contingent. I follow custom in calling these the 'alethic' modes, without implying anything precise about their meanings. We will use four shorthands for the alethic modes, as follows:

ABS for 'absolute';

NEC for 'necessary', 'necessarily', 'with necessity';

POS for 'possible', 'possibly', 'with possibility';

CON for 'contingent', 'contingently', 'with contingency'.

We will think of 'necessary', 'necessarily' and 'with necessity' as the same

mode but applied in different ways; likewise with ‘possible’ etc. and ‘contingent’ etc.¹

As for the different ways of applying a mode: we can distinguish two main ways that Aristotle used. In the first way he would discuss the premises and conclusions of syllogisms, with phrases like

(7.2) Let every *C* be possibly a *B*.

Since he is discussing a syllogism here, this must be shorthand for

(7.3) Let it be assumed or posited that every *C* is possibly a *B*.

So the posited premise is the categorical sentence ‘Every *C* is a *B*’ but with the mode ‘possibly’ added to it. We say that a predicative sentence got by adding a mode to a categorical sentence is a ‘modal sentence’.

In his second usage Aristotle uses modes as classifiers, or more precisely as classificatory descriptions of predicative sentences. For example he says

(7.4) Let the negative premise be necessary.

Here ‘necessary’ describes a class of modal sentences; it is not being used as a word inside a modal sentence.

One might assume (as some writers have done) that Aristotle classified a modal sentence as ‘necessary’ when it contains the word ‘necessary’, and likewise as ‘possible’ if it contains the word ‘possible’, so that his two usages contain the same information. But this doesn’t work, for more than one reason. In the first place consider the two modal sentences

(7.5) Some *B* is not necessarily an *A*.

Sentence (7.5) is equivalent to (i.e. means the same as)

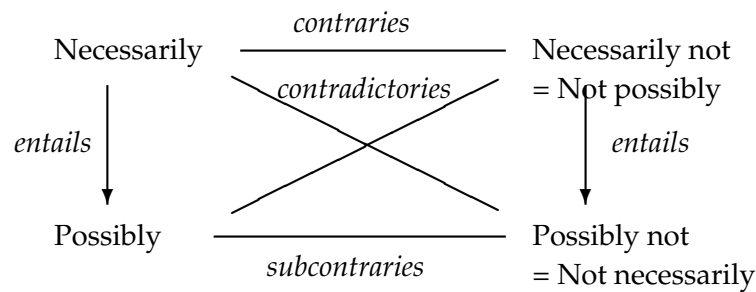
(7.6) Some *B* is possibly not an *A*.

Since these two sentences are equivalent, they should have the same modal classification. But should it be ‘necessary’ as in (7.5) or ‘possible’ as in (7.6)?

¹Theodorus in his Arabic translation of *Prior Analytics* uses the word *mumkin* both for ‘possible’ and for ‘contingent’, copying the ambiguity of Aristotle’s text. As alternatives to *mumkin* he also uses the verb ‘it is possible’ (*jāza* or *qad yajūzu* or *qad yumkinu*), or ‘it is able to’ (*yastata‘u*), or the participle *jā’iz*, or the adverbial phrase *bi-l-imkān*. Again these phrases can be found at any of several places in the sentence. Ibn Sīnā in *Short Epitome in Logic* uses *mumkin* but says nothing to distinguish one-sided possibility from two-sided possibility; by the time of *Twenty Questions* he will usually distinguish them, as in Section 11.2 below.

Aristotle himself was aware that bringing together negations and modes raised some problems of interpretation, with implications for logic. So he devoted two sections (xii and xiii) of his *De Interpretatione* to ways of resolving the ambiguities created by the presence of both a negation and a mode. These two sections spawned various discussions by commentators, including Alexander of Aphrodisias, Themistius and Ammonius Hermiou in Roman Empire times.²

Perhaps Aristotle's clearest contribution in these sections is a sort of Modal Square of Opposition along the following lines:³



(7.7)

This diagram is from Aristotle *De Interpretatione* xiii, 22a23–31, slightly rearranged. Aristotle writes ‘to be’ after each of the phrases in the diagram, presumably to indicate that the negations and modes are read as qualifying the copula of a sentence. In the English phrase ‘not possible not to be’, each negation or mode X qualifies the phrase P that comes immediately after it (in other words, P is within the ‘scope’ of X). So for example the first ‘not’ negates ‘possible not to be’, the word ‘possible’ qualifies ‘not to be’, and the second ‘not’ negates ‘to be’. Fortunately similar scope rules tend to apply in all of Greek, Arabic and English.⁴

We turn to a second reason why a necessary sentence doesn’t have to be the same thing as a sentence containing the mode ‘necessary’. Alexander of Aphrodisias in his essay *On conversion* [14] draws a distinction between two ways in which a mode can be attached to a categorical sentence so as to qualify the predicate. In the first way, the mode is added to the sentence

²The choice of language is relevant. These commentators wrote in Greek. Modern discussions of the same material in English are in Ackrill [2] p. 149, Hintikka and Moravcsik [82] and Whittaker [221] chapters 12, 13.

³For the meanings of ‘contradictory’, ‘contrary’ and ‘subcontrary’ see BELOW.

⁴But note that in Arabic *laysa ba’d* (‘not some’) tends to be read as meaning ‘not all’, i.e. ‘some not’, as Ibn Sīnā points out at *Syllogism* [124] i.4, 37.10f.

alongside the predicate; we say that the mode is ‘added to’ (*mazīd ‘alā*) the predicate. In the second way, the mode counts as being a ‘part’ (*juz’*) of the predicate; we will say that the mode is ‘incorporated’ into the predicate. These two ways of attaching the mode can lead to syntactically identical sentences, but there is a difference in how they are parsed. In the first case the predicate is the same word or phrase as it was before the attachment, but in the second case the predicate is taken to be the original predicate together with the mode.

In both cases the resulting sentence is predicative. But in the second case and not the first case, the resulting sentence is categorical, because the added mode is absorbed into the predicate. Hence in the second case the resulting sentence can have operations of categorical syllogisms applied to it. Alexander was well aware that this fact allows categorical conversions (cf. Section 4.5 above), but there is no sign that he applied it to derive modal syllogisms. (That would come later with Ibn Sīnā; see Section 15.1 below.) The editors of [14] make a case (on their page 264) that Alexander was more interested in arguing that a mode incorporated in the predicate is no longer a mode, so that for example if the incorporated mode is ‘necessarily’ then we can’t classify the sentence as a ‘necessary’ modal sentence.

Alexander also points out that if we incorporate a mode into the predicate of an existential affirmative categorical sentence, for example to get

(7.8) Some human possibly-runs.

then *i*-conversion yields

(7.9) Something that possibly-runs is a human.

Sentence (7.9) can’t be classified as a possible sentence, even if we count the ‘possibly’ as a mode, because the mode qualifies the subject and not the predicate.

We will put the notions of this section to practical use. Let us say that a modal sentence is in ‘modal normal form’ if its mode qualifies its predicate and is not qualified by a negation. For example sentence (7.5) above is not in modal normal form but (7.6) is in modal normal form. *If a modal sentence is in modal normal form, then we use the mode of the sentence as its classification.* We use the same classification for any sentence equivalent to the modal sentence. For example we classify (7.6) as a possible sentence; and since sentence (7.5) is equivalent to (7.6), we count (7.5) as a possible sentence too, not a necessary one. Or consider

(7.10) Nobody possibly lives for ever.

This is equivalent to

(7.11) It is true of everybody that they necessarily don't live for ever.

which is in modal normal form (though rather heavy in English) and hence is a necessary sentence. So the equivalent sentence (7.10) is also classified as necessary. I believe that this definition agrees with the modal classifications that Aristotle gives in *Prior Analytics* i.9–22.

7.2 Naming modal sentences and modal syllogisms

By a 'modal syllogism' we mean a syllogism in which at least one of the premises is classified by a mode other than ABS. In Section 7.3 we will extract from *Short Epitome in Logic* [112] 6.8–8.1 a list of modal syllogisms accepted by Ibn Sīnā. Ibn Sīnā's way of identifying these syllogisms is indirect, using dominance. We want a direct way of describing them, with as little ambiguity as possible.

Ibn Sīnā assumes in *Short Epitome in Logic* that each modal syllogism is the result of attaching modes to premises and conclusion of a categorical syllogism. We call this categorical syllogism the 'underlying categorical syllogism', and we will give it its Latin name (for instance *Barbara*). The underlying categorical syllogism is a first step towards identifying the modal syllogism.

The only modes that Ibn Sīnā mentions in *Short Epitome in Logic* are the alethic modes ABS, NEC and a conflation of POS and CON.⁵ He runs through a list of all the formal categorical moods. (Since each productive categorical premise-pair has a unique conclusion, we could also describe Ibn Sīnā as listing the formal productive categorical premise-pairs and labelling them to find productive modal premise-pairs.) For each mood and each assignment of alethic modes from the list ABS, NEC, POS/CON to the premises of the mood, he gives information about what mode can be attached to the conclusion. The information that he gives in *Short Epitome in Logic* doesn't distinguish between ABS and POS/CON, but it does tell us when the conclusion is classified as NEC.

⁵*mumkin* in Ibn Sīnā can mean either possible or contingent; cf. Section 7.1 on Aristotle's blurring of this distinction. I have no general criterion for telling whether Ibn Sīnā means possible or contingent at any particular occurrence of *mumkin*. Baeck [15] p. 25 reports that Ibn Sīnā 'tends not to use the one-sided possible'. Chatti [34] 258, after studying many examples, concludes that Ibn Sīnā 'uses very rarely the bilateral possible proposition in his valid moods, which contain mostly universal possible premises and conclusions'.

This leaves some ambiguity about when we should take the conclusion to be classified as ABS and when as POS/CON. But fortunately there is a sensible way of resolving the ambiguity when one of the premises is classified as ABS and the other as NEC. This is to invoke Ibn Sīnā's use of the metaphor that premises in a syllogism are parents of the conclusion, so that properties of the conclusion are inherited from the parents in accordance with dominance. In some cases with premises ABS and NEC, the dominance indicates that the conclusion is NEC, and there is no ambiguity. But if the dominance tells us that the conclusion is not NEC, then by the metaphor it has to be either ABS or NEC, so it must be ABS.⁶ (In a few cases Ibn Sīnā states the dominance but then contradicts it, as described in Section 2.2 above. In Section 7.3 we will start to make sense of this in the context of the Modal Challenge.)

In short Ibn Sīnā says enough to show us, for each of the fourteen quantified categorical moods and each assignment of ABS to one premise and NEC to the other, which of ABS and NEC is assigned to the conclusion. Section 7.3 will give the resulting list at (7.23). We name each modal syllogism in the list by naming the underlying categorical mood, and then in parentheses the modes classifying the minor premise, the major premise and the conclusion.

For example suppose we have a modally decorated syllogism

- Absolutely, no *C* is a *B*.
 (7.12) Necessarily, every *A* is a *B*.
 Therefore necessarily, no *C* is an *A*.

This syllogism adds labels to the categorical syllogism *Camestres*:

- No *C* is a *B*.
 (7.13) Every *A* is a *B*.
 Therefore no *C* is an *A*.

The labels added to the first and second premise and the conclusion are 'absolutely', 'necessarily' and 'necessarily' respectively, all of them outside the scope of negations; abbreviated these labels are ABS, NEC and NEC. Then we can identify the modal syllogism (7.12) as

- (7.14) *Camestres*(ABS,NEC,NEC).

Likewise the premise-pair of (7.12) can be written

- (7.15) *Camestres*(ABS,NEC).

⁶This is the hunch described in Section 5.3 above.

Ibn Sīnā himself never used the notation described above. But all the information that this notation contains about a modal syllogism could have been reached in a reasonable way by a mature enough reader of *Short Epitome in Logic* [112] 6.8–8.1. For example the notation *Celarent*(ABS,NEC,NEC) describes a modal syllogism with underlying categorical syllogism

- Every *C* is a *B*.
 (7.16) No *B* is an *A*.
 Therefore no *C* is an *A*.

and in the modal syllogism the sentences are classified in turn as absolute, necessary and necessary. These three modes can be added to the appropriate categorical sentences in any way that makes the resulting modal sentences have modal normal form. In the second premise and the conclusion the negation is in the first word of the sentence, so the safest place to put the mode 'necessary' is before the word 'no'. There are no constraints on where to put 'absolute'. Thus, with adjustments to 'necessary' and 'absolute' to fit the grammar, we can read *Celarent*(ABS,NEC,NEC) as the following modal syllogism:

- Every *C* is absolutely a *B*.
 (7.17) Necessarily no *B* is an *A*.
 Therefore necessarily no *C* is an *A*.

WARNING: A similar notation is in use for naming modal syllogisms and modal premise-pairs in the western Scholastic tradition. But in the Latin West it's normal to write the major premise before the minor premise, and hence to name the classifying mode of the major premise before that of the minor premise. In this book we put the minor premise first in agreement with Ibn Sīnā's normal usage, which is clear already in *Short Epitome in Logic*.

Could there be a valid modal syllogism which becomes an invalid categorical syllogism when we remove the modes? A valid modal syllogism with this property will not appear in Ibn Sīnā's list, since Ibn Sīnā considers only the modal syllogisms got by attaching modes to valid categorical syllogisms. Similarly, is there a productive modal premise-pair that becomes sterile when the modes are removed? If there is, Ibn Sīnā will miss it.

The logical methods available to Ibn Sīnā when he wrote *Short Epitome in Logic* were certainly not adequate to answer these questions. But it would have been open to him to say that if there were any rogue modal syllogisms or rogue modal premise-pairs of this kind, then logicians would

surely have come across one. This is similar to an argument used today for justifying the assumption that Peano Arithmetic is consistent.⁷

There are no examples of modal sentences written out in *Short Epitome in Logic*. But there are plenty in *Twenty Questions*, including some forms of modal sentences that Ibn Sīnā himself is introducing into syllogistic logic. Among the examples in *Twenty Questions*, Ibn Sīnā gives some affirmative modal sentences with the mode at the end and some with it in the middle:

(7.18) Every *C* is a *B* with necessity.
([115] vi, 87.3)

(7.19) Some white thing is with necessity an animal.
([115] ii, 83.20)

He gives some modal sentences with the mode at the beginning, but these are exactly the modal sentences that are negative:

(7.20) With necessity not every *C* is a *B*.
([115] xiv, 98.14)

(7.21) With necessity some *C* is not an *A*
and every *B* is an *A* with necessity.
So it follows that with necessity *C* is not *B*.⁸
([115] vi, 87.6f.)

In *Twenty Questions* Ibn Sīnā gives an example of a syllogism whose major premise is ‘Nothing that can reason is a crow’, and comments

(7.22) It is known that the major premise is necessary, because denying of ‘crow’ to anything that in any way fits the description that ‘it reasons’ is a necessary denial.
(*Twenty Questions* [115] xi, 95.2f.)

From this comment it seems that the necessity that Ibn Sīnā is referring to is the necessity that certain individuals are not crows; it doesn’t involve the mode of possibility expressed by the word ‘can’. But note that the major premise itself doesn’t mention any necessity. This illustrates a way in

⁷Chapter 12 in [98] (Corollary 12.2.4 in the current revision) has more to say about such rogue syllogisms from the point of view of today’s logic.

⁸The first premise is *bi-l-darūrati laysa ba‘du j a*, illustrating how *laysa ba‘d* ‘not some’ often means ‘some not’ or ‘not all’, as in Footnote 4 to this chapter.

which sentences can be characterised as 'necessary' not because they state that something is necessary, but because the sentences are true and would remain true if 'necessarily' was added to them. Likewise with modes of possible and contingent. In all such cases I will say that the sentences are 'self-classifying'. We will return to self-classifying sentences in BELOW.

When a quantified negative sentence is written with a mode at the beginning, as (7.20) and (7.21), there is an uncertainty about whether the mode qualifies the quantifier. Passages like (7.22) can be used as evidence that Ibn Sīnā intends the mode to qualify what follows the quantifier, i.e. that certain individuals don't have a certain property. Likewise when a quantified affirmative sentence is written with a mode at the end, as (7.18), nothing in the sentence indicates whether the mode qualifies the quantifier.

The examples above from *Twenty Questions* illustrate that Ibn Sīnā is moving into questions about the interpretation of modal (or self-classifying) sentences. We have very little evidence from the Gap period about the methods that Ibn Sīnā used for answering such questions. But there is some evidence in the form of what Ibn Sīnā calls the *dictum de omni*. We will come back to this in Section 7.4 below, after assembling a list of modal syllogisms that Ibn Sīnā considers valid in *Short Epitome in Logic*.

7.3 The *Short Epitome* list of modal syllogisms

The previous section has given us a notation for identifying modal syllogisms that involve the labels ABS, NEC, POS and CON. In this section we will use this notation to list those modal syllogisms that have one premise labelled ABS and one premise labelled NEC, and that are valid according to the listing of syllogisms in *Short Epitome in Logic* [112] 6.8–8.1. Our chief tool is the information that this passage of *Short Epitome in Logic* gives us on the dominance for necessity; see Section 5.3 on dominance.

Aristotle doesn't assume that every categorical syllogism can be turned into a valid modal syllogism by adding labels that include POS or CON; for safety we shouldn't ascribe this assumption to Ibn Sīnā either, at least at the stage of his development represented by *Short Epitome in Logic*. For this reason there is some uncertainty about what modal syllogisms Ibn Sīnā in *Short Epitome in Logic* counts as valid when one of the premises carries a label POS or CON. We will come back to modal syllogisms with POS or CON at BELOW.

We can now list the twenty-four modal syllogisms with one premise

ABS and the other premise NEC that *Short Epitome in Logic* claims are valid, as follows. The column ‘Aristotle’ reports the conclusion labels of the syllogisms that Aristotle himself counts as valid in *Prior Analytics*, and the column ‘Ibn Sīnā 1’ lists those that *Short Epitome in Logic* endorses as valid, except for the four where *Short Epitome in Logic* gives two opinions on the labelling of the conclusion. For these four the columns ‘Ibn Sīnā 1’ and ‘Ibn Sīnā 2’ report Ibn Sīnā’s first and second opinions respectively. In Section 2.2 we saw a reason for assuming that in the cases where Ibn Sīnā gave two opinions, the second opinion records his considered belief, even at this early stage in his logical development.

#	Premise-Pair	Aristotle	Ibn Sīnā 1	Ibn Sīnā 2
1	<i>Barbara</i> (ABS,NEC)	NEC	NEC	
2	<i>Barbara</i> (NEC,ABS)	ABS	ABS	
3	<i>Celarent</i> (ABS,NEC)	NEC	NEC	
4	<i>Celarent</i> (NEC,ABS)	ABS	ABS	
5	<i>Darii</i> (ABS,NEC)	NEC	NEC	
6	<i>Darii</i> (NEC,ABS)	ABS	ABS	
7	<i>Ferio</i> (ABS,NEC)	NEC	NEC	
8	<i>Ferio</i> (NEC,ABS)	ABS	ABS	
9	<i>Cesare</i> (ABS,NEC)	NEC	NEC	
10	<i>Cesare</i> (NEC,ABS)	ABS	ABS	NEC
11	<i>Camestres</i> (ABS,NEC)	ABS	ABS	NEC
12	<i>Camestres</i> (NEC,ABS)	NEC	NEC	
13	<i>Festino</i> (ABS,NEC)	NEC	NEC	
(7.23) 14	<i>Festino</i> (NEC,ABS)	ABS	ABS	NEC
15	<i>Baroco</i> (ABS,NEC)	ABS	ABS	NEC
16	<i>Baroco</i> (NEC,ABS)	ABS	NEC	
17	<i>Darapti</i> (ABS,NEC)	NEC	NEC	
18	<i>Darapti</i> (NEC,ABS)	NEC	ABS	
19	<i>Felapton</i> (ABS,NEC)	NEC	NEC	
20	<i>Felapton</i> (NEC,ABS)	ABS	ABS	
21	<i>Datisi</i> (ABS,NEC)	NEC	NEC	
22	<i>Datisi</i> (NEC,ABS)	ABS	ABS	
23	<i>Disamis</i> (ABS,NEC)	ABS	NEC	
24	<i>Disamis</i> (NEC,ABS)	NEC	ABS	
25	<i>Bocardo</i> (ABS,NEC)	ABS	NEC	
26	<i>Bocardo</i> (NEC,ABS)	ABS	ABS	
27	<i>Ferison</i> (ABS,NEC)	NEC	NEC	
28	<i>Ferison</i> (NEC,ABS)	ABS	ABS	

Ibn Sīnā's claims in (7.23) (counting the conclusion as NEC in those cases where he offers two opinions) have a pleasing symmetry: in every case the mode of the conclusion depends at most on the figure of the underlying categorical syllogism and whether the major or minor premise has the label NEC. Aristotle's claims lack this symmetry in second and third figures.

(7.23) also agrees with Street's listing of the modal syllogisms that Ibn Sīnā recognises as valid, cf. Street [204] p. 160.⁹

⁹As in footnote 14 to Chapter 2, we interpret Street's *L* as 'inevitable' or *lā budda*, i.e. as

The list (7.23) includes nine moods where Ibn Sīnā disagrees with Aristotle, as follows.

#	Premise-pair	Aristotle	Ibn Sīnā
10*	<i>Cesare</i> (NEC,ABS) <i>Prior Analytics</i> i.10, 30b18–24 <i>Prior Analytics</i> i.10, 30b24–31 <i>Prior Analytics</i> i.10, 30b31–40	ABS	NEC
11*	<i>Camestres</i> (ABS,NEC) <i>Prior Analytics</i> i.10, 30b18–24 <i>Prior Analytics</i> i.10, 30b24–31 <i>Prior Analytics</i> i.10, 30b31–40	ABS	NEC
14*	<i>Festino</i> (NEC,ABS) <i>Prior Analytics</i> i.10, 31a14–17	ABS	NEC
(7.24) 15*	<i>Baroco</i> (ABS,NEC) <i>Prior Analytics</i> i.10, 31a14–17	ABS	NEC
16	<i>Baroco</i> (NEC,ABS) <i>Prior Analytics</i> i.10, 31a14–17	ABS	NEC
23	<i>Disamis</i> (ABS,NEC) <i>Prior Analytics</i> i.11, 31b31–33	ABS	NEC
25	<i>Bocardo</i> (ABS,NEC) <i>Prior Analytics</i> i.11, 32a4–5	ABS	NEC
18	<i>Darapti</i> (NEC,ABS) <i>Prior Analytics</i> i.11, 31a31–33	NEC	ABS
24	<i>Disamis</i> (NEC,ABS) <i>Prior Analytics</i> i.11, 31b12–19	NEC	ABS

The references to *Prior Analytics* given in the column below ‘Premise-pair’ are explained as follows. When Aristotle declares that a syllogism has conclusion labelled NEC, he should give evidence that the premises entail a candidate conclusion labelled NEC. When Aristotle declares that a syllogism has conclusion labelled ABS, he should give evidence that the premises are not strong enough to entail a candidate conclusion labelled NEC. In both cases (7.24) lists, below the name of the syllogism, the reference or references in *Prior Analytics* where Aristotle gives such evidence. The criteria for strength of candidate conclusion stated in Section 4.4 need

alethic necessity. In *Short Epitome in Logic* there are no references to temporal permanence and the word *dā'im* ‘permanent’ is absent; so at this early date Ibn Sīnā was still reading NEC as alethic necessity.

to be updated here to allow for the strength of the modal label; NEC counts as stronger than ABS.¹⁰

So Ibn Sīnā's claims in (7.24) put him under two obligations: to answer the arguments put forward in the indicated passages of *Prior Analytics*, and (for the first seven syllogisms) to prove that the premises entail a conclusion labelled NEC. In *Short Epitome in Logic* there is no hint of how he can meet these obligations, but by the time of *Twenty Questions* he will be well on his way to dealing with them.

We pick out the cases of *Cesare*(NEC,ABS,NEC), *Camestres*(ABS,NEC,-NEC), *Festino*(NEC,ABS,NEC) and *Baroco*(ABS,NEC,NEC) (the four marked with an asterisk in (7.24)) for special attention. By the time he wrote *Twenty Questions*, Ibn Sīnā was aware that he had no satisfactory Aristotelian proofs for these syllogisms, so he would need to innovate. Perhaps he was already aware of this problem when he wrote *Short Epitome in Logic*. Also his first move towards dealing with these four syllogisms in *Twenty Questions* was to argue for a revised form of absolute sentence; although he doesn't say so, this move is a sensible response to Aristotle's argument at *Prior Analytics* i.10, 30b18–24. Accordingly I will refer to these four syllogisms as the Modal Challenge, and to Aristotle's three arguments at *Prior Analytics* i.10, 30b18–40 as (Aristotle's) Modal Refutations, as in Section 2.2 above.

To make sense of the list (7.23) we need to be able to write down examples of the syllogisms listed. But unfortunately *Short Epitome in Logic* gives inadequate clues to the structure of sentences with modal labels added, either in terms of meaning or in terms of syntax. Here is a typical example:

- (7.25) Every *C* is a *B*;
 and no *B* is an *A* however it is (*kayfa kāna*).
 So no *C* is an *A* likewise (*ka-dhālika*).
 (*Short Epitome in Logic* [112] 6.13)

This is a formal categorical syllogism in mood *Celarent* in first figure, but with some extra text added to the second (i.e. major) premise and to the conclusion. The mode added to the conclusion is given as 'likewise', which has to refer back to the extra text in the major premise, and this fits with the fact that the dominance for necessity in the first figure lies with the major premise. But Ibn Sīnā could have explained this better, and he could also have mentioned that the minor premise can take a mode too.

¹⁰See Hodges [98] Theorem NUMBER for a check that in this modal context there is still a unique conclusion.

However, it would be rash to infer from (7.25) that Ibn Sīnā thinks of the modes as being added at the *end* of the sentences. Examples (7.19)–(7.21) are counterexamples, and *Twenty Questions* contains many more counterexamples. We will see in Section 10.4 that al-Fārābī used a template for modal sentences, to show what ingredients they should contain, and that Ibn Sīnā developed his own template for the same purpose, very likely during the Gap period. But Ibn Sīnā didn’t use his template to impose a rigid rule on where to put his modes.

7.4 Modes in al-Fārābī and Ammonius

We have covered all the main information that can be drawn from *Short Epitome in Logic*. But before we say farewell to the book, we should try to make something of Ibn Sīnā’s definition of modes in *Short Epitome in Logic*:

The modes of propositions are three: necessary (*wājib*), contingent (*mumkin*) and impossible (*mumtanī*).¹¹ The necessary is (7.26) as ‘The human is an animal’, the impossible is as ‘The human is a stone’ and the contingent is as ‘The human is a writer’.
(Badawi [112], modes defined at 5.5f.)

Several things about this passage are odd. In the first place, all the three examples of sentences are self-classifying—they contain no modes. Self-classifying sentences do appear elsewhere in Ibn Sīnā’s logical writings; we noted one in Section 7.2 above. But they are rare and very atypical.

Secondly, the modes that Aristotle mentions in his treatment of syllogisms are ‘necessary’, ‘possible’ and ‘contingent’, not ‘impossible’. Perhaps ‘impossible’ derives from Aristotle’s opening words to Section xii of *De Interpretatione*:

... we must consider how negations and affirmations of the possible to be and the not possible are related to one another, and of (7.27) .the admissible and not admissible, and about the impossible and the necessary.
(*De Interpretatione* [20] vol. 1 p. 34, 21a34–37)

If so then it is likely to have reached Ibn Sīnā from al-Fārābī’s *Syllogism*, in particular from al-Fārābī’s treatment of ‘matters’, from which we can

¹¹*mumtanī*^c is an alternative Arabic word for ‘impossible’, found already in an Arabic text of the 8th century.

extract three kinds of matter, namely necessary, impossible (*mumtani*^c) and possible:

- (7.28) Using necessary [matter] ... is like saying 'Every human is an animal' Using impossible [matter] is like saying 'Every human is a stone' Using possible [matter] is like saying 'Every human is white'.
- ([59] 16.13–16, cf. [36] p. 121)

Again the example sentences are self-classifying. Also the context in which al-Fārābī makes this statement has nothing to do with modes. Rather, al-Fārābī is about to use these three 'matters' as an alternative approach to the logical relationships in the square of opposition.

This use of the three matters to explain the square of opposition appears already in the lectures of Ammonius Hermiou on Aristotle's *De Interpretatione* at the Academy in Alexandria in around AD600. Ammonius also uses the same three modal names for the matters. Two pictures may help to explain what is going on here:

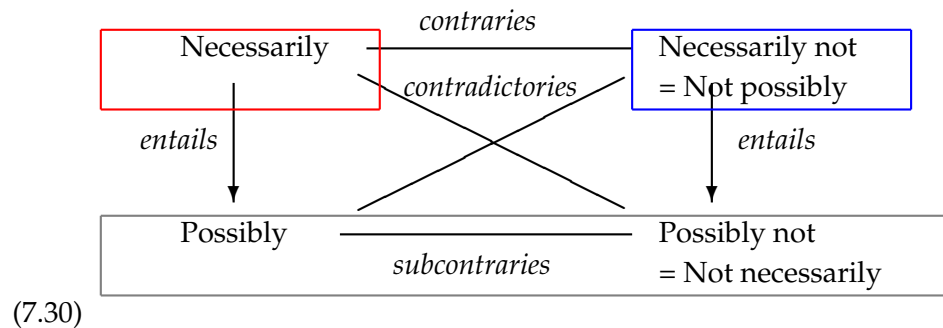
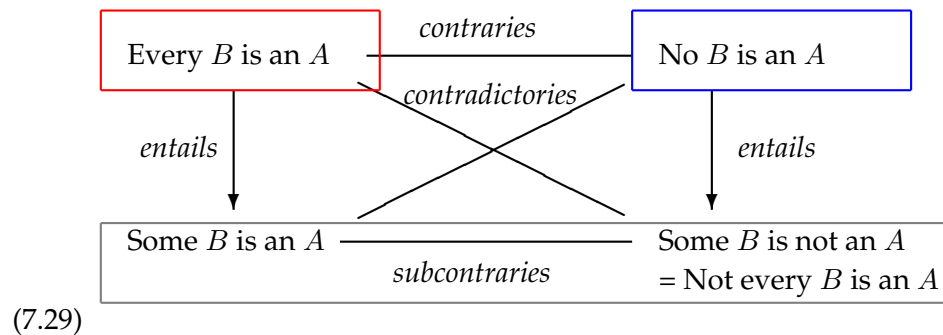


Diagram (7.30) is the same as (7.7) except that I have added three rectangles to represent Ammonius's three matters. These matters are actually classes of matters in Alexander's sense. The relations between the matters can be read off from the logical relations. Since 'Necessarily' is contrary to 'Necessarily not', no Alexander matter is in both Ammonius necessary matter and Ammonius impossible matter. Since 'Necessarily' is contradictory to 'Possibly not', and 'Necessarily not' is contradictory to 'Possibly', an Alexander matter that is not in either Ammonius necessary matter or Ammonius impossible matter makes both 'Possibly' and 'Possibly not' true, and hence is in Ammonius possible matter. Likewise no matter is in more than one rectangle; so the Ammonius matters form a partition of the Alexander matters. Going in the other direction, from these relations between the three Ammonius matters we can deduce the italic relations between the modes.

Possibly it was Ammonius who first noticed that the same relationships hold in the diagram (7.29). Perhaps it was also Ammonius who inferred from this that the logical facts expressed in the diagram (7.30) are the same as those expressed in (7.29), so that the modal names of the matters shown in (7.30) can be justifiably carried over from one diagram to the other.

Thirdly, Ibn Sīnā seems to have taken the isomorphism between the two diagrams (7.29) and (7.30) too literally and jumped to the meaningless conclusion that modes and matters are really the same thing. This seems to have been a temporary juvenile confusion, since in *Middle Summary in Logic* and later works¹² Ibn Sīnā is vigorously clear that modes and matters are not the same thing. It's the example of immaturity in *Short Epitome in Logic* that was briefly introduced in the last paragraph of Section 4.2 above.

7.5 Al-Fārābī's tolerance and *muhmal* sentences

A reader of al-Fārābī's *Syllogism* might easily get the impression that the book has nothing to say about modal logic, and so contributes nothing to the sections of *Short Epitome in Logic* that do mention modal logic. But that would be a misunderstanding. Al-Fārābī did explain a kind of theory of modal logic in his *Syllogism*, but from an unfamiliar point of view. His starting-point, given categorical logic, was that many natural language sentences can be viewed as categorical sentences with extra words added or assumed as a kind of decoration that implies some adjustment of the mean-

¹²*Middle Summary in Logic* [114] 75.2–16, *Deliverance* [131] 29.10–12, *De Interpretatione* [123] 112.14f

ing of the underlying categorical sentence. For example, towards the end of Part 18 of *Syllogism* he mentions the following different ways in which we might interpret 'Every *B* is an *A*':

- (i) Every *B* is an *A* because it is a *B*.
(42.16, see note 74 on page 171 of [36])
- (ii) *B* is the cause of *A*.
(43.14; 44.2 with 43.17f, 48.5; 52.4,9,14;)
- (iii) Every *B* is an *A* and vice versa.
(44.16)
- (7.31) (iv) Necessarily every *B* is an *A*.
(43.3, cf. [58] 103.8)
- (v) Whenever and wherever a thing is a *B* it is an *A*.
(44.8, cf. [58] 103.6)
- (vi) The *B* is an *A*.
(42.19)

In Part 18 of *Syllogism* al-Fārābī gives some thoughts on these sentences, though the passage is cramped and unclear. But some manuscripts of al-Fārābī's *Syllogism* attach two further Parts, on 'Transfer' and on 'Jurisprudential syllogisms', that express a much clearer view under the name of 'tolerance' (*musāmaḥa*).¹³

Tolerance for al-Fārābī means that

- (7.32) It is not appropriate to seek the same kind of exact investigation in all matters; but each thing should be investigated as is appropriate for its subject-matter.
(Al-Fārābī *Syllogism* [59] 60.14–16, [36] p. 160)

In support of this view he quotes Aristotle's *Nicomachean Ethics* 1094b11–14 and 1094b23–27. He adds that

- (7.33) [Tolerance] is very helpful in the art of jurisprudence and many other arts, where it is natural to make much use of tolerance about what is known. But this [practice] should not extend beyond these arts.
(Al-Fārābī *Syllogism* [59] 61.7–9, [36] p. 161.)

¹³[124] includes these two added parts. Some other manuscripts, including a manuscript of *Short Syllogism* translated by Rescher [186], also contain these two parts. Chatti and Hodges [36] pp. 21–24 give one possible account of the order in which al-Fārābī wrote these various texts. Lameer [151] pp. 13f–18, 185–198 offers a different account.

The implication for sentences like those in (7.31) above is that they can be handled using the normal rules for categorical logic, but with a light touch. Apparently it never occurs to al-Fārābī that with new kinds of sentence one might look for new logical rules that they obey precisely.

Al-Fārābī picks out sentence (vi) in (7.31) for special attention. At [59] 53.1–9 he points out that if a *muhmal* sentence is written with a definite article instead of a quantifier, say as ‘The *B* is an *A*’, then this will blur the line between concrete instances and large classes. This blurring can be ‘useful’ through tolerance, by allowing us to treat something that is only known to be true in a special case as if it was true universally. Ibn Sīnā’s rejection of this view is brutal:

[One should] beware of *muhmal* sentences and not make any
(7.34) use of them at all.
(*Short Epitome in Logic* [112] 13.3).

This opinion serves as a useful date marker. After *Short Epitome in Logic* Ibn Sīnā took a more positive view of *muhmal* sentences in logic, starting in the Gap period. On this change of view see Section 9.2 below.

Part III

The Gap ($\leq 1000-1011$)

Chapter 8

Dating the Gap period

In Section 1.3 above we introduced the Gap period. This is our name for a period running from the finishing of *Short Epitome in Logic* (which in Section 4.2 we cautiously dated to no later than 994) to around the writing of *Twenty Questions* and the dictation of *Middle Summary in Logic* (whose dates we will assign to 1012–1013 in Section 11.1 below). The period is a gap only in the sense that there are no published and currently available logical writings of Ibn Sīnā that we have evidence for placing within the period. But by comparing the logical content of *Short Epitome in Logic* with that of *Middle Summary in Logic* we can see that the Gap period must have been one of the most productive periods in Ibn Sīnā’s logical development. For example it contains what in Section 1.2 we described as the Hypothetical Breakthrough, a broad and deep development of new hypothetical logics based on the wholly *muttaṣil* logic of *Short Epitome in Logic*; Chapter 9 below is devoted to exploring the Hypothetical Breakthrough. We will see in Chapter 10 that the Gap period also contains Ibn Sīnā’s first moves to collect material that would be helpful for redesigning modal logic. Ibn Sīnā never lost sight of this material, and it continued to be a kind of signature of his approach to modal logic for the rest of his career.

In Section 8.3 below we will argue that there was a major development of Ibn Sīnā’s hypothetical logic during the first part of the Gap period, after the writing of *Short Epitome in Logic* and before Ibn Sīnā had written the main source behind the logic part of al-Barakāt’s *Kitāb al-muṭabar*. There was some mild development of hypothetical logic before the writing of *Twenty Questions* and *Middle Summary in Logic* at the end of the Gap period. But in Chapter 14 we will find that after this the next advances that

we know of in Ibn Sīnā's hypothetical logic appeared first in *Syllogism* in the early 1020s.

The development of Ibn Sīnā's predicative logic, and above all his 2D logic, seems more complicated than that of his hypothetical logic. But this may be partly because the predicative logic continued to develop through the Jurjān period and later, and we have a much richer supply of logical texts from these times than we have from the Gap period when the Hypothetical Breakthrough took place. Maybe this difference will fade as more manuscripts from the early Gap period become available. But for the present we have what we have.

However, Ibn Sīnā's 2D logic does have some features that make its history inherently more complex than his hypothetical logic. First, Ibn Sīnā's wholly *muttaṣil* logic was a natural development from categorical logic.¹ By contrast Ibn Sīnā's predicative logic makes use of the work of various earlier logicians (Aristotle, Theophrastus, Alexander, al-Fārābī among others) in several ways, some of them quite indirect. Chapter 10 below will examine these sources in earlier logic. And second, Ibn Sīnā found that his development of 2D logic during the Jurjān period required him to challenge the foundations of Aristotle's modal logic, as in Chapters 11 and 12 below.

8.1 Dating *Twenty Questions* and *Middle Summary in Logic*

This section is mostly based on Section 2 of Hodges [95].

Anybody who reads both *Twenty Questions* and the sections of *Middle Summary in Logic* dealing with syllogisms must be impressed by the large number of resemblances between these two works in aims, content, style and vocabulary. For a first example, simple but striking, consider the words that Ibn Sīnā uses to name Aristotle in various works on syllogisms:

¹We can see this from the number of times similar developments appeared in Western logic, in Ramus REF, Wallis [220] and Boole [28]

	Aristotle	The Philosopher	The First Teacher
<i>Epitome of the Principles of Logic</i>	0	1	0
<i>Short Epitome in Logic</i>	0	0	0
<i>Twenty Questions</i>	1	26	0
<i>Middle Summary in Logic</i>	0	44	0
<i>Shorter Summary in Logic</i>	0	4	3
<i>Guidance</i>	0	0	0
<i>Syllogism</i>	0	0	26
<i>Pointers and Reminders</i>	0	0	1

The repeated use of ‘The Philosopher’ is a feature that singles out *Twenty Questions* and *Middle Summary in Logic*. (In *Short Epitome in Logic* Aristotle is referred to as ‘he’.)

A second resemblance between *Twenty Questions* and *Middle Summary in Logic* is in their treatment of the ‘foundations’ (*uṣūl*, singular *aṣl*) of modal logic. The word *aṣl* occurs quite often in Ibn Sīnā’s writings, in several contexts. For example it is used for principles of sciences in general, as at *Shorter Summary in Logic* [131] 5.8, 137.12. Also when a phrase W is converted into a phrase X, W (the ‘original’) is called *aṣl*, as at *Syllogism* [124] ii.1, 77.1.4; i83.8; 85.3. The usage that ties *Twenty Questions* to *Middle Summary in Logic* is different from both of these. Rather, an *aṣl* is a principle that justifies a modal syllogism, and there are frequent references to rival principles used by different logicians. Thus for example in *Twenty Questions* [115]:

- 83.2 ‘on it he [Aristotle] built many foundations’ (i.e. properties of absolutes)
- 87.4 ‘this foundation’ (on the meaning of ‘possible’)
- 89.2 ‘his [Aristotle’s] foundations’ (on the meaning of absolutes)
- 89.10 ‘his [Aristotle’s] foundations’
- 92.18 ‘we verify this foundation’ (on the meaning of ‘necessary’, cf. 92.10 on dictum de omni)
- 95.10 ‘the foundations’
- 98.21 ‘their foundations’

Here for comparison are passages from the part of *Middle Summary in Logic* that deals with modal syllogisms:

- 80.15 ‘the foundation that we determined’
- 132.14 ‘his [Aristotle’s] foundations’

- 132.18 'these foundations'
- 133.11 'built on a rotten foundation' (the bad foundation is a conversion)
- 137.17f 'the foundations that we stated'
- 138.1 'we posit a foundation'
- 139.1 'our foundation'
- 140.3 'his foundations'
- 144.3 'his foundations'
- 145.8 'our foundations'

From these two sets of quotations it would be reasonable to infer that *Twenty Questions* and the modal syllogistic part of *Middle Summary in Logic* share a common programme, namely to rewrite modal syllogistic so that it rests on sounder foundations than the ones that Aristotle relied on. The quotations from *Middle Summary in Logic* carry a strong suggestion that Ibn Sīnā thinks he has replaced some of Aristotle's foundations by something better.

But caution: there is a third logical work of Ibn Sīnā that uses very similar phrases. This is *Syllogism* [124], mainly in Book iv:

- ii.2, 90.6 'difference between permanent and necessary are foundations that you will have to remember for yourself'
- iii.1, 138.6 'according to their foundation'
- iii.2, 142.5f 'according to this foundation'
- iv.4, 215.7 'their foundation'
- iv.4, 215.11 'their foundations'
- iv.5, 216.5 'their foundations'
- iv.5, 218.10 'the preceding foundations'
- iv.5, 218.12 'their foundations'
- iv.5, 220.5 'the foundations that you adopted'
- iv.5, 221.16f 'their foundations'
- iv.5, 221.18 'our foundations'
- iv.6, 225.17 'their foundations'

Do these quotations from *Syllogism* commit us to arguing that the relevant parts of *Syllogism* were written together with the modal syllogisms of *Middle Summary in Logic*?

I think yes and no. The foundations discussed in *Twenty Questions* and *Middle Summary in Logic* are relevant to modal syllogisms as a whole. By contrast the foundations discussed in *Syllogism* are nearly all related to modal syllogisms containing statements of possibility or contingency. This is what we would expect if Ibn Sīnā felt that his new foundations in *Middle*

Summary in Logic worked well in general but left some problems related to modal syllogisms that involve possibility or contingency. In fact Ibn Sīnā's treatment of modal syllogisms in the later parts of *Syllogism* Book iv is not much different from the corresponding parts of *Middle Summary in Logic*, and it is possible that Ibn Sīnā had *Middle Summary in Logic* by him when he wrote that part of *Syllogism*.

It should be added that the 'foundations' discussed in *Twenty Questions* and *Middle Summary in Logic* need not have been due to either Aristotle or Ibn Sīnā. For example Question viii of *Twenty Questions* is known to be a critique of an argument used by al-Fārābī, almost certainly in his *Long Commentary on Prior Analytics*, to justify a conversion.

A third resemblance between *Twenty Questions* and the syllogistic part of *Middle Summary in Logic* consists of two passages, Question xix in *Twenty Questions* and the opening paragraphs of the section on Hypothetical Syllogisms in *Middle Summary in Logic* ([114] 145.17–147.10). In both passages Ibn Sīnā observes that mathematicians, including Euclid, make many claims that are naturally read as hypothetical sentences. He points out that the syllogisms endorsed by traditional Peripatetic logic all have conclusions that are predicative and not hypothetical. So he announces a programme: to find syllogisms that will prove hypothetical conclusions. He adds that he himself has already completed this programme. Taken as a whole this is an extraordinarily brazen statement, and not one likely to win him many friends among logicians. Given the substantial content that the two passages share, it's fair to describe them as near-doublets, though no sentences are copied from one to the other.²

In these two passages Ibn Sīnā claims to have made a breakthrough in hypothetical logic, both by finding a novel and significant research problem, and by finding what he seems to regard as a comprehensive solution to the problem. The passage in *Middle Summary in Logic* stands at the beginning of a longish section ([114] 145.17–163.6) on recombinant hypothetical logics. In Section 9.1 below we will see that the same recombinant hypothetical logics appear also in Abū l-Barakāt's *Al-kitāb al-mu^ctabar*, in a part which is believed to be based on Ibn Sīnā's writings from 1000 or a little

²There are some similarities of language: compare *Twenty Questions* 103.12 *istakhrājnā* and *Middle Summary in Logic* 147.10 *yukhrajū*. *Syllogism* also has some related passages, though none come close to being doublets of those in *Twenty Questions* and *Middle Summary in Logic*. Thus *Syllogism* vi.6, 356.10 *istakhrājnā*, and *Syllogism* v.1, 231.8 'Many of the claims made in mathematics, physics or metaphysics are *muttaṣil* or *munfaṣil* hypothetical [propositions]'.

later. (See Section 8.2 below.) Thus we have two separate accounts of this body of work that Ibn Sīnā did in hypothetical logic after *Short Epitome in Logic*. We will call this achievement the Hypothetical Breakthrough, as already mentioned above.

A fourth resemblance between *Twenty Questions* and *Middle Summary in Logic* is that both works refer to 'commentaries' that Ibn Sīnā wrote earlier. Thus:

- (8.1) We have already shown this in our commentaries (*shurūḥunā*).
(*Middle Summary in Logic* [114] 117.13)
- (8.2) ... as we showed in our commentary (*sharḥunā*) on the *Analytics*.
(*Twenty Questions* [115] 96.6; cf. 97.1, 21.)

These quotations gain interest from the fact that there are no such references to an earlier commentary by Ibn Sīnā in any of his *Shorter Summary in Logic*, *Syllogism* or *Pointers and Reminders*. Also in (8.1) above al-Jūzjānī reports Ibn Sīnā making a similar reference to his earlier commentaries when al-Jūzjānī first met him in 1013. Gutas [74] p. 99f adds to the picture by providing evidence that *The Available and the Valid*, written by Ibn Sīnā when he was twenty-two years old and now long since lost, could have been written in the form of a commentary.³

A possible fifth resemblance is that *Twenty Questions* and *Middle Summary in Logic* show the same development of the word *muṭlaq* ('absolute'). We examine this development in Section 8.3 below. CHECK THIS.

The title of *Twenty Questions* is given in the [115] edition as 'Twenty questions on logic, which people of that time asked him, finding them testing'. This is surely wrong, because nothing in the work resembles a question put to Ibn Sīnā by somebody else, and it would be hard to claim that the work is written in an explanatory style. There are a few questions in the text, but they seem to be addressed by Ibn Sīnā to himself.⁴ Some remarks point out issues that need clarification—presumably in some future work,

³Street [207] p. 99 holds that the commentary mentioned in (8.2) must be *Syllogism*, thus pushing the date of *Twenty Questions* forward into the 1020s. I think Street's argument for taking the commentary cited by Ibn Sīnā to be *Syllogism* is that Ibn Sīnā in *Twenty Questions* [115] 97.21 refers to his commentary as a *tafsīr*, which Street understands to imply an 'extensive treatment'. But at *Twenty Questions* [115] 83.6f Ibn Sīnā refers to the explanations of Themistian absolute and Alexandrine absolute as *tafsīr*, and these are each a sentence or so.

⁴For example in Question i there are several questions asked, but they are at the end of the Question with no attempt to answer them, and the final sentence of the Question is an

since the required clarification is not given in *Twenty Questions* itself. But the title in [115] is easy to discount, since the copyist may have been misled into connecting *Twenty Questions* with the episode that al-Jūzjānī reports at *Autobiography* [117] 74–81, where a copy of *Deliverance* (a work published in the late 1020s, containing *Shorter Summary in Logic* as its logical part, cf. Section 1.2 above) caused some scholars in Shīrāz to send Ibn Sīnā questions about it, which he answered very promptly.⁵ Street [207] p. 99 rightly discourages connecting *Twenty Questions* with this episode, since the length of *Twenty Questions* is much less than the story implies. But in any case ‘questions’ could equally well mean questions which the research itself is required to answer. There are a number of such ‘questions’ in al-Rāzī’s commentary [181] on *ʿUyūn al-ḥikma*.

The arguments above in this section place *Twenty Questions* and *Middle Summary in Logic* close together in time, but they say nothing about which was written first, or whether they could have been composed at the same time. The only evidence we have for their relative dating lies in their contents, and in particular the discussion of predicative syllogisms in *Twenty Questions* ii–xviii and *Middle Summary in Logic* [114] 99–145.16. There are a number of smallish details that we can compare between these two passages. My present impression is that the majority of these passages swing the evidence in favour of *Twenty Questions* being notes that Ibn Sīnā wrote for himself in preparation for composing *Middle Summary in Logic*. But I hope other scholars will soon add their own analyses. For illustration I give three examples (i)–(iii).⁶

instruction from Ibn Sīnā to himself to do some work (‘Let us demonstrate’, *Twenty Questions* [115] 82.21). The rhetorical question at Question xvii, 101.7f is a criticism of Aristotle by Ibn Sīnā himself.

⁵The attempt to relate *Twenty Questions* to *Shorter Summary in Logic* rather than to the very much closer *Middle Summary in Logic* may be the result of historical confusions suggesting that *Middle Summary in Logic* and *Shorter Summary in Logic* might be the same work. Kalbarczyk [146] clears up these confusions.

⁶There is a surprising discrepancy between *Twenty Questions* and *Middle Summary in Logic* at *Twenty Questions* Question vii, 87.15–17. Here Ibn Sīnā says that the sentence ‘Every *B* is an *A* with necessity’ doesn’t convert to a necessary sentence. But at *Middle Summary in Logic* 114.13 he says the opposite: that universal affirmative necessary sentences convert to necessary existential sentences; this was Aristotle’s position. One might suspect that this is evidence that *Middle Summary in Logic* was written before *Twenty Questions*, and in the interval between them he changed his mind about this conversion. Not so: if we read on in *Middle Summary in Logic*, we find that at 118.8–10 he says that the conversion need not be a necessary sentence, giving the same example that he used in *Twenty Questions*. As elsewhere, he gives Aristotle’s view and then his own which contradicts Aristotle. I think

(i) The first example is a passage from *Twenty Questions*, namely the instance of *dictum de omni* listed in Section 10.4 below as DDO.1:

- /92.10/ What is said of the all [i.e. *dictum de omni*] and the syllogism is that every subject [individual] in which there is found what belongs to the nature of the subject [term], and what is described by [the subject term], that content is found in it. In effect [the content] is not related to the essence of 'coloured white', for example in the sense that it is coloured white alone, but rather to the subject [individuals] that have it. And there is no doubt that the definitions [of the subject individuals?] are compatible with whiteness, whether or not the description [of whiteness] follows from [the definitions]. When you say 'every white thing' or 'every *B*', [the subject individual] has to fall under the [subject] term, regardless of whether it is white or /92.15/ *B* with necessity or without it. So when [the sentence] expresses a judgment made with necessity or possibility or affirmation or denial, [the content] is applied to each individual that fits the description given by the subject term and that is posited [as a subject individual], regardless of whether (for example) [the subject individual] is white with or without necessity.
(8.3) (Twenty Questions [115] x, 92.10–17.)

One feature of this passage—or more strictly of my tentative translation of this passage—is the large number of places where text is added between square brackets. These are mostly places where Daneshpazhuh's Arabic text is ambiguous about what is being referred to. Without the material in square brackets there would be a number of occurrences of 'it', where the reader would have to choose whether 'it' is the subject term, the subject individual, the predicate, the content⁷ or some other feature. This kind of ambiguity is most likely to arise if Ibn Sīnā is writing for himself; he will probably know at once what he was thinking when he wrote. It is totally unsuitable in an explanatory text.

(ii) In Section 10.4 I list what I take to be the separate occurrences of Ibn Sīnā's *dictum*. There is one in *Twenty Questions* (namely DDO.1 in (i) above) and there are nine in *Middle Summary in Logic*. Moreover there are no mentions of time (*waqt* or *zamān*) in the occurrence in *Twenty Questions*,

the fact that he leaves Aristotle's view uncorrected in *Twenty Questions* tends to show that he wrote *Twenty Questions* for himself.

⁷See Section 6.1 above for the notion of content (*ḥukm*).

while every one of the occurrences in *Middle Summary in Logic*, and most occurrences in later works, mention time in connection with the predicate. Time is already a key ingredient of Ibn Sīnā's notions of modality in *Middle Summary in Logic*. These statistics suggest that the occurrence of the *dictum* in *Twenty Questions* is a preparatory example, and those in *Middle Summary in Logic* form an early working out of the details.

(iii) Another statistic that points the same way is the lengths of Ibn Sīnā's lists of 'necessary predications', as discussed in Section 10.2 below. See Footnote 11 of that section.

8.2 *Kitāb al-mu^ctabar* and other evidence for the Gap period

In 2016 Jules Janssens published a paper [143] entitled 'Abū al-Barakāt al-Baghdādī and his use of Ibn Sīnā's *al-Ḥikma al-^cArūḍīya* (or another work closely related to it) in the logical part of his *Kitāb al-mu^ctabar*'. The work of Ibn Sīnā that Janssens cites in his title, *al-Ḥikma al-^cArūḍīya*, survives in a single incomplete manuscript, Uppsala 364, which is missing the parts corresponding to the end of *De Interpretatione*, the whole of *Prior Analytics* and the beginning of *Posterior Analytics*. So it lacks Ibn Sīnā's account of syllogistic logic. Gutas [74] pp. 86–93 mentions the possibility that other surviving works may contain quotations or paraphrases of some of the missing material.⁸

Janssens's paper, after examining the relevant texts, concludes that 'there is overwhelming evidence that Abū al-Barakāt's sections on dialectics, sophistry, rhetoric, and poetics, as contained within the *Mu^ctabar*, are largely inspired by and indebted to Ibn Sīnā's *Ḥikma*, or, at least, a text very close to it'.⁹ (He draws on work of Renate Wüsch [224] and Geert Jan van Gelder and Marlé Hammond [219].)¹⁰

⁸Among them is *Epitome of the Principles of Logic*, which is reported to be in the manuscript Mishkāt 1149 in Tehran. I was able to consult this manuscript (my thanks to Mohammad Esmaeili for his help) and found that it omits the parts corresponding to *De Interpretatione* and *Prior Analytics*.

⁹Janssens cites the manuscript *Nuruosmaniye* 4894 as a witness to this text 'very close to' that of *Ḥikma*. Unfortunately I have no access to *Nuruosmaniye* 4894.

¹⁰In a recent preprint 'La Rhétorique du Kitāb al-Mu^ctabar d'Abū l-Barakāt al-Baghdādī Première partie : Sources avicenniennes et innovations personnelles' Janssens adds to the overwhelming evidence by making a close comparison between al-Barakāt's section on Rhetoric in his *Kitāb al-mu^ctabar* and texts known to be close to Avicenna's view of rhetoric in the early 1000s. This comparison gives a valuable insight into how far al-Barakāt was

Janssens's results make it likely that Abū l-Barakāt, when writing his account of syllogistic logic in *Al-kitāb al-mu^ctabar*, made use of what Ibn Sīnā wrote about this logic during the Gap period. In his final paragraph Janssens says 'I think we cannot exclude the possibility that the *Mu^ctabar*'s first three sections on logic might have been strongly inspired by Ibn Sīnā's *Ḥikma*'. Happily, enough facts are known about the development of Ibn Sīnā's logic during the Gap period for us to begin checking this.

First consider hypothetical logic. In Section 1.2 we met Ibn Sīnā's Hypothetical Breakthrough, but this was before we could explain what this Hypothetical Breakthrough was in terms of the development of Ibn Sīnā's logic. Al-Barakāt's *Kitāb al-mu^ctabar* will allow us to identify the Hypothetical Breakthrough, and in Section 8.3 below we will do our best to date it.

Section 6.1 described wholly *muttaṣil* logic, using the account in *Short Epitome in Logic* together with some clarifying remarks in *Middle Summary in Logic*. The account of this logic in *Short Epitome in Logic* is essentially complete; the remarks in *Middle Summary in Logic* are an improvement in the exposition. But *Short Epitome in Logic* says nothing to suggest that wholly *muttaṣil* logic can be extended or modified in interesting ways. Now al-Barakāt's book *Kitāb al-mu^ctabar* knows wholly *muttaṣil* logic and gives a good description of it. But *Kitāb al-mu^ctabar* also describes some six or seven other hypothetical logics that can be seen as developments from wholly *muttaṣil* logic; we will examine them in Chapter 9 below. If we move on to *Middle Summary in Logic*, we find it describes wholly *muttaṣil* logic briefly, with a remark that wholly *muttaṣil* logic has already been studied in earlier work. But *Middle Summary in Logic* also includes an account of the new hypothetical logics that *Kitāb al-mu^ctabar* described, in more detail than *Kitāb al-mu^ctabar* gave. So it appears that these new logics were studied together after wholly *muttaṣil* logic had been introduced, and before the source of the logic of *Kitāb al-mu^ctabar* was written. Ibn Sīnā's discovery of this new group of hypothetical logics is what we are calling the 'Hypothetical Breakthrough'. The evidence from *Kitāb al-mu^ctabar* suggests that the Breakthrough was broadly complete by around 1005.

There are some features of hypothetical logic that are reported in *Middle Summary in Logic* but not mentioned in *Kitāb al-mu^ctabar*. One of these is the programme for the Hypothetical Breakthrough, namely to find all the logical rules that yield a hypothetical conclusion; this is emphasised in both *Middle Summary in Logic* and *Twenty Questions*. Since *Kitāb al-mu^ctabar*

prepared to deviate from his source or template.

describes the same rules for hypothetical logic as *Middle Summary in Logic* does, the natural explanation is that Ibn Sīnā hadn't thought of this programme when he first developed the Hypothetical Breakthrough, but later he added it to his description of the Hypothetical Breakthrough logics, adopting it after the event as a suitable aim for the research that led to the Breakthrough. *Kitāb al-mu'tabar*'s source was too early to mention this programme.

The following remark of al-Jūzjānī throws some light on the shortage of surviving works from the Gap period:

- (8.4) When I appealed to [Ibn Sīnā] to compose long works and commentaries, he referred to the commentaries he had written and books he had composed in his native country.¹¹ I had heard, however, that these were widely dispersed and that people who owned a copy of them Withheld them [from others]; as for him, it was not his habit to save a copy for himself, just as it was not his habit to make a clean copy from his holograph or transcribe [a holograph] from his rough draft; he would only either dictate or himself write the copy and give it to the person who had commissioned it from him.
([118] 2.2–7; trans. Gutas [74] p. 31.)

One should treat al-Jūzjānī's statement with a little caution. Al-Jūzjānī certainly felt he deserved credit for ensuring that Ibn Sīnā's research was properly motivated, recorded and preserved.¹²

There are a small number of unpublished manuscripts that may help to fill the gap. For example the essay on propositions with modes, that Ibn Sīnā refers to at *Twenty Questions* [115] xii, 97.20, is very likely the *Bayān dhawāt al-jihā* for which Gutas [74] p. 438 finds two manuscripts in

¹¹Given our dating in Section 8.3 below, Ibn Sīnā was probably developing the Hypothetical Breakthrough after leaving Bukhārā but still within greater Khorāsān. But he is not likely to have changed his editorial habits when travelling from Bukhārā to Gurgānj. 'His native country' could be read as Khorāsān.

¹²In [his prologue to *The Cure*, al-Jūzjānī] represents himself as the instigator and constant goad of the project, and if this was indeed the case, he must have considered it his right to introduce the finished product and could hardly have tarried to do so.' (Gutas [74] p. 113.) Likewise in *Autobiography* pp. 44f al-Jūzjānī claims credit for getting Ibn Sīnā to create *Middle Summary in Logic* and dictate it to him; but it seems just as likely that Ibn Sīnā himself, in the light of his radical progress in logic during the Gap period, had already decided that this was the right time to write a definitive account for the record. Al-Jūzjānī could have been unaware that Ibn Sīnā already had a programme for the book, in the form of *Twenty Questions*.

Cairo. This work is very likely to be the work that in *Twenty Questions* [115] 97.20 Ibn Sīnā refers to as 'our essay on propositions with modes'. Most of these unpublished manuscripts are beyond my reach under present circumstances, but I hope this book provides a framework that will help future workers on this corner of logical history.

8.3 Dating the Hypothetical Breakthrough

In the middle of the part of his *Syllogism* that deals with hypothetical syllogisms, Ibn Sīnā includes a personal note about his own involvement with this topic. He tells us that he came across a poorly written book on hypothetical syllogisms

about eighteen years after we had worked out this part of science.

(8.5) *wa ba'da an kunnā istakhrajnā hādhā al-juz'a min al-ʿilmi bi-qarībin min thamāniya ʿashara sinatan.*

Syllogism [124] vi.6, 356.9–11

There are three questions that we need to answer here. First, when did Ibn Sīnā write this remark? Second, what did he mean by 'we had worked out this part of science'? And third, what is he measuring with the number eighteen?

As to the first question: according to al-Jūzjānī's Preface to *Cure*, Ibn Sīnā worked on the logical parts of *Cure* (i.e. those corresponding to the *Organon* after his move from the prison at Fardajān to Hamadhān nearby, and he was still working on them when he left Hamadhān to join Alā' l-Dawla in Iṣfahān; he completed the logical parts of *Cure* in Iṣfahān.¹³ Ibn Sīnā's escape from the prison in Fardajān can be dated to 1024, and accordingly Gutas [74] p. 107 places the composition of *Syllogism* in 1024. It could possibly have been 1025; 1024–5 will serve for our calculations.

Turning to the second question, there are enough pointers in (8.5) to allow us to identify 'this part of science' with the Hypothetical Breakthrough described in *Twenty Questions* Question xix and in *Middle Summary in Logic* 145.17–163.6 (cf. the 'third resemblance' described in Section 8.1 above.)

¹³See [118] 3.10–12. In [117] 64f al-Jūzjānī confirms that the logical parts were completed in Iṣfahān.

Finally the third question is what period of time Ibn Sīnā is describing as ‘about eighteen years’. To the best of my knowledge the published literature contains three incompatible answers to this question. The first answer (as in Gutas [74] p. 109) is the length of time between his achieving the Hypothetical Breakthrough and his coming across a poor book on hypothetical logic. The second answer (Shehaby [197] p. 159) is the length of time between Ibn Sīnā’s completing the Hypothetical Breakthrough and his writing *Syllogism* vi.6. The third answer (Di Vincenzo [42] p. 69) is the length of time that it took Ibn Sīnā to complete the Hypothetical Breakthrough.

I argue that the third answer doesn’t fit either the grammar or the historical context of (8.5). As to the grammar: the verb *kunnā istakhrajnā* is doubly perfective, which should tell us that Ibn Sīnā has in mind the completion of the task, not the period of working on it. As to the historical context: given the self-glorifying tone of *Twenty Questions* Question xix and *Middle Summary in Logic* 145.17ff, which is still perceptible in (8.5), it would be odd to find Ibn Sīnā emphasising that a task took him many years, rather than that he achieved it when he was still young.

Gutas [74], p. 109 including footnote 13, rejects the second answer. He argues that the language of (8.5), and in particular the use of the verb *istakhrajnā*, matches what Ibn Sīnā says in his *Autobiography* about what he was doing between the ages of sixteen and seventeen.¹⁴ Eighteen years later he was between the ages of thirty-four and thirty-five. Given the rough consensus that Ibn Sīnā was writing *Syllogism* in 1024/5, this would place Ibn Sīnā’s birth in about 990, which is far too late (cf. our calculations in Chapter 1). However, this argument depends on Gutas’s claim that the Hypothetical Breakthrough occurred when Ibn Sīnā was between the ages of sixteen and seventeen. On our dating this claim must be wrong: *Short Epitome in Logic* matches the logic that Ibn Sīnā was doing in his late teens, and it contains no hint of any of the hypothetical logics that Ibn Sīnā built on top of wholly *muttaṣil* logic to complete the Hypothetical Breakthrough.

The first and second answers differ only in the end-point that they put on the eighteen years. The second answer puts this end-point around 1024/5, and the first answer puts it more vaguely a few years earlier, say

¹⁴Gutas also compares Ibn Sīnā’s claim (in (8.5) to have ‘worked out this part of science’ with his claim in *Autobiography* [117] p. 30f that around the age of eighteen ‘Everything which I knew at that time is just as I know it now; I have not added anything to it to this day’. But the passages are not comparable; that in (8.5) is about recombinant hypothetical logics, and there is no evidence to suggest that Ibn Sīnā was aware of these logics when he was around eighteen.

1021/2. The starting-point of the eighteen years is when the Hypothetical Breakthrough was completed, and this will be 1006/7 for the second answer and about 1003/4 for the first answer. I think both these results are possible, but we can cautiously nudge the date towards the first answer by considering what we know about the development of Ibn Sīnā's hypothetical logics during the Gap period.

By the dates that we have gathered so far, the Gap period runs from at the latest 994 (the date of *Short Epitome in Logic*) to 1012 (the date of *Twenty Questions*). This is a period of eighteen years. In Section 8.3 we saw that al-Barakāt's *Al-kitāb al-mu'tabar* has the effect of dividing the Gap period into two parts. In the earlier part the Hypothetical Breakthrough was begun and finished, but there is no sign that Ibn Sīnā had begun thinking about the programme of developing the notion of an absolute sentence. He must have started work on this programme sometime during the later part, since the programme is well under way in both *Twenty Questions* and *Middle Summary in Logic*. From this description it seems that the earlier part achieved more than the later part, suggesting that the earlier part was longer than the first half of the Gap period. That might place the source of the logic of *Al-kitāb al-mu'tabar* at around 1004, in line with the first answer above.

We do have some other information that might eventually help to fix the date of the source of the logic part of *Al-kitāb al-mu'tabar*. For example Gutas [74] p. 88 reports that at the end of *al-Ḥikma* in the manuscript Uppsala 364 there is a note 'Ibn Sīnā wrote this book in the year 391' (i.e. Gregorian 1001). This is compatible with the logical source of *Al-kitāb al-mu'tabar* being no later than 1004. But Gutas observes that Ibn Sīnā is not known to have given absolute dates for any of his own works. The search for further clues continues.

Next consider modal logic. We will see below (for example in the introduction to Chapter 10) that the development of modal logic to meet the Modal Challenge was probably already under way in *Twenty Questions*, but not in a solid enough form for al-Barakāt to report it convincingly. This makes it probable that the development of modal logic became serious after hypothetical logic had already gone through the Hypothetical Breakthrough. Also *Kitāb al-mu'tabar* shows no awareness of the Themistian or Alexandrine absolutes.

In sum, the evidence of *Kitāb al-mu'tabar* suggests that we should split the Gap period into two parts: a first part which includes the Hypothetical Breakthrough, and a second part in which Ibn Sīnā begins to experiment with features of his new modal logic. Al-Barakāt's source belongs to the

first part. For the date separating these two parts I have nothing to offer beyond the phrase in the title of Janssens's paper [143]: 'use of Ibn Sīnā's *al-Ḥikma al-ʿArūḍiyya* (or another work closely related to it)'. Let me suggest this date lies somewhere between 1000 and 1005 inclusive.

Chapter 9

Extending wholly *muttaşil* logic in the Gap period

9.1 Adding *munfaşil* premises

.

9.2 Adding *muhmal* sentences

.

9.3 Proofs from parts of terms

Chapter 10

Sources of new temporal modes in the Gap period

By the end of the Gap period (around 1012 or 1013) Ibn Sīnā had assembled, mostly from earlier logicians, tools that he would use to build the basis of a new modal logic. In this chapter we review what he says about these tools themselves, and how he continues to refer to them in later works. Chapter 11 will turn to the question of how he applied the tools in creating the new modal logic.

This division between chapters is convenient for exposition, but it has the unfortunate effect of suggesting that Ibn Sīnā assembled the tools during the Gap period and then applied them in the following Jurjān period. To correct that suggestion, let me say straight away that the available texts don't allow us to point to any moment when Ibn Sīnā switched from assembling the tools to applying them. The development seems to have happened more gradually and not always with a clear sense of direction. We must hope that future scholars will be able to refine the historical details.

10.1 Pseudosyllogisms and Janus sentences as sources of temporal modes

In *Prior Analytics* i.4–6 Aristotle uses a number of things that—at least when we have decoded his laconic descriptions—look like material categorical syllogisms, but on closer inspection they are not syllogisms, because their 'conclusion' is not entailed by their premises. I will call these fake syllogisms 'pseudosyllogisms'. They form part of Aristotle's machinery for

proving that a given formal categorical premise-pair is sterile. Their connection with sterility will be described in detail in [100]; but it is not what interests us here. Our present interest is in how Aristotle constructs these pseudosyllogisms. The description below describes his procedure more fully than Aristotle himself does, by using facts of logic to fill the cracks in his account. This will force us to introduce some new terminology.

Aristotle begins with a formal premise-pair Φ in one of the three figures, and finds its four candidate conclusions.¹ Then he chooses an instantiation I of the three term letters in Φ ,² such that the premises of $\Phi[I]$ are both true. At least one of the candidate conclusions $\Phi[I]$ will be true (for example exactly one of the material sentences $((e)(B, A))(I)$ and $((i)(B, A))(I)$ will be true); he chooses a candidate conclusion θ which is true under I . The pseudosyllogism that he is constructing will have as premises the instances under I of the two premises in Φ . We write $(\theta)(I)$ for the instance of θ under I . In general there is no reason why $(\theta)(I)$ should be entailed by the premises of the pseudosyllogism. If it is not entailed by them, we complete the pseudosyllogism by writing $(\theta)(I)$ as its 'pseudoconclusion' (though both Aristotle and Ibn Sīnā sometimes call it the 'conclusion', as for example below in Section 11.2 Example 1).

For Ibn Sīnā in *Twenty Questions*, the interest of Aristotle's pseudosyllogisms is that Aristotle believes that their pseudoconclusions are true sentences. In *Prior Analytics* i.8–22 Aristotle sometimes uses the same machinery where the formal premise-pairs consist of alethic modal sentences, not necessarily categorical. It's not at all clear that what Aristotle is doing here makes logical sense. But in *Twenty Questions* Question ii Ibn Sīnā is not interested in that. For him the value of Aristotle's alethic pseudosyllogisms is the same as with the categorical ones, namely that Aristotle believes that their premises and pseudoconclusions are all true.

In some of Aristotle's examples, a sentence of the pseudosyllogism is categorical and at face value it is false. Ibn Sīnā infers that in these cases Aristotle is using the categorical sentence to mean something different from its face value. Ibn Sīnā understands this situation in the light of Section 5.5 above, which argued that categorical sentences may need to be interpreted as containing unspoken conditions. In this way Ibn Sīnā collects up examples of Adding Careful Conditions. They come with the recommendation that they were taken from the text of Aristotle.

¹See (4.16) in Section 4.4 above, and the discussion following it.

²On instantiations see Section 4.5 above.

We turn to examples. In *Prior Analytics* i.9, 30a37–30b6 (a passage pointed out by Ibn Sīnā at *Twenty Questions* ii, 83.20–22) Aristotle discussed first figure modal syllogisms in which the minor premise is necessary and the major premise is absolute, i.e. categorical. Aristotle stated that neither *Darii*(NEC,ABS) nor *Ferio*(NEC,ABS) has a conclusion that is a necessary sentence, justifying this by the remark

- (10.1) As for the terms, they [can be] ‘moving’, ‘animal’ and ‘white’.
(Theodorus [139] 215.11, translating *Prior Analytics* i.9, 30b5f.)

Commentators have had difficulties making sense of this remark. But from Aristotle’s usual practice it’s sensible to make a first guess that Aristotle is forming two pseudosyllogisms. For the case of *Darii*(NEC,ABS) we guess that he is forming the pseudosyllogism

- Some white thing is necessarily an animal.
(10.2) Every animal is moving.
(Pseudoconclusion:) Possibly no white thing is moving.

For the case of *Ferio*(NEC,ABS) he is forming the pseudosyllogism

- Some white thing is necessarily an animal.
(10.3) No animal is moving.
(Pseudoconclusion:) Possibly every white thing is moving.³

Here is how Ibn Sīnā manages to agree with the readings (10.2) and (10.3) while maintaining the requirement that the sentences ‘Every animal is moving’ and ‘No animal is moving’ are both true. He has in mind the thesis of Aristotle that we examined in Section 5.5 above. Aristotle pointed out that when we speak, we often leave unspoken some details of what we meant. For logical purposes we need to make these details explicit. In the case of ‘Every animal is moving’, the speaker who believes this is true is presumably adding a tacit condition ‘at some time’. Likewise the speaker who says ‘No animal is moving’ is presumably intending this in the sense ‘Every animal is at some time not moving’. By this addition Ibn Sīnā has put into our

³Striker [210] p. 118 accepts these readings, but drops Aristotle’s usual requirement that all the sentences of the pseudosyllogisms are true. This may be because at face value the sentences ‘Every animal is moving’ and ‘No animal is moving’ are both false. She may be attributing to Aristotle the weaker requirement that in each pseudosyllogism it is imaginable—and hence logically possible—that all three sentences are true together, i.e. in the same imaginable situation. This would be evidence that the two premises of (10.2) don’t entail the conclusion that necessarily some white thing is moving, and similarly with (10.3).

hands two new sentence forms, 'Every B is at some time an A ' and 'Every B is at some time not an A '.

This explanation of Ibn Sīnā's treatment of *Prior Analytics* i.30a37–30b6 is confirmed by Ibn Sīnā's remark

- (10.4) Sometimes [the time] is not determinate, like the time (*waqt*) at which an animal moves.
(*Twenty Questions* [115] ii, 83.13)

and also by a later remark in *Twenty Questions*:

- (10.5) [Aristotle] used [the sentence] 'No horse is moving absolutely, and every animal is moving absolutely' in the sense that for every animal there is a time (*waqt*) at which it moves voluntarily, and for every horse there is some time at which it doesn't move.
(*Twenty Questions* [115] ix, 89.4–6)

At *Twenty Questions* [115] ii, 83.23f Ibn Sīnā invokes another of Aristotle's proofs of nonentailment to make the same point:

- (10.6) Also in another place he took the sentences 'Every animal is sleeping' and 'Every animal is awake' to be both true together.

Ibn Sīnā is citing *Prior Analytics* i.22, 40a36–38. He cites the same passage a little more accurately in *Middle Summary in Logic*:

- (10.7) [Aristotle] uses the fact that 'Every horse is asleep' and 'Every horse is awake' are both true together, i.e. at two [different] times. In dealing with absolute propositions one has to consider the time (*waqt*) and the condition that creates the contradiction, and this is difficult because of the different times (*zamān*) in the two cases.
(*Middle Summary in Logic* [114] 109.10–12.)

In short, in *Twenty Questions* ii and ix Ibn Sīnā is inviting us, on the authority of Aristotle, to interpret 'Every B is an A ' as 'Every B is sometimes an A ', and 'No B is an A ' as 'Every B is sometimes not an A '.

A few lines before the passage of *Twenty Questions* ii quoted at (10.4), Ibn Sīnā has told us:

- (10.8) ... the affirmative absolute [sentence] has two interpretations (*tafsīr*), the interpretation of Alexander, in which it is affirmed that the affirmation is at a time and is not permanent, and the interpretation of Themistius, which allows [the impermanence] but doesn't demand it.
(*Twenty Questions* [115] ii, 83.6f.)

In other words Ibn Sīnā is telling us that Themistius interpreted ‘Every B is an A ’ as saying that every B is at some time an A , and Alexander interpreted it as ‘Every B is an A at some time, and not an A at some other time’. Ibn Sīnā gives us no clue as to where in the writings of Themistius and Alexander he found these interpretations, and I have never seen such a passage. But probably Ibn Sīnā’s intention here is not to write about the history of logic, but to choose memorable names for the two interpretations. The connection with anything that Alexander or Themistius wrote may be tenuous. Later, when Ibn Sīnā has promoted the interpretations into new forms of sentence, he will suggest names for most of the sentence forms that he has introduced, as in his *Easterners*.⁴ At this later stage he will abandon the references he makes in *Twenty Questions* ii to Aristotle, Alexander and Themistius.

The passages of Aristotle quoted in (10.4) to (10.7) above, read as Ibn Sīnā read them, might be taken as evidence that Aristotle himself sometimes interpreted ‘Every B is an A ’ in what Ibn Sīnā calls the Themistian interpretation—though Footnote 3 above shows a different way of reading them. Also we would need further evidence before concluding that Aristotle also recognised the Alexandrine interpretation. I don’t see any place where Ibn Sīnā offers such evidence. But it does leap to the eye that the relationship between the Themistian interpretation and the Alexandrine is closely analogous to the relationship between propositions classified as possible and propositions classified as contingent. Compare:

- (Contingent:) Every B is possibly an A and possibly not an A .
 (10.9) (Alexandrine:) Every B is sometimes an A and sometimes not an A .

The formula followed in both cases is: Given the sentence ‘Every B is an A ’, replace it by

- (10.10) Every B is with mode M_1 an A , and with mode M_2 not an A .

where M_1 and M_2 are the same or similar modes. We can describe sentences of the form (10.10) as ‘Janus’ sentences, after the Roman god Janus who had two faces that looked in opposite directions.⁵

⁴See also Section 11.3 below.

⁵In later Arabic logic, Janus sentences were often described as ‘compound’ (*murakkab*), as for example by al-Kātibī in his *Shamsiyya*: ‘There are seven compound propositions’ ([209] pp. 36f). In fact al-Kātibī recognises many more than seven compound propositions, but here is restricting himself to the predicate sides (cf. Section 10.4 below) of sentences in the predicative logic of *Shamsiyya*. I don’t know any Arabic word that means the same as this restriction of ‘compound’.

As far as I am aware, Ibn Sīnā in his writings before the late *Pointers and Reminders* mentioned only two Janus sentences, and these were the two in (10.9). In both of these examples the two modes M_1 and M_2 are the same. It seems likely that Ibn Sīnā introduced the Alexandrine sentence in (10.9) on the basis that the corresponding Themistian sentence has in some way the same logic as the sentence

(10.11) Every B is possibly an A .

When we come to Ibn Sīnā's views on possibility in Chapter 13 below, we will ask what this 'same logic' could be.

But Ibn Sīnā does give a Janus sentence in *Pointers and Reminders* where the two modes are different:

Every B is, for as long as it fits the description B , an A , but it is
(10.12) not permanently [an A].

(*Pointers and Reminders* [135] i.7.4, 145.8)

(To bring this to the form (10.10), note that 'not permanently an A ' can be paraphrased as 'sometimes not an A '.) This particular form of Janus sentence became a regular feature in post-Avicennan logic from Fakhr al-Dīn al-Rāzī onwards:

Those [sentences] which assert that the predicate continues to be true [of the subject individual] for as long as the subject description applies [to the subject individual] but is not true [of the subject individual] for as long as the essence of the subject individual exists; these sentences are the anti-permanent conventional sentences. They are a compound of two restrictions: firstly that [the predicate] is true for as long as the subject description applies, and secondly that it is not true for as long as the essence exists.
(10.13)
(Al-Rāzī, *Mulakhkhaṣ* [178] 165.16–166.2)

Al-Rāzī and his successors also mention other examples of Janus sentences with $M_1 \neq M_2$, none of them as far as I know found in Ibn Sīnā. This is not the place to ask why these later logicians thought these asymmetrical Janus sentences were worth studying. But we should ask why Ibn Sīnā himself mentioned the Janus sentence in (10.12). Fortunately the answer is known; see Section BELOW REF.

We will write t for the Themistius interpretations of affirmative absolute sentences as above, and w (short for *wujūdī*) for the Alexander interpretations of these sentences. When Ibn Sīnā turns to thinking of new sentence

forms rather than new interpretations of old sentence forms, we will apply these notations *t* and *w* to the new sentence forms.

The places where Ibn Sīnā used passages from Aristotle to justify new interpretations of categorical sentences include the following:

- (10.14) (al-Barakāt, *Al-kitāb al-mu^ctabar* iii, 121.3–8;
Twenty Questions ii, 83.20–22; ix.89, 4–6;
Middle Summary in Logic 109.10–12;
Deliverance 40.5;
Syllogism i.5, 39.12f, 41.3, 43.17.

Included in (10.14) is a passage of al-Barakāt’s *Al-kitāb al-mu^ctabar*, although this is not a work of Ibn Sīnā. In this passage al-Barakāt says:

- (10.15) ... no one who conceives things accurately on hearing it would accept ‘No person is laughing’ as an absolute statement because [each man] is not laughing at some times, while he would accept that ‘Every person is laughing’ because [each man] is laughing at some times. ... Reflect on these words, and how they fall in with comprehensibility and conceivability—dispense with all they go on about, and ascertain the correctness of Aristotle’s doctrine in his extremely clear words that do without the subtleties used above.
(al-Barakāt, *Al-kitāb al-mu^ctabar* iii, 121.3–8), trans. based on Street [206] pp. 569f.)

Street op. cit. believes that al-Barakāt in this passage is attacking subtleties of Ibn Sīnā. In [97] I gave reasons for thinking that if Street is right in taking Ibn Sīnā to be al-Barakāt’s target (which is very plausible), then the passage or passages of Ibn Sīnā that al-Barakāt is attacking must be earlier than *Twenty Questions*. (For example in his writings from *Syllogism* onwards, Ibn Sīnā agrees with al-Barakāt about the interpretation of universal negative sentences in Arabic.) If we set this alongside Janssens’s evidence for his suggestion that ‘the *Mu^ctabar*’s first three sections on logic might have been strongly inspired by Ibn Sīnā’s *Ḥikma*’ (see Section 8.3 above), it looks as if al-Barakāt’s attack might be evidence for what Ibn Sīnā was saying in or before 1005.

Now al-Barakāt’s passage is missing most of the context in *Twenty Questions* ii. He knows nothing of the interpretations by Themistius and Alexander. Also he shows no knowledge of Aristotle’s discussions of sleeping

horses or moving animals; I suspect that if he had read the passages that Ibn Sīnā cites in *Twenty Questions* ii, his touching faith in Aristotle's 'extremely clear words' would have taken a knock. The texts that we have invite us to suggest that in 1005 Ibn Sīnā had noticed evidence that Aristotle sometimes interpreted 'No B is an A ' in a way that included a reference to time, but he hadn't yet realised how this could help him to develop a new modal logic.

Returning to *Twenty Questions*: in Question ii Ibn Sīnā has introduced new interpretations of some categorical sentences. In Question iii he will begin to draw out some properties of the sentences under these interpretations, starting with the fact that e -conversion fails for them. It will soon become clear that Ibn Sīnā needs to pin down more precisely what these new interpretations say. So we shouldn't be surprised that at the end of the Gap period Ibn Sīnā has begun to use devices from earlier logic that help him to be specific about sentence meanings. These devices will occupy the rest of this chapter.

10.2 Theophrastus's *tathlīth* and the six Necessary Predications

None of the interpretations of 'Every B is an A ' and 'No B is an A ' that Ibn Sīnā attributes to Aristotle, Alexander or Themistius in *Twenty Questions* Question ii carries any implication of necessity.⁶ But Ibn Sīnā was aware that some earlier logicians had found and classified some interpretations of 'Every B is an A ' that they believed did express kinds of necessity. Like the interpretations that Ibn Sīnā attributes to Alexander and Themistius, these interpretations were expressed in terms of times—but 'all times' rather than 'some times'.

Sharples [196], Fortenbaugh et al. [63] and Huby [105] all assemble the early evidence for these and related discussions. Sharples [196] traces the theme as far back as Plato, who in his *Phaedo* 103d discusses the proposition that snow, for as long as it continues to be snow, can't admit heat. Other examples are found in Aristotle's *De Interpretatione*. Alexander of Aphrodisias credits Theophrastus with listing, in his own *Prior Analytics* (now lost), different ways of using time to express necessity. From Alexander

⁶Street [207] pp. 104f disagrees, adding 'it is necessary that' to these interpretations. The word 'necessary' does occur in *Twenty Questions* ii, but not in any context that justifies adding it to these interpretations.

we know that Theophrastus's list had at least three items in it. (The relevant passages of Alexander's *Commentary on Prior Analytics i* are quoted in Greek and English at items 100A and 100B in Fortenbaugh et al. [63] and discussed in Huby [105].)

Later sources for Theophrastus's list tend to say that the list consists of three items. For example the following version is from a Hebrew precis apparently taken from an Arabic translation of Themistius's *Commentary on Prior Analytics*.

- (10.16) Theophrastus says that the necessary is said of three things: the first [N1], and the most befitting the term [necessary], is that which exists perpetually all the time, as in our saying "The heavens are ungenerated." The second [N2] is that which does not exist perpetually, but rather exists of necessity as long as the subject exists, like the existence of soul in man and heat in fire. The third [N3] is the existent thing when it exists, of which it is said that it exists of necessity, like the sitting of Reuven when he sits, or the walking in that which walks, when it walks.⁷
(Trans. Rosenberg and Manekin [190] p. 94)

Al-Fārābī had the list as follows, with no mention of Theophrastus:

- (10.17) 'Necessary' is used homonymously in three meanings. First, it means what exists permanently without interruption in the past or future. Second, what exists in a subject for as long as the subject exists (*mā dāma huwa mawjūdān*), like blueness in the eye and flatness in the nose. Thirdly, what exists in a subject for as long as it, the attribute, exists itself. For example, standing in Zayd is necessary as long as it lasts, i.e. as long as Zayd is standing.
(Al-Fārābī, *Short Treatise on De Interpretatione* [54] 162.15–19 trans. Zimmermann [227] p. 247)

Ibn Sīnā was aware of the list; he refers to it as the 'triad' (*tathlith*) at *Syllogism* i.3, 27.6 and i.5, 41.5. In *Syllogism* i.3 he tells us that 'some people'

⁷Surprisingly Themistius adds 'The [absolute] is said in the same ways as the necessary is said, for we say that movement belongs to the sun ..., and cold [belongs] to snow ..., and speech [belongs] to a speaker when he speaks ...' ([190] p. 95, their translation slightly adjusted.) Section 11.3 NtpF below will try to make sense of this addition.

recognise three cases:

- One is that [the individual satisfying] B is an A permanently, the second is that it is [an A] while it continues to fit the description B , and the third is [that it is an A] while it continues to fit the description A .
 (10.18) (Syllogism [124] i.3, 27.2–5)

However, he regarded the triad as blurring important distinctions, and he put more energy into compiling his own list of (usually six) 'necessary predications'.

We can illustrate the kind of blurring that Ibn Sīnā objected to. At *Commentary on Prior Analytics i*, 36.25–28 Alexander quotes Theophrastus as contrasting the false proposition that every literate being is always human (without further qualification) and the proposition that every literate being is human '*ést' án ê_i* literate'. This passage appears in item 100A in both Fortenbaugh et al. and Huby, but their translations of the Greek are different as follows:

- (Fortenbaugh p. 187) ... man of necessity to every literate being, while it is literate.
 (10.19) (Huby p. 88) ... man necessarily of every literate being ... while there exists a literate being.

Barnes et al. [8] p. 93 translate 'Man holds by necessity of every grammarian so long as he is a grammarian', agreeing with Fortenbaugh against Huby.⁸

Sharples p. 90 translates '[Man] necessarily applies to everything grammatical only as long as the grammatical exists', which neatly reproduces another ambiguity in the Greek—namely between 'there are grammatical individuals' and 'the (aforementioned) grammatical thing exists'. Thus we have three possible versions of this Theophrastan interpretation of 'Every B is an A ' expressing a necessity:

- (1) Every [individual satisfying] B is an A for as long as it is a B .
 (10.20) (2) Every B is an A for as long as there is a B .
 (3) Every [individual satisfying] B is an A for as long as [this individual] exists.

⁸I suggest that Alexander's text supports Huby, because it goes on to say that Theophrastus justified his contrast by pointing out that 'There are not always literate beings', which is more directly relevant to the Huby reading.

Ibn Sīnā made no use of version (2), but he took the distinction between versions (1) and (3) to be a major discovery of his own.⁹ Thus:

it is not the case that when

Every *C* is a *B* at every time when *C* is true of it.

then

- (10.21) Necessarily while [its] essence continues to exist, [each individual satisfying *C*] is a *B*.

You already know this. But our predecessors weren't able to agree with us about this, given their examples and usages.

(*Pointers and Reminders* [135] 111.2–4)

The two quoted sentences that Ibn Sīnā in (10.21) tells us his predecessors were unable to distinguish correctly are respectively (1) and (3) from (10.20), with term letters *C* and *B* in place of *B* and *A*. Ibn Sīnā obscures this with his curious phrase about the 'essence' (*dhāt*) of the individual *C*; I come back to this phrase later in this section.

From *Twenty Questions* onwards, Ibn Sīnā regularly lists his own choice of ways in which 'Every *B* is an *A*' can be interpreted as expressing a form of necessity in terms of time. This topic is introduced in *Twenty Questions* Question x, 89.11–91.6, where Ibn Sīnā lists four temporal interpretations of 'Every *C* is an *A*'. The other two surviving logical works of the Jurjān period also give classifications of sentences that express necessity in a temporal form, and similar lists appear in four later works. Altogether these lists are found in the following seven places:

- (10.22) *Twenty Questions* [115] x, 89.11–91.6
Middle Summary in Logic [114] 105.6–107.3,
Deliverance [131] 35.2–36.13 (translated [5] pp. 28–30),
Syllogism [124] i.4, 31.14–33.10,
Demonstration [125] 120.15–121.23,
Easterners [116] 68.6–20,
Pointers and Reminders [135] i.4.2, 88.9–89.11 (translated [110] pp. 92f)

⁹Ibn Sīnā is claiming here to have been the first to note that an example of NP3 need not be an example of NP2 (in the notation of the list NP1–NP6 later in the present section). Ammonius [11] 153.13ff had distinguished between NP1, NP2 and NP4. But his only examples of NP3 were equally examples of NP2. For example a fire is hot only while it is a fire; but the fire exists when and only when it is a fire.

At the head of the list in *Deliverance* Ibn Sīnā writes

- (10.23) Necessary predication takes six forms, which all have
 permanence (*dawām*) in common.
 (*Deliverance* [131] 35.2)

This justifies our description of these lists as lists of the 'six Necessary Predications'—though the length of the list is not six in all cases.

We can summarise Ibn Sīnā's six Necessary Predications as follows.

- NP1** Everything that is a *B* is an *A* at all times.
NP2 Everything *x* that is a *B* is an *A* at all times when *x* exists.
NP3 Everything *x* that is a *B* is an *A* at all times when *x* is a *B*.
NP4 Everything *x* that is a *B* is an *A* at all times when *x* is an *A*.
NP5 Everything *x* that is a *B* is an *A* at all those times τ at which $\phi(x, \tau)$.
 [Here ϕ is a condition specifying a definite time τ for each *x*. For example *A* might say that *x* is an eclipsed moon, with $\phi(x, \tau)$ saying that the earth lies between the moon and the sun at time τ .]
NP6 Everything *x* that is a *B* is an *A* at all those times τ at which $\phi(x, \tau)$.
 [Here ϕ is a condition which doesn't specify a definite time τ for each *x*.]

As I read them, Ibn Sīnā's seven lists contain the following necessary predications:¹⁰

	NP1	NP2	NP3	NP4	NP5	NP6
<i>Twenty Questions</i>	×	✓	✓	✓	✓	×
<i>Middle Summary</i>	✓	✓	✓	✓	✓	✓
<i>Deliverance</i>	✓	✓	✓	✓	✓	✓
<i>Syllogism</i>	✓	✓	✓	✓	✓	✓
<i>Demonstration</i>	✓	✓	✓	✓	✓	×
<i>Easterners</i>	×	✓	✓	✓	✓	✓
<i>Pointers</i>	✓	✓	✓	×	×	×

¹⁰The list in *Twenty Questions* is the only list from the Gap and Jurjān periods which lists four items; the other lists from this period list six items. This might indicate that the list in *Twenty Questions* is an earlier form of the list than those in *Middle Summary in Logic* and *Deliverance*, and is still escaping from Theophrastus's triad.

Already in *Middle Summary in Logic* Ibn Sīnā is looking for useful descriptions of the new interpretations, but he continues to innovate with the names of modal sentence forms for the rest of his career. Section 11.3 below will have more to say about this. For the present we note:

NP2 *Middle Summary in Logic* describes these as ‘necessary’ (*ḍarūrī*) at *Middle Summary in Logic* [114] 105.6–9 and as ‘paradigmally necessary’ (*ḍarūrī ḥaqīqī*) at *Middle Summary in Logic* [114] 105.9. The former, which Ibn Sīnā continues to use for NP2 for the rest of his career, may be shorthand for the latter. We use *d* to name NP2.

NP3 In *Middle Summary in Logic* [114] 115.16–116.2 these are described as ‘useful for the sciences’ and as ‘customary’ (*muta‘āraf*). In al-Rāzī and later logicians this second word is simplified to ‘conventional’ (*‘urfī*). But in *Easterners* [116] 14.1 Ibn Sīnā adopted the name ‘entailed’ (*lāzim*) for these sentences; we use this name, abbreviated to *ℓ*. Section 14.3 will consider what Ibn Sīnā meant by this name.

10.3 Logical properties of the six Necessary Predications

In Chapters 11 and 12 below we will examine the logical forms and rules of the sentences of Ibn Sīnā’s temporal predicative logic. As a preparation for this we will now consider some of the logical properties of Ibn Sīnā’s necessary predication sentences.

The first logical property of these sentences is that they are two-dimensional in the sense of Section 6.1 above. (See in particular Footnote 3 of that section.) This means that their terms are 2-ary, i.e. true or false of *pairs* of things; the pairs consist of an object and a time. We need to be careful here: if the reference to a time is quantified, as in

(10.24) Every *B* is at some time an *A*.

then the second term can be taken to be not *A* but ‘at some time an *A*’, which is a one-dimensional term as in categorical logic. (This is an example of ‘incorporating’ a temporal mode in a term.) To show that terms are genuinely 2-ary (i.e. two-dimensional) we have to show that in reasoning with the term, we need to be able to handle the term separately from the quantification over time. Ibn Sīnā makes the following statement, which

settles the matter:

- (10.25) [At a time] when there are some *B*s and moreover the *B*s don't satisfy the description *A*, then *at that time* (*fī dhālika l-waqt*) there is nothing that is both a *B* and an *A*.
(*Syllogism* [124] iii.1, 134.1f, cf. iii.1, 133.11–13)

If the reference to a time is hidden inside the terms *A* and *B*, then we have no way of saying that it is the same time in both terms. In modern logic this point can be covered by writing *A* as $A(x, \tau)$ where x is a variable for an object and τ is a variable for a time, and similarly with *B*. Thus $A(x, \tau)$ is true of the pair (a, α) if we get a true sentence by reading x as referring to the object a and τ as referring to the time α . Using *A* and *B* as 2-ary terms in this way, (10.25) can be expressed by:

- (10.26) For any time τ , if there are some x such that $B(x, \tau)$ but there is no x such that $B(x, \tau)$ and $A(x, \tau)$, then there is no x such that $B(x, \tau)$ and $A(x, \tau)$.

There are several logical notations that can be used to refine (10.26), but they have in common that their syntax shows that the terms *A* and *B* are 2-ary. Ibn Sīnā was restricted to writing his sentences in Arabic or Persian, and this prevented him using the notation (x, τ) to make a term 2-ary.¹¹ This limited his progress with temporal logic.

We can extract a second logical property from the way in which Ibn Sīnā distinguishes between NP1 sentences and NP2 sentences. In *Deliverance* Ibn Sīnā tells us that in NP1 sentences

- (10.27) the predication is permanent, it never ceased nor will it cease, as when we say: "Allah the Exalted is alive".
(*Deliverance* [131] 35.3f)

In other words the implied meaning, for the quoted sentence, is that Allah is alive at τ for every time τ . (Or as we will say, Allah is 'eternal'.) He contrasts this with the NP2 sentence

- (10.28) "Every human is necessarily an animal", i.e. every person is permanently an animal . . . though he is not permanently and unconditionally [an animal] in such a way that he would be an animal without ever having not been [an animal] and without ever ceasing to be one, before he comes into existence or after he goes out of existence.
(*Deliverance* [131] 35.5–8.)

¹¹Without this or an equivalent notation it was almost impossible to avoid overload between 2-ary and 1-ary terms.

(We will fill in the dotted part of this quotation in a moment.) Ibn Sīnā is using the fact that every human has a finite life span, so that there are times before she exists and times after she has ceased to exist. During these times of nonexistence, Ibn Sīnā tells us, the human is not an animal.

What general logical fact is Ibn Sīnā intending to illustrate with this example of an NP2 sentence? The example should be compared with his statement

- (10.29) Being dead is incompatible with being an animal.¹²
(*De Interpretatione* [123] 100.14)

and also with

- (10.30) If the icosahedron doesn't exist then it doesn't satisfy any description, because being nonexistent it can't satisfy any description.
(*De Interpretatione* [123] 80.1f.)

and

- (10.31) An unsatisfied [subject] doesn't satisfy the description 'white colour' or have any affirmative property.
(*Syllogism* [124] iii.1, 135.9.)

Clearly Ibn Sīnā sees some difficulty in ascribing a property to something that doesn't exist. It's not clear how this difficulty passes over to situations in time, where an object can go in and out of existence. Here I take a deep breath and suggest that we can ascribe to Ibn Sīnā the belief that

- (10.32) For every individual x and every time τ , if x is a B at time τ then x exists at time τ . (Or for short, you don't have properties when you don't exist.)

This might be too strong: if we take B to be the term 'nonexistent', the sentence (10.32) could be self-contradictory. In an ideal world Ibn Sīnā would have noticed this point and written something to deal with it. But there is no sign that he did. I can imagine him imposing what in *Syllogism* [124] i.5, 41.13 he calls an 'affirmative condition', perhaps that B is an affirmative term. But I think the best we can do is to accept the guess (10.32) and hope that we avoid contradictions.

¹²If being dead implies you are not an animal, then arguably it must also imply that you don't exist. Ebbesen [44] collects and discusses Peripatetic claims of this kind.

We turn to the passage replaced by dots in (10.28). Restoring this passage gives the sentence

- (10.33) “Every human is necessarily an animal”, i.e. every person is permanently an animal, **for as long as his essence (*dhāt*) continues to exist**, though he is not permanently and unconditionally [an animal] in such a way that he would be an animal without ever having not been [an animal] and without ever ceasing to be one, before he comes into existence or after he goes out of existence. (*Deliverance* [131] 35.5–8.)

The last part of this sentence (from ‘though he is not’ onwards) serves to clarify what is meant by the phrase ‘his essence continues to exist’. Namely, the phrase carries the same information as ‘he (the human) continues to exist’.

This talk of essences appears first in *Twenty Questions, Middle Summary in Logic* and *Deliverance*, at the end of the Gap period and through the Jurjān period. It takes one of two forms. In the first form Ibn Sīnā makes no distinction between saying

- (10.34) The subject individual exists (*mawjūd*).

and saying

- (10.35) The [individual] essence of the subject individual exists (*mawjūd*).

In the second form he makes no distinction between saying

- (10.36) The subject individual satisfies the term *A* at a time.

and saying

- (10.37) The [individual]¹³ essence of the subject individual satisfies the term *A* at a time.

This indifference between ascribing existence or properties to the subject individual and to its individual essence can be seen at *Middle Summary in*

¹³At *Twenty Questions* Question x, 91.15–22 Ibn Sīnā distinguishes three essences: (1) the essence of a term *B*, (2) the essence of an individual that is necessarily a *B* and (3) the essence of an individual that is at least once a *B*. On my understanding the essence referred to in (10.32) and (10.34) is the individual essence of the relevant individual, as in (2) or (3) but not (1). This reading is confirmed by (11.16) in Chapter 11.

Logic [114] 106.1–6, *Deliverance* [131] 46.1f and *Syllogism* [124] iii.1, 128.9–11 and iv.2, 196.7.

In logical contexts this same indifference saves us from needing to explore the ontological properties of essences. Of course one can ask why Ibn Sīnā felt a need to mention individual essences in this context. Perhaps it was a stylistic device to emphasise the importance that Ibn Sīnā attached to the notions expressed in (10.32) and (10.34).¹⁴

There remain NP4, NP5 and NP6. NP4 was one of the three items in Theophrastus’s triad; see [N3] in (10.16) above. Ibn Sīnā regularly mentions it as one of the Necessary Predications (except in *Pointers and Reminders*). But he never associates it with any form of reasoning.

NP5 and NP6 illustrate a different kind of connection between necessity and time, namely that some states of affairs can be described as being necessary at such-and-such a time. We can refer to this kind of necessity as a ‘temporary necessity’. The difference between NP5 and NP6 is that an NP5 sentence specifies a definite time when the necessity applies, whereas an NP6 sentence leaves this time indefinite.

In one of the earliest lists of Necessary Predications, that in *Middle Summary in Logic*, Ibn Sīnā tells us that in some NP5 sentences the condition mentions a temporary cause which compels the subject individual to be an *A* and also opposes this:

(10.38) There are four subdivisions of these necessities. . . . Another [of the four] is where the condition is an opposition (*ʾitirāḍ*) and a cause which compels the judgment at a determinate time
(*Middle Summary in Logic* [114] 105.18–20.)

This should probably be put alongside al-Barakāt *Kitāb al-muʿtabar* ii.3 (‘On modes of propositions’), where he discusses temporary necessity in terms of the causes which act at a given time; for example it is impossible for the planets to rise or set at a time when no cause is able to compel them.¹⁵ Bearing in mind Janssens’s claim about the dependence of *Kitāb al-muʿtabar* on an early work of Ibn Sīnā, the parallel between *Kitāb al-muʿtabar* and *Middle Summary in Logic* could be evidence that Ibn Sīnā was already thinking

¹⁴Or perhaps it was a reminder to the reader that syllogisms are performed in the intellect, so that their essential components must be ideas. If I perform a syllogism about the Zagros mountain range, the range is not literally in my mind, but its individual essence is an idea that I can think.

¹⁵At *Kitāb al-muʿtabar* ii.3, 82.15–18. Could ‘opposition’ in (10.35) be a technical term, say for a temporary decoupling of a cause from its effect?

of putting NP5 and NP6 into the Necessary Predications early in the first decade of the 11th century. Ibn Sīnā continued to include NP5 in the Necessary Predications at least up until *Easterners*, but it is noticeable that he never came near forming a logic of temporary necessity.

10.4 Semantic expansions and al-Fārābī's *dictum de omni*

This section and the next contain material from [94]. SOME OF THIS SECTION NEEDS A MAJOR REWRITE.

The part of al-Fārābī's *Long Commentary on Prior Analytics* (for short, *Long Commentary*) which comments on *Prior Analytics* book i is unfortunately missing. What we know about it comes mostly from quotations and discussions by Ibn Sīnā, Ibn Rushd and Maimonides.

There is evidence that some passages in Ibn Sīnā's *Syllogism* contain material from *Long Commentary*. For example at *Syllogism* [124] iv.4, 210.4–211.2 Ibn Sīnā quotes an argument which he attributes to 'the outstanding [scholar]' (*al-fāḍil*); see Street [205] for convincing evidence that this is Ibn Sīnā's name for al-Fārābī. Ibn Rushd discusses the same argument, saying that it is due to al-Fārābī. Since the argument is about conversions and is not found in those logical works of al-Fārābī that we do have, the missing part of *Long Commentary* is much the most likely source. *Twenty Questions* viii attacks a very similar argument—both arguments are unusual in trying to base a conversion on the phrase *min jiha* ('from the point of view that' or 'insofar as'). If all these relationships are solid, then Ibn Sīnā must have studied al-Fārābī's *Long Commentary* some time before he wrote *Twenty Questions*.

And in that case it's a priori likely that Ibn Sīnā read in *Long Commentary* four passages that Ibn Rushd quotes in his *Maqālāt* [108] at 128.5–7, 129.6f, 133.2–4 and 146.9f.¹⁶ The four passages are said to give the meanings of universal sentences; they are best regarded as verbal blueprints for constructing the meanings of universal predicative sentences. Al-Fārābī describes them as 'said of all' (Arabic *maqūl 'alā l-kull*, in Latin *dictum de omni*, or for short *dictum*).¹⁷ We can assume with confidence that Ibn Sīnā was aware of these passages in *Long Commentary*, because in his own logical writings from *Twenty Questions* onwards he gives very similar but more complicated explanatory sentences, several of which he calls *maqūl 'alā l-*

¹⁶The first three of these passages are from *Maqāla*.1.3.5, which I believe is not known in Latin translation. The fourth is from *Maqāla* 1.3.6, which in Latin translation is *Quaesitum* 6.

¹⁷The plural of *dictum* is *dicta*.

kull. In Section 10.5 below we will list twenty-five examples from Ibn Sīnā. But for the present we stay with al-Fārābī's versions of the *dictum*.

The versions of al-Fārābī's *dictum* quoted by Ibn Rushd at [108] pp. 128 and 146 give abstract forms for the meaning of 'Every *B* is/isn't an *A*', as follows:

(10.39) [The *dictum de omni* stipulates that:] *A* is made a predicate, either affirmatively or negatively and with whatever mode the predication happens to be made, of everything that is described as a *B* in such a way that *B* is affirmed as actually true of this thing.

(10.40) [The *dictum de omni* is that:] The predicate extreme *A* is affirmed or denied, either as *wujūdī* or as necessary or as possible, of everything that is an actual or possible or necessary *B*.

Besides the four *dicta* mentioned above, there is another passage from Ibn Rushd from which we can deduce that al-Fārābī also used a version of the *dictum* that allowed the subject term *B* to be negated:¹⁸

(10.41) Abū Naṣr [al-Fārābī] thought that the meaning of the *dictum de omni* makes some pairs of negative premises productive. ... For example 'What is not an animal is not a human' combined with 'The statue is not an animal' yields that the statue is not a human. (Ibn Rushd [108] 154.17–19)

Since (??) allows the subject term to be negated, but neither of (10.36) and (10.37) allows this, we can see that al-Fārābī used different *dicta* to describe different classes of sentence form, some more inclusive than others.

The following is my attempt (in [94]) to extract from Ibn Rushd's exam-

¹⁸There are some ad hoc examples of negated subject terms already in Aristotle, but as far as we know, it was Ibn Sīnā in *Syllogism* vi.2 who first developed a logic in which negation of the subject term is fully allowed; see [99] in preparation. Boole [29] pp. 138–141 credits De Morgan with describing a logical system which extends categorical syllogistic and allows every occurrence of a term to be negated. In fact work of Leibniz, later than Ibn Sīnā but independent, implies that Leibniz had already made this advance; cf. Malink and Vasudevan [159].

presents a most inclusive version of al-Fārābī's *dictum*:

Everything said to	be not be	actually necessarily possibly	a <i>B</i>
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is said to	be not be	actually necessarily possibly	an <i>A</i>
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As the diagram shows, the *dictum* can be broken into two 'sides' whose structures are quite similar to each other. The first side finishes with the subject term *B* and the second finishes with the predicate term *A*, so it would be natural to speak of the 'subject side' (*jānib al-mawḍūʿ*) and the 'predicate side' (*jānib al-maḥmūl*). We don't have enough of al-Fārābī's text to see whether he uses this terminology or something like it. But Ibn Sīnā certainly does when he adopts his own refinement of the *dictum*. In *Syllogism* [124] i.3, 19.16–21.12 he discusses the meaning of the quantifier in a quantified predicative sentence, finishing with the remark

This takes care of the subject side [of the sentence]. As for the
(10.42) predicate side ...
(*Syllogism* [124] i.3, 21.12f.)

In *Syllogism* i.3, 21.13–26.5 Ibn Sīnā turns to describing the meanings of several temporal additions that could be put in the predicate side, including those of the Necessary Predications. This use of 'side' appears briefly in both *Twenty Questions* (Question viii, 88.8 'the subject side') and *Middle Summary in Logic* (102.12 'the predicate side', 217.1 'the subject side').

Each of the two sides of the *dictum* can contain a mode, showing that al-Fārābī allows the universal sentence to have two modes, one on the subject term *B* and one on the predicate term *A*. This innovation will be a

feature of Ibn Sīnā's new temporal sentence forms.¹⁹ Also al-Fārābī allows the sentence to quantify over all individuals that are 'actual' (*bi l-fi'l*) *B*s, regardless of the mode on *A*.²⁰

Al-Fārābī's *dictum* is limited to describing the meanings of universal sentences (affirmative or negative). But Ibn Sīnā tells us that the meaning of an existential predicative sentence can be inferred from the corresponding universal sentence by simply changing the quantifier. For example 'Some person is an *A*' means the same as 'Every person is an *A*' but applied only to some people.²¹

Before we move on from al-Fārābī's *dictum* to Ibn Sīnā's *dictum*, this is an appropriate moment to take note of some habits of Peripatetic philosophers when they specify the meaning of a phrase.

Firstly, they would sometimes take the phrase and add other words around it until the meaning was unambiguous. A standard example was that we disambiguate 'Ajax' by expanding it to 'Ajax the son of Oeleus of Locris' or 'Ajax the son of Telamon of Salamina'.²² We can refer to this style of pinning down the meaning as 'semantic expansion'.

The method of semantic expansion lends itself to explaining things in conversation, where the other person can say 'Got it, thanks' or 'Could you explain it a bit more, please?'. But in a book the author has to guess what is the right amount to add in order to make the meaning clear. The result is that different explanations of the same meaning, even in the same book, tend to be fuller or thinner by chance. For example about half of the *dicta* that we have from Ibn Sīnā say that the universal quantifier is over things that are actual *B*s; the remaining *dicta* say nothing about being an actual *B*.

¹⁹Some modern scholars have taken *Prior Analytics* i.13, 33a25–30 as evidence that Aristotle already allowed modalisation of the subject. But Theodorus's Arabic translation of this passage can be read in other ways; possibly the Arabic retains a sense that was lost in the Greek tradition. There is slightly stronger evidence that Alexander in his *On Conversion* [14] allowed modalisation of the subject, and this may have influenced al-Fārābī. Hodges [94] Section 3 discusses these texts.

²⁰This seems to go against the common statement that al-Fārābī ampliates in modal sentences; see BELOW on ampliation.

²¹*Middle Summary in Logic* [114] 53.4f. It's important to note that Ibn Sīnā replaces the entire relativised quantifier 'Every person' by the relativised quantifier 'Some person'. In first-order logic the sentence that comes from $\forall x(Bx \rightarrow Ax)$ by the corresponding replacement is $\exists x(Bx \wedge Ax)$. By contrast, replacing the unrelativised quantifier $\forall x$ by the unrelativised quantifier $\exists x$ gives $\exists x(Bx \rightarrow Ax)$, which expresses the quite different statement that either something is not a *B* or something is an *A*, or both.

²²As in Porphyry [43] p. 44 (64.10-21).

The default understanding has to be that Ibn Sīnā always intends 'actual *Bs*' but often omits to mention it. Anybody who wants to use Ibn Sīnā's *dicta* as evidence for his views would be well advised to consult all the examples together.

Secondly, when explaining the meaning of a sentence *S*, Peripatetic philosophers tended to describe what is going on in the mind of a person who is (either aloud or purely mentally) intending *S*. So for example a person who intends 'This is a horse' mentally describes the relevant thing as a horse, and as a result the explanation of the meaning contains the phrase 'is described as a horse'. (That accounts for the 'is said to be' in the picture above.) This style of explanation can come into conflict with the alternative style (which we also meet in Peripatetic philosophers) of describing the conditions under which the sentence *S* counts as true. For example the truth conditions for 'This is a horse' don't include anything about anybody making a description.

But the mentalistic style of explanation can have advantages. For example when describing the state of mind of a person intending *S*, we can add negative information about what this person is *not* intending, so as to say that some possibilities are left open by *S*. This can be trickier with the truth-conditional style, because a negative statement there is easily misunderstood as saying that the sentence *S* rules out certain possibilities.

For example Ibn Sīnā's *dictum* at *Middle Summary in Logic* 108.21f describes the subject term (which the context shows is *B*) as selecting

- (10.43) each individual that is described by the subject term however it is (*kayfa kāna*), permanently or not permanently.

This could be read as saying that there are two kinds of universal necessary sentence, one of them applying the predicate to every individual that satisfies the subject term permanently, and the other applying the predicate to every individual that satisfies the subject term for only a limited period of time. In fact what (10.14) says is that the person stating or thinking the sentence has in mind that the predicate is true both of individuals that are always *Bs* and individuals that are only sometimes *Bs*. In other words the speaker or thinker intends 'at least sometimes a *B*'.

10.5 Ibn Sīnā's *dictum de omni*

There is a curious form of sentence that occurs frequently in Ibn Sīnā's logical writings from *Twenty Questions* onwards. The details of the sentence

vary from one occurrence to another, but the overall form and style are quite distinctive. They are clearly modelled on al-Fārābī's *dicta*, and Ibn Sīnā himself sometimes refers to them as 'said of all' (*maqūl 'alā l-kull*, Latin *dictum de omni*). We will refer to the sentence in its various forms as the *dictum*, or as Ibn Sīnā's *dictum*.

The exact number of examples of the *dictum* in Ibn Sīnā's logical writings is open to personal preference. But for the sake of definiteness I counted twenty-seven examples, as follows:

	<i>Twenty Questions</i> [115]	DDO.1: x, 92.10–17;
	<i>Middle Summary in Logic</i> [114]	DDO.2: 100.16–19;
		DDO.3: 100.19–101.1;
		DDO.4: 102.17–19; ;
		DDO.5: 103.7–11;
		DDO.6: 103.12–16;
		DDO.7: 108.7–11;
		DDO.8: 108.21f;
		DDO.9: 131.2–6;
		DDO.10: 132.10–12;
	<i>Deliverance</i> [131]	DDO.11: 41.8–12;
		DDO.12: 42.10–12;
		DDO.13: 43.12–14;
		DDO.14: 66.6–9;
(10.44)		DDO.15: 133.6–10;
	<i>Guidance</i> [121]	DDO.16: 79.3–5;
	<i>Syllogism</i> [124]	DDO.17: 26.18–27.2;
		DDO.18: 31.14–32.1;
		DDO.19: 33.11–14;
		DDO.20: 33.15–34.6;
		DDO.21: 127.11f;
	<i>Easterners</i> [116]	DDO.22: 64.3–6;
		DDO.23: 64.14–16;
		DDO.24: 68.6–8;
		DDO.25: 69.12–14;
	<i>Pointers and Reminders</i> [135]	DDO.26: 72.17–73.3, Inati para. 226;
		DDO.27: 93.9–14, Inati paras. 280, 282 ²³

²³The Inati paragraph numbers are printed in the margins of Inati [110]; they are the page

Every *dictum* describes the structure of a universal predicative sentence, sometimes in general and sometimes only in some special cases. The description can be extended to existential predicative sentences by writing 'Some *B*' in place of 'Every *B*' and to singular sentences by writing a proper name in place of the quantified subject term. (*Muhmal* sentences are more complicated; see BELOW, REFS.) Also from *Short Epitome in Logic* onwards, Ibn Sīnā makes recombinant *muttaṣil* sentences borrow their structures from predicative sentences, so that the *dictum* can be applied at least to these hypothetical sentences.

See the discussion around (10.39) CHECK above for the distinction between the subject side and the predicate side of a *dictum*.

The *dicta* form a stable thread throughout the second half of Ibn Sīnā's logical career, from *Twenty Questions* in around 1012 to *Pointers and Reminders* in around 1030. They must surely be worth studying, for example to see in what ways they control Ibn Sīnā's constructions and interpretations of modal sentences.

For example the subject side of a *dictum* tells us something about the class of individuals that is quantified over in a predicative sentence that is a modalised version of "Every *B* is an *A*". Some of the *dicta* tell us that the quantification ranges over the individuals that are *B*s (i.e. that satisfy the term *B*) 'in act' (*bi l-fʿl*).²⁴

For this reason I devote most of this section to offering translations—some more tentative than others—of all twenty-seven *dicta* in the hope that someone will pick them up and run with them.

For example note that al-Fārābī allows both the subject and the predicate to be modalised independently of each other. At *Twenty Questions* [115] x, 91.10f Ibn Sīnā considers the sentence

(10.45) Everything that is with necessity a *B* is with necessity an *A*.

The necessity condition on the subject is unusual and I don't know of any other place where Ibn Sīnā suggests it. But in Question xiii he implies that a universal *ℓ* premise should be read with the condition 'at some time' on the subject. Later he will apply this condition for all temporal predicative sentences, and his wording in (10.47) below suggests to me that he already applied it in *Twenty Questions*, though he forgot to tell us so. The syllogism

numbers from the edition of Dunyā [136].

²⁴Namely the DOI listed below with numbers.

is:

- (10.46) Every *C* is a *B* with necessity.
 Every *B* is an *A* while it continues to fit the description *B*.
 So *C* will be an *A* with necessity.
 (*Twenty Questions* [115] xiii, extracted from 98.2–8)

Since the supporting argument is given entirely in temporal terms, the ‘necessity’ here has to be read as *d.* At 98.4 Ibn Sīnā tells us

- (10.47) The content in the major premise is just about what is a *B* at some particular time.

applying a temporal condition to the subject.

DDO.1. /92.10/ What is said of the all [i.e. *dictum de omni*] and the syllogism, it is that every subject [individual] in which there is found what belongs to the nature of the subject, and what is described by it, there is found in it that content; as if it is not related to the essence of ‘coloured white’, for example in the sense that it is coloured white alone, but rather to the subject [individuals] that have it. And there is no doubt that its definitions, from the aspect of whiteness agreeing, the description attaches to them or doesn’t attach. So when you say ‘every white thing’ or ‘every *B*’, [the subject individual] has to satisfy the [subject] term, regardless of whether [the individual] is white or /92.15/ *B* with necessity or without it. Then when [the sentence] expresses a judgment made with necessity or possibility or affirmation or denial, it is applied to each individual that fits the description given by the subject term and that is posited [as a subject individual] where it is (for example) white with necessity or without necessity.

(*Twenty Questions* [115] x, 92.10–17)

DDO.2. /100.16/ The meaning of the phrase ‘every *B*’ is: each individual thing that is according to the intellect described as being in act *B*. This can be understood in two ways. The first of the two is: each individual thing that according to the intellect is a *B* and the intellect has assumed that.

(*Middle Summary in Logic* 100.16–19)

DDO.3. The second is: each individual thing that according to the intellect is a *B*, where its being described as a *B* is something it has in act and not under some interpretation, /100.20/ and the intellect has assumed that—or has not assumed it, given that *B* is definitely true of it in act /101/ at some time.

(*Middle Summary in Logic* 100.19–101.1)

DDO.4. while it is not described as being a *B* or that the loss of its description as an *A* when it loses the description *B* is obligatory, as with the sentence

(10.48) Every moving thing is a body.

i.e. everything that is described in the intellect in act as moving, that thing also is described together with that as being a body and if there remains true of it the description that it is a body even if the description of its being moving is lost.

(*Middle Summary in Logic* 102.17–19)

DDO.5. And as for in terms of the thing that is compelled by study that is verified and more general, namely that it is the meaning of the sentence 'Every *B* is an *A*', i.e. is **what is described as in act *B* of course it doesn't make clear when, given that there is to it that in act a time regardless of whether it is at the time of the content or another time and not determined**—that thing is described as being in act an *A* also, /103.10/ but we don't know when the *A* is described as being a *B* /85.15/ or at another time before it or after it or simultaneous with it in some interval and not at another or in every **interval during which it exists** (*mudda*), because our phrase 'It is an *A* in act' is more general than all of those.

(*Middle Summary in Logic* 103.7–11)

DDO.6. And likewise in the negative the sentence 'No *B* is an *A*' really means 'Those things that are described, in act in the intellect, as being a *B* in any way and at any time, given that they are described as being a *B*, no such thing is described as being an *A*, but we don't know whether it is an *A* at a particular time or permanently, regardless of whether it satisfies (*ma^ca*) this description of it at some specific time, or at every time /103.15/ in agreement with (*ma^ca*) its being a *B* permanently, or with (*ma^ca*) its being a *B* at some time at which it is a *B*, or before or after that [time].

(*Middle Summary in Logic* 103.12–16)

I gave up trying to make an exact translation of DDO.6, and particularly of the three occurrences of *ma^ca* 'together with' in it. See the discussion of NtsC in Section 11.2 below.

DDO.7. And know that when you say ‘Every B is an A ’ it is as if you say that everything fitting the description B in any way and without explaining how that is, it can be found for it A without any explanation of when it is found, then whenever it is found, $[A]$ is true [of it], and the condition only requires the necessity which we intend from the sense of the proposition, so it generalises what was restricted in the absolute to mention of the content only. /108.10/ There is added in the necessary and possible a mention of the **content** together with its being necessary or possible, and likewise in the negative.

(Middle Summary in Logic 108.7–11)

DDO.8. And as for the universal necessary [sentence], it is what is judged with negation or affirmation of the predicate, permanently for each individual that is described by the subject term however it is, permanently or not permanently.

(Middle Summary in Logic 108.21f)

DDO.9. [For proving $Barbara(t, d.d):$] ... the sentence ‘Every B is an A ’ means that everything described as a B and posited on B , in whatever way it is described as $[a B]$ and posited $[on B]$, with necessity or without necessity—given that it is described in this way, it is described also as being with necessity an A . What is moving permanently doesn’t differ from what is moving at a particular time /131.5/ in that it is moving and is described as moving [at some time]; and C is posited on B , so [a thing described as a C] is included in the content [of the second premise]. When this aspect is understood, the outcome is that anyone who disagrees with the Philosopher on this topic is mistaken.

(Middle Summary in Logic 131.2–6)

DDO.10. If C is put as B /132.10/ there attaches to it what is said about every B , or B is put as C there attaches to it what was said of every C /52b15/ on the condition which we mentioned **about what was said about the universal**, that its meaning is that everything that is described by the subject term and posited on the subject in any way as a necessary description or an unnecessary one, [every such thing] is described by the predicate or denied of the predicate.

(Middle Summary in Logic 132.9–12)

DDO.11. And the sentence 'Every B is an A ' in the absolute means that every one of the things described in the intellect or reality as being a B , regardless of whether it is described as being a B permanently, or is described /41.10/ as being a B at some time after it is not a B , this thing is described as being an A , we don't know when, whether as soon as it is described as B or at some other time, and whether permanently or not permanently; this is according to the opinion of Theophrastus.²⁵
(*Deliverance* 41.8–12)

DDO.12. The sentence 'Every B is an A necessarily' means that every single thing that is described by the intellect as a B , whether permanently or not, this thing is permanently, so long as its essence is instantiated the same(?), described as an A .
(*Deliverance* 42.10–12)

DDO.13. The meaning of the sentence 'Every B is an A with possibility' is that for every single thing described as being a B , in any way, A is affirmed of that thing without necessity, and when this affirmation is assumed to apply to some object in hand, no impossibility results from this.
(*Deliverance* 43.12–14)

DDO.14. ... the sentence 'Every B is an A with necessity' or 'With necessity no B is an A ' means that every single thing described as a B and posited as a B and which is a B at some time, necessarily or not necessarily, perpetually or not perpetually, this thing is described as permanently at every time an A , or it is not described as an A at any time at all.
(*Deliverance* 66.6–9)

DDO.15. What is said of 'everything' in this case is not the same as in the Book of Syllogism. In fact the meaning of something said of 'everything' in this case is that it is said of each individual thing in every time, so long as it continues to be described by what was put as subject [term] with it because the universals of demonstration are necessary and unchanging. And the universally quantified in this case has conditions added to it; in fact it needs to be predicated of /133.10/ each individual at every time; and besides that it is a primary statement.
(*Deliverance* 133.6–10)

²⁵ At *Deliverance* [131] 39.10 Ibn Sīnā clarifies that this is also the view of Themistius. At *Deliverance* 41.13 he tells us that the opinion of Alexander about the subject part doesn't disagree with that of Themistius.

DDO.16. Every *A* is a *B*, i.e. every one of the things described in fact as an *A* is permanently or at some time whatever time it is is described as being a *B* however it is without identification of a time.

(*Guidance* 79.3–5)

DDO.17. So it is now clear from this that the sentence ‘Every *B* is an *A*’ means that each separate one of the things described and assumed to be actually a *B*, permanently or not permanently, is also described as an *A* without referring to when, or to which of the three segments of time it is in.

(*Syllogism* 26.18–27.2)

DDO.18. /31.14/ The best thing we can do now is to talk about universally quantified affirmative necessary propositions. The meaning of the sentence ‘Every *B* is an *A* with necessity’ is that each separate one of the things that are described as actually a *B*, whether it is permanently a *B* or a *B* at a certain time, this thing is described as an *A*, not just while it is described as a *B*, and not at a determinate time; but rather so long as the essence of this thing which is described as a *B* is satisfied.

(*Syllogism* 31.14–32.1)

DDO.19. [1.4.15] The affirmative universally quantified possibility proposition is as when one says ‘Every *B* is an *A* with possibility.’ It has two meanings. The first is that each thing that is described as a *B*, either permanently or not permanently, is not necessarily an *A* and not necessarily not an *A*—where [the proposition] is not considered under a condition, and it makes no reference to the thing being [an *A*] at some specific time, or to its having to be never [an *A*], or to its being tied to [being an *A*] permanently.

(*Syllogism* 33.11–14)

DDO.20. The second meaning is that each one of the things that are described as being actually a *B*, either permanently or not permanently, is [an *A*] at some time, where this time is assumed to exist for it, i.e. a future time which is specific for it; this time is assumed insofar as it is possible for that content to be true at it and [possible] for it not to be, it is not a future at a time defined at the outset (?). So in this case the subjects are subjects of a special kind, [namely] they are the subjects which are described as *B*s at some determinate time, and the future under consideration is the future of a defined time that is under consideration. So the phrase ‘Every *B*’ is not

used of everything that is described as a *B* at any time, and this is contrary to what we assumed [in the first meaning?].

(*Syllogism* 33.15–34.6)

DDO.21.

(10.49) Everything that is either with necessity or without necessity a *B*, given that it is a *B*, regardless of whether that is temporarily or permanently, is an *A* with necessity and permanently.

then [it follows that] *C* is included in whatever is said of every [*B*]. Likewise when we say

(10.50) Every *B* [is a *C*].

(*Syllogism* 127.11f)

DDO.22. [Subject.1] When you say

(10.51) *B* is a *C*.

it means /64.3/ that any [idea] that fits the description *B* and is determinately a *B*, whether or not it is satisfied, and whether its satisfaction is possible or impossible, given that it is counted as actually fitting the description *B*, **without adding /64.5/ that it is a *B* permanently, or [is a *B* but] not permanently** — this idea fits the description *C*. And analogously with denial.

(*Easterners* 64.3–6)

DDO.23. [Predicate.1] When you say

(10.52) *B* [is a] *C*.

what this means is that everything fitting the description *B* does in actuality fit the description *C*. The meaning doesn't include an addition to the effect that the thing fits the description [*C*] permanently, or that it doesn't fit it permanently, /64.14/ or that it fits [the description *C*] sometime (?) when it fits the description *B*, or at some other time, whether or not one of the **two times** [in the subject and in the predicate] is determinate (as with eclipses of the moon), or indeterminate (like when a human breathes).

(*Easterners* 64.14–16)

DDO.24. Next we consider the universally quantified necessary proposition. /68.6/ This is like when you say

(10.53) Necessarily every B is a C .

meaning that everything that fits the description B in actuality, **regardless of whether it fits the description B permanently or not permanently**, each such thing fits the description that for as long as its essence is satisfied, it is a C . An example is when you say

(10.54) Necessarily everything that moves is a body'.

(*Easterners* 68.6–8)

DDO.25. Next we consider the congruent proposition. /68.12/ This is like when you say

(10.55) Every B is a C .

meaning that it is a C **sometime when it is a B** , but without adding that it is a C permanently for as long as it is a B , or that it is so but not permanently [for as long as it is a B]. Next we consider the predetermined proposition. /68.14/ This is like when you say

(10.56) Every moon is eclipsed.

or

(10.57) Every star rises.

(*Easterners* 69.12–14)

DDO.26. The predicative affirmation is as when we say 'The human is an animal' meaning that the thing which we suppose in our mind to be a human—regardless of whether it exists in the real world or not—we have to suppose that it is animal and we judge of it in that it is animal, without adding when or in what circumstances, but rather in a way that includes the dated, the bounded and the opposites of these two.

(*Pointers and Reminders* 72.17–73.3)

DDO.27. Know that when we say "Every C is a B " we don't mean that [the class] 'all of C ' is a B , or that the universal C is a B . Rather we mean

that each one of the things that fit the description C , whether it does so in mental assumption or in the world (*wujūd*), or whether it fits that description permanently or non-permanently, but however it is—this thing fits the description that it is a B without adding that it fits this description at such-and-such a time (*waqt*) or in such-and-such a situation (*ḥāl*). All of this is more specific than its fitting the description [B] absolutely. This is what is understood when we say “Every C is a B ” without adding any mode.²⁶ (*Pointers and Reminders* [135] 93.9–14.)

²⁶This translation is mine, but I consulted the translations of Inati [110] paras 280, 282 and Street [208] p. 20 Text 1.

Part IV

Jurjān (1012–1015)

Chapter 11

The Jurjān period: dates and (dt) logic

Section 11.1 will summarise the evidence for the dating of the beginning and end of the Jurjān period, i.e. the period of Ibn Sīnā's intense activity in Jurjān. There is little new to be said here. The period runs from 1013 to 1015. The main new point is that in Section 8.1 we have dated *Twenty Questions* to the beginning of the Jurjān period. In fact *Twenty Questions*, which we date to around 1012, sets a programme for the logical research of this period.

This programme is mainly directed to predicative logic, and it replaces the sentences of categorical logic by new sentences that are temporal and two-dimensional. The temporal nature of these new sentences is clear from their use of expressions such as 'so long as' and 'sometimes', and Ibn Sīnā himself was obviously aware of this.¹ But there is little evidence that he understood their two-dimensional nature; in fact we have to apply some knowledge of modern logic to track what is happening with the temporal parameters. These temporal and two-dimensional sentences form what we refer to as 'Ibn Sīnā's predicative 2D logic', though we sometimes drop the 'predicative' when it's clear from the context.

Of Ibn Sīnā's logical writings from before *Twenty Questions*, the only ones that (on present evidence) make any reference to logical properties of time are those that discuss the wholly hypothetical sentences and the other hypothetical sentences developed from them in the source of al-Barakāt's formal logic. These make no direct contribution to predicative logic. The turning-point appears precisely in Question ii of *Twenty Questions*, where

¹But see Section 6.3 on the ambiguity between times and situations.

Ibn Sīnā scans the writings of Aristotle for clues to different ways of constructing categorical sentences.

In Section 11.3 we review the main new temporal sentences introduced in the Jurjān period. An important subset of these sentences consists of the sentences that we describe as *d* (for *dā'im*, 'permanent') or as *t* (for 'Themistian absolute'). We can consider these sentences as forming a self-contained logic with a clear meaning; we call it (*dt*) logic.

As Ibn Sīnā introduces the new sentence forms, he looks for names for them. Some of the names that he chooses seem to have some metaphysical or ontological implications; though it is not clear what these implications amount to, since in every case the sentence form can be defined in purely logical terms without any reference to metaphysics. Section 11.4 is about some of these names that suggest ontological content.

11.1 The Jurjān period tied to history

It was during his stay in Jurjān² that Ibn Sīnā first met Abū 'Ubayd al-Jūzjānī, who was to become his faithful disciple and amanuensis until Ibn Sīnā's death in 1037.

The dividing-line between Ibn Sīnā's *Autobiography* and its continuation by al-Jūzjānī is closely tied to two events in recorded history. The first event, mentioned by Ibn Sīnā himself in his *Autobiography*,³ is the death of Qābūs ibn Wushmagīr, the ruler of Jurjān who was deposed and frozen to death by members of his family in the winter of 1012/3. The second event, mentioned early in al-Jūzjānī's continuation of the *Autobiography*, is the killing of Hilāl ibn Badr ibn Ḥasanūyah in April 1015.⁴

After the death of Qābūs, Ibn Sīnā moved from Jurjān to Dihistān where he was 'very ill', then back to Jurjān where he met al-Jūzjānī. There he dictated to al-Jūzjānī the *Middle Summary in Logic* and wrote *Shorter Summary in Logic* (which al-Jūzjānī tells us, at [117] pp. 74–77, was later incorporated in *Deliverance* as its part on logic). Al-Jūzjānī adds that '[Ibn Sīnā] wrote many works during this stay in Jurjān, such as the first part of the (medical) *Canon*, and the summary of Ptolemy's *Almagest*'. At some time after the death of Hilāl Ibn Sīnā moved to Rayy where he wrote *The Return*.⁵ A

²Actually his second visit to Jurjān, cf. *Autobiography* [117] 42.1f. But his first visit is not associated with any writing on logic, so I ignore it.

³Ibn Sīnā [117] p. 42f.

⁴*Autobiography* [117] 42.2f, 50.1–3.

⁵*Autobiography* [117] 44.3–8, 48.9–50.1, 74.9.

busy two years!

Alexander Kalbarczyk [146], [147] examined the treatment of Aristotle's categories in *Middle Summary in Logic* and *Shorter Summary in Logic*. He showed that the two treatments are radically different, and that *Shorter Summary in Logic* has to be the later of the two works. As far as the material on formal logic goes, the most significant difference between *Middle Summary in Logic* and *Shorter Summary in Logic* is that in *Shorter Summary in Logic* Ibn Sīnā points out—with emphasis—that second figure premise-pairs consisting of broad absolute sentences are sterile, but this fact is not mentioned in either *Middle Summary in Logic* or *Twenty Questions*. (See Section ?? below for more details.) This makes it almost certain that *Middle Summary in Logic* and *Twenty Questions* were written at least months earlier than *Shorter Summary in Logic*. The other differences in logical content between *Middle Summary in Logic* and *Deliverance* are relatively slight, though the two books have very little text in common.

Even granting that Ibn Sīnā was a fast worker, it seems safest to put *Middle Summary in Logic* near the beginning of the Jurjān period and *Shorter Summary in Logic* near its end. So we should place the two works respectively at the beginning and the end of the second stay in Jurjān, i.e.

- (11.1) *Middle Summary in Logic*, 1013.
Deliverance, 1015.

It must have been clear to Ibn Sīnā, at least by the time he wrote *Short Epitome in Logic*, that his own list of the valid syllogisms of alethic modal logic (as reported in Section 7.3) disagreed at several points with that of Aristotle and with those of the leading commentators on Aristotle. He will also have been aware that Aristotle in *Prior Analytics* mentioned explicitly some of the modal syllogisms that Ibn Sīnā believed were valid, and that Aristotle claimed to give arguments showing that they couldn't be valid. But when Ibn Sīnā wrote *Short Epitome in Logic* he showed no signs of having a methodology for resolving conflicts of this kind.

The position was different by the time he came to write *Twenty Questions*, around the beginning of the Jurjān period. By this time he had already started to make specific comparisons between his own logical views and those of Aristotle. One feature of these comparisons was an interest in the justifications that he or Aristotle offered for their respective positions. In this sense he had started to move attention from the logical facts (for example about the forms of conversions or the validity of syllogisms) to the 'foundations' (*uṣūl*) that would support these facts. Interestingly it was

soon after the end of the Jurjān period that Ibn Sīnā started to formulate theories about the structure of logic as a science and the 'subject-matter' (*mādda*) of this science. See REF.

We also learn in *Syllogism* i.2 that one of the causes of difference of opinion in logic is

- verbal expressions and the occurrence of differences in what
(11.2) these expressions are used to mean.
(*Syllogism* [124] i.2, 17.8f.)

This could be in part a reference back to his own experience when tackling his differences of opinion with Aristotle about modal logic. Throughout the Jurjān period one of his main approaches in logic was to find new meanings for Aristotle's alethic modal predicative sentences (treating 'absolute' as a mode), and then study what logical entailments stayed valid when the alethic sentences were given their new meanings. The new meanings could be expressed by new temporal predicative sentences, which I will refer to as 'Ntp' sentences for short. But it's important to note that even after Ibn Sīnā had formulated versions of these Ntp sentences, he still normally stated the relevant entailments using alethic rather than temporal sentences. This continued to be true up to the end of his career, and it is one of the main facts about Ibn Sīnā's mature modal predicative logic that demand an explanation.

The next section will examine four examples of Ibn Sīnā's use of Ntp sentences to develop a new two-dimensional (2D for short) modal logic. After Ibn Sīnā had introduced an Ntp sentence, he tended to add further properties of the sentence as they occurred to him. This may explain why our description of these sentences spreads through several sections of this and the next chapter. For the present we begin with broad outlines.

Here is a list of names and labels of the four Ntp sentences that we will meet in the next section. The names are taken from Ibn Sīnā's texts (some of them later than the Jurjān period), but the single-letter labels are mine.

d: *ḍarūrī*, *dā'im*, paradigmatic necessary, necessary, permanent.

ℓ: *lāzim*, *muta'āraf*, entailed, conventional.

t: *muṭlaq 'āmm*, broad absolute, Themistian absolute, Theophrastan absolute.

w: *wujūdī*, *muṭlaq khāss*, impermanent, narrow absolute, paradigmatic absolute, Alexandrine absolute.⁶

These are the four main temporal forms of Ibn Sīnā's 2D modal predicative logic. They control the predicate sides—except for quality—of the sentences with these forms. Later Ibn Sīnā will add a fifth form which he calls *muwāfiq* (and we abbreviate to *m*), to provide contradictory negations for the ℓ sentences.

Each of these forms can be combined with any of the four categorical forms of quality and quantity, labelled *a*, *e*, *i*, *o*. Thus the combination of categorical form *a* with temporal form *d* is written (*a-d*), the combination of categorical form *e* with temporal form ℓ is written (*e- ℓ*), and so on.

11.2 Verifying discourses and new foundational features

In the remainder of this and the next two chapters, we survey some examples of new logical vocabulary and new concepts, which appeared during the Jurjān period. I call attention to a phrase that is typical of this period: 'verifying discourse' (*qawl muḥaqqiq*). This phrase makes its entrance at *Middle Summary in Logic* [114] 132.1 and 144.11. We also meet the word 'verify' (*ḥaqqāqa*) referring to discourse that could be thought of as verifying in a similar way, at *Twenty Questions* [115] x, 92.18 and *Middle Summary in Logic* [114] 107.5, 109.5 and 125.9.

These passages have several things in common. In the first place, all of them describe the justification of items that today we would describe as 'metatheory', for example not syllogisms but rules for constructing or checking syllogisms. In this sense they are all about foundations; in fact the passage at *Twenty Questions* x, 92.18 explicitly speaks of 'verifying this foundation'. And secondly, the passages all justify claims that Ibn Sīnā made which contradict Aristotle. For example *Twenty Questions* x, 92.18 and *Middle Summary in Logic* 132.1 both support Ibn Sīnā's position on the Modal Challenge; *Middle Summary in Logic* 107.5 insists against Aristotle that the categorical sentences in syllogisms should be interpreted in a way that involves time; *Middle Summary in Logic* 109.5 explains why categorical sentences interpreted in this way cannot be contradictory negations of each other; *Middle Summary in Logic* 125.9 explains why the second figure

⁶I believe that Street means by 'special absolute' ([204] p. 132) the same as I mean by 'narrow absolute'. (I use the contrast narrow/broad to express 'meaning that is true of fewer individuals' as opposed to 'meaning that is true of more individuals'.) 'Paradigm' is my translation of *ḥaqīqī*, as justified in Section 11.4 below. Inati [110] and Ahmed [5] translate *ḥaqīqī* in this context as 'real', though I am less clear what they mean by 'real'.

contains no productive premise-pairs consisting of absolute sentences; and at *Middle Summary in Logic* 144.11 Ibn Sīnā returns to the Modal Challenge, arguing a little bizarrely that the absolute premises of these syllogisms can all be understood as affirmative. (This is a reference to *w* sentences, as we will see in Subsection 11.3.1 below.)

And thirdly, as we will see in Chapter 12, all the claims that Ibn Sīnā makes in these six ‘verifying’ passages about the productivity or sterility of premise-pairs are claims that can’t be justified using Aristotle’s methods. This forces Ibn Sīnā to rely on hands-on proofs for these results about syllogisms.

We turn to four examples of new foundational features.

Example 1: *Productive categorical premise-pairs that become sterile when the categorical sentences are reinterpreted as Alexandrine absolutes.*

In (11.3) below, Ibn Sīnā aims to use Aristotle’s method of pseudoconclusions to show that the second-figure premise-pair *Camestres* becomes sterile if its two premises are read according to the Alexandrine interpretation of categorical sentences (for details of which see Section 10.1 above and Subsection 11.3.1 below). This is correct, and Ibn Sīnā manages to say enough true things to allow us to deduce it by Aristotle’s method. But as it is cogently explained by Alexander,⁷ the method is justified only for categorical logic. When the new Ntp sentences are involved, a further explanation is needed, which Ibn Sīnā never provides. Note also that Ibn Sīnā reverts to Aristotle’s unfortunate terminology and uses ‘conclusion’ both for logical conclusions and for pseudoconclusions. Thus he finds himself saying, at the end of (11.3), that because the premise-pair has appropriate ‘conclusions’ (i.e. pseudoconclusions) it need not have a ‘conclusion’ (i.e. logical conclusion)!

⁷Alexander of Aphrodisias [7] 89.29–92.22, trans. Barnes et al. [8].

(11.3) On the fact that there is no syllogism in second figure from two paradigmatic absolutes.^a What we say about this is what the Philosopher said about the same question in relation to pairs of possible [propositions]. Namely, that the absolute just overlaps the possible in matter, /88.20/ so that it is in one consideration absolute and in another consideration possible. So the absolute which is of this kind is like the possible in that there is no conclusion from it,^b and that is because it is possible to make an affirmative universal absolute sentence and a negative universal absolute sentence [true] together, as we showed. Then when it is about a single subject [individual], it can be that it affirms and denies universally, I mean from a pair of absolutes. This can also be about two subject [classes], one of the two predicated of the other. ... [The Philosopher] used

No horse is moving absolutely, and every animal is moving absolutely.

I.e. /89.5/ every animal has a time at which it moves voluntarily, and every horse is at some time not moving. These two [sentences] are the negative and affirmative absolutes, and in this case the [pseudo]conclusion is affirmative.^c If the two poles are taken to be [different] species of animal, the [pseudo]conclusion is negative. In both cases [the pseudoconclusion] is necessary. So therefore there doesn't have to be anything following from two [paradigmatic] absolutes in the second figure.

(*Twenty Questions* [115] ix, 88.18–89.8.)

Note a. See Section 11.4 for the use of 'paradigmatic absolute' to name the Alexandrine interpretation. Since an Alexandrine sentence entails the corresponding Themistian sentence, a proof that *Camestres* with both premises read as Alexandrine (i.e. *w*) is sterile implies that *Camestres* with both premises read as Themistian is also sterile. The Themistian version is a premise-pair in (*dt*) logic.

Note b. Ibn Sīnā notes that Aristotle in *Prior Analytics* i.17 takes *Camestres* with two possible premises to be sterile, and he hopes to carry over Aristotle's justification of this claim to paradigmatic absolutes. The key passage is *Prior Analytics* i.17, 37a38–37b10, where Aristotle tries to prove the sterility

by pseudoconclusions (which incidentally he calls 'conclusion' at 37b2f). I suspect that Ibn Sīnā has at the back of his mind an idea that any pair of pseudosyllogisms to prove sterility in the possibility case can be translated into an equivalent pair in the paradigmatic absolute case by translating different possible situations into different times.

Note c. Ibn Sīnā borrows 'horse' and 'animal' from Aristotle's text, and claims that the pseudoconclusion in this case will be the universal affirmative 'Every horse is an animal'. But a suitable pseudoconclusion for the temporal language, spelled out more thoroughly, would be 'Everything that is sometimes a horse is an animal at every time when it exists'. Fortunately this is true, as Ibn Sīnā realises later; see Note d below.

Note d. Compare Aristotle's remark at *Prior Analytics* i.17, 37b9 that 'necessarily no horse is a human', which I think Ibn Sīnā may be right in reading as an attempt at a negative pseudoconclusion; he copies it in his own sentence here.

Example 2: *Camestres*(ABS,NEC) and some similar alethic premise-pairs have a conclusion of the form *d* (paradigmatic necessary) when their absolute premise is interpreted as of the form *t* and their necessary premise is interpreted as of the form *d*.

As with Example 1, Ibn Sīnā gives a verdict here that disagrees with that of Aristotle. We saw in (7.21) that Aristotle claimed an absolute (i.e. categorical) conclusion for *Camestres*(ABS,NEC), whereas in *Short Epitome in Logic* Ibn Sīnā claimed a necessary conclusion. This and the similar alethic premise-pairs form the heart of what we called the Modal Challenge, and Ibn Sīnā in the Jurjān period is responding to that challenge by proving a *d* conclusion when the two premises are read as *t* and *d* in the appropriate order. (Soon after the Jurjān period he gives a complete solution to the challenge; see Section 15.2 below.)

Here is the proof that Ibn Sīnā gives in *Middle Summary in Logic*. He gives simultaneous proofs for several similar syllogisms, but without loss of generality we can suppose that he has in mind the alethic syllogism *Camestres*:

(11.4) No *C* is an *A*. Every *B* is an *A*. Therefore no *C* is a *B*.

He gives essentially the same proof also in *Twenty Questions* [115] Question xvi, 100.5–8 (in terms of possible and necessary) and *Deliverance* [131] 67.7–68.2.

We will prove that the conclusion is necessary permanent^a when *C* and *B* are different in such a way that for one of them, /132.5/ all the individuals under it are said to be necessarily and without exception *As*, or else the same with ‘not *As*’; and the other term is such that every individual under it is of one of two kinds: either^b it has necessarily the opposite property to those of the first term, or it is possible that it has the opposite property, if the absolute (11.5) is taken in the first [i.e. Alexander’s] reading or in the second [i.e. Themistius’s] reading in such a way that it is not ruled out that at least one individual is an *A* or not an *A* [as appropriate]. Then [we say that] between the two natures *B* and *C* there is an essential distance^c (*bawn dhātī*). [In this case] neither of [the terms] applies to the other at all, but rather each of the two is denied of the other with necessity,
(*Middle Summary in Logic* [114] 132.4–9.)

Note a: I.e. necessary in the sense of permanent, in other words *d*.

Note b: The ‘either’ case is redundant since it can be deduced from the ‘or’ case. The ‘or’ case is phrased as if it involved possibility (‘possible’, ‘not ruled out’), but this is probably careless phrasing since the title of this section states that it is about syllogisms with absolute and necessary premises. However, this phrasing does draw attention to Ibn Sīnā’s belief that possible sentences are in some sense close to absolute sentences.

Note c: ‘Essential distance’ in line 132.8 seems to be Ibn Sīnā’s new expression for the idea of the method in his proof. In *Twenty Questions* xvi, 100.8 he calls the same idea ‘essential and necessary distinction’ (*farq dhātī ḍarūrī*) and in *Deliverance* 68.1 it becomes ‘essential difference’ (*khilāf dhātī*).

The reason for the new vocabulary is that the method he has in mind is not an Aristotelian one. Consider the Aristotelian proof for *Camestres*(ABS, ABS,ABS). It can be explained as using the fact that the predicates of the two premises (*e*)(*C*, *B*) and (*a*)(*A*, *B*) have the same term letter but it is negated in only one of these premises. Not-*B* versus *B* is the starkest form of contradiction. But when we interpret the sentences as temporal, the term *B* acquires a time parameter, and this needs to be handled with time quantifiers in both premises. So not-*B* versus *B* becomes Sometimes-not-*B* versus Always-*B*. These phrases still contradict each other, but to see this we need

to analyse the phrases down to the level where the quantifiers live. The situation is very much like that of the 'parts of terms' in Section 9.3 above. In both cases the extra analysis of terms makes the method non-Aristotelian.

Example 3: *Barbara(NEC,ABS) has a conclusion of the form d when the necessary premise is interpreted as of the form d and the absolute premise is interpreted as of the form ℓ .*

The exposition in (11.8) below is truly dreadful. I haven't made any attempt to allocate the blame for this either to Ibn Sīnā himself or to the textual tradition.⁸

Ahmed [5] p. 155 note lxxix raises a doubt about the cogency of the syllogism as reported in (11.8). In fact Ibn Sīnā himself in *Twenty Questions* raises a similar doubt, taking the premise-pair to be

- (11.6) Every C is a B permanently.
Every B is an A throughout the time while it is a B .

Ibn Sīnā's doubt is that B in the first premise is said to be permanent, whereas in the second premise taken on its own, there is nothing to imply that B is permanent. See BELOW for further discussion of this doubt.

I think the syllogism is good, given the excellent hands-on proof that Ibn Sīnā states in *Twenty Questions* as follows.

- (11.7) In this case C will be an A with [paradigmatic] necessity, because C is a B and every B is, for as long as it is a B , an A . So a C is an A for as long as it is a B . But C is permanently a B , so it is permanently an A .
(*Twenty Questions* [115] xiii, 98.8f.)

But note that the method of this proof of the syllogism is not Aristotelian, as witness the fact that it doesn't follow any of the routes described by Street [204] pp. 139–142 'Rules used in proofs'.

Besides the proofs at (11.7) and (11.8), Ibn Sīnā also proves this syllogism at *Syllogism* [124] iii.1, 128.16–129.4 (a decent hands-on proof together with a scientific example).

⁸See Footnote 6 in Section 1.2 above on Ahmed's policy for his translation of *Deliverance*.

... when the major premise is absolute, and the time of its application is while the essence of the subject [individual] continues to be described as it is described,^a then the conclusion is necessary, because [every] *C* is a *B* permanently.^b It was already described as a *B* for as long as it continues to be a *B*.^c So it is an *A*, /67.1/ and so *C* is permanently an *A*. In this case it can happen that the conclusion is necessary and the major premise is absolute.^d
 (11.8) (*Deliverance* [131] 66.11–67.1.)

Note a: The major premise is the second premise, and Ibn Sīnā is saying that this premise as interpreted has the form (*a-ℓ*). In other words, each subject [individual] of the major premise is an *A* at every time in which this individual is a *B* (i.e. every time in which it is described as the subject term describes it). ‘Be described as it is described’ is an unfortunate phrasing; see the end of Section 10.4 above for these uses of ‘is described’.

Note b: ‘*C* is a *B* permanently’ is missing its quantifier, but we can safely add ‘Every’ and take the underlying categorical mood as *Barbara*. So the first (i.e. minor) premise as interpreted is ‘Every *C* is a *B* permanently’ i.e. everything that is sometimes a *C* is a *B* at every time when it exists; this is a sentence of the form (*a-d*).

Also ‘a *B* permanently’ requires that *B* is a 2-ary term with a time parameter. There is a risk of overloading here, since nothing in the first premise requires *C* to be 2-ary.

Note c: The sentence as it stands is true but unhelpfully trivial. Logical common sense can be restored either by amending the first *B* to *A* to get the required meaning as in Note a, or by deleting this sentence altogether as probably a note added in the margin by a reader.

Note d: Since the broadening of the notion of ‘absolute’ seems to be entirely in the hands of Ibn Sīnā himself, the last clause ‘The major premise is absolute’ should probably be regarded as a definitional stipulation and not a logical discovery.

Example 4: *The contradictory negation of a sentence of the form ℓ.*

This example is not from the Jurjān period but from late in Ibn Sīnā's career. I included it to illustrate that Ibn Sīnā's use of the Ntp sentences continues to the end of his career as an active logician. The contradictory negation that Ibn Sīnā gives for an ℓ sentence in (11.9) is confirmed by *Easterners* [116] 82.22f (though this text in *Easterners* needs some exegesis too).

(11.9) The sentence 'Every C is a B ' is true just when every single [individual satisfying] C is a B at every period or moment of time in which C is true of that individual.^a Accordingly when [the individual] is described at some time as being a C necessarily or not necessarily,^b and at that time it doesn't satisfy the description B ,^c this sentence ['Every C is a B '] is false.
(*Pointers and Reminders* [135] i.5.2, 110.11–14.)

Note a: The categorical sentence 'Every C is a B ' has contradictory negation 'Not every C is a B ' (or 'Some C is not a B '). But here Ibn Sīnā gives it a different contradictory negation, which shows that he is treating the sentence not as categorical but as an interpretation of a categorical sentence. The interpreted sentence has the form that he describes as *lāzim*, which is ℓ in our notation.

Note b: The phrase labelled for Note b gives the temporal mode on the subject term of a sentence, so it describes the subject side but without the quantifier. The next phrase (labelled c) gives the complete predicate side. These two phrases together provide a sentence of the form m but without its quantifier, so that it serves for any of $(a-m)$, $(e-m)$, $(i-m)$ or $(o-m)$.

Note c: The sentence that he gives here is a sufficient condition for the ℓ sentence to be false. But the whole of Section i.5.2 of *Pointers and Reminders* is about contradictory negations, and the contradictory negation of a sentence ϕ is a sentence whose truth is a necessary and sufficient condition for ϕ to be false. So we can guess that the sentence described in Note b as of the form m is intended to be a contradictory negation of the ℓ sentence, though we have to supply the quantifier.

Before leaving this section, we note an interesting statistic. Before the end of the Gap period Ibn Sīnā shows no particular interest in the class of absolute sentences. But in *Twenty Questions*, when he starts to enlarge the class of absolute sentences to include some that involve temporal modes

and hence are not categorical, his interest in them spikes. The frequency of the word *muṭlaq* in *Twenty Questions* is over twenty-five times its frequency in *Short Epitome in Logic*, measured by the number of occurrences per printed page.⁹

11.3 Two-dimensional sentence forms introduced

In this section we will give a systematic description of the four kinds of Ntp sentence that Ibn Sīnā introduces in the Jurjān period (or possibly late in the preceding Gap period). These four kinds are *t*, *w*, *d* and *ℓ*. Each of these kinds comes in four versions, an *a* version, an *e* version, an *i* version and an *o* version, and each of these versions is formed by adding a temporal mode (specified by the kind) to the predicate of a categorical sentence of the form *a*, *e*, *i* or *o*. For the addition of this temporal mode to make sense, the predicate must be changed to be 2-ary, with an object parameter and a time parameter.

Thus for the kind *t* with subject term *B* and predicate *A* we will introduce four new forms $(a-t)(B, A)$, $(e-t)(B, A)$, $(i-t)(B, A)$ and $(o-t)(B, A)$; and similarly for *w*, *d* and *ℓ*. This will yield $4 \times 4 = 16$ new Ntp forms. But as tends to happen with Ibn Sīnā, the reality is more complicated. Each of these sixteen forms is reached by a series of approximations as two refinements are added to the simplest version. Ibn Sīnā doesn't put these refinements into a fixed order, but I will do so for the sake of exposition. Thus for example the simplest version of $(e-ℓ)(B, A)$ will be its first approximation, followed by second and third approximations; the third is the most refined. Also the *a* forms have a fourth approximation, to incorporate an existential augment.¹⁰

Peripatetic logicians tend to concentrate on the four new affirmative universal Ntps, as for example $(a-d)$ and $(a-t)$, on the basis that the existential or negative sentences $(e-d)$, $(i-d)$ etc. can be inferred from the new affirmative universal sentences. But both English and Arabic have some irregularities in their treatment of scopes. So we want a notation that avoids these irregularities. This becomes much easier if we adopt the modern no-

⁹In the 15 pages of Badawī's edition of *Short Epitome in Logic* the word occurs twice, a frequency of 0.13 per page. In the 24 pages of Daneshpazhuh's edition of *Twenty Questions* the word occurs 81 times, a frequency of 3.37 per page. 3.37 divided by 0.13 is 25.9.

¹⁰Thus the *a* forms generate $4 \times 16 = 64$ approximations, and each of *e*, *i*, *o* generates $3 \times 16 = 48$ approximations, giving a total of 208. Fortunately Ibn Sīnā seems to regard only the sixteen most refined approximations as the genuine ones that determine the syllogistic rules for the use of the forms.

tation of variables (with x, y etc. for object variables and τ, σ etc. for time variables). But Ibn Sīnā didn't have the notation of variables in logic, so I will also use a notation that translates what he said or could have said in Arabic.

11.3.1 Broad absolute, t

In Ibn Sīnā's logical texts that are available to us, a kind of sentence called 'broad absolute' (*muṭlaq ʿāmm*) appears first in *Syllogism* i.5.¹¹

(11.10) Consider affirmative universally quantified propositions, [read in the] broad absolute meaning, such as the sentence "Every B is an A ". This sentence excludes two possibilities. The first of them is that necessarily some B is not an A , and the second is that it [just] happens that some B is never an A . An affirmative proposition [stating that its predicate holds] either permanently, or definitely at some time, counts /47.5/ as broad absolute.
(*Syllogism* i.5, 47.2–5)

Here Ibn Sīnā defines the broad absolute reading of $(a)(B, A)$ by saying what states of affairs it excludes. It excludes two states of affairs, namely that some B is necessarily not an A , and that some B is never an A but not by necessity. As Ibn Sīnā will imply later at *Pointers and Reminders* i.5.2, 108.12, these two states of affairs combine to form a simpler state of affairs, namely that some B is permanently not an A ; and it follows that the broad absolute is the contradictory negation of this, namely that

(11.11) Every B is at least once an A .

This sentence (11.11) is equivalent to what Ibn Sīnā in *Twenty Questions* Question ii called the 'Themistian interpretation' of $(a-t)(B, A)$, where $(a-t)(B, A)$

(11.12) has two interpretations, the interpretation of Alexander, in which it is affirmed that the affirmation [of A] is at a time and is not permanent, and the interpretation of Themistius, which allows [the impermanence] but doesn't demand it.
(*Twenty Questions* [115] ii, 83.6f.)

In other words the Themistian interpretation says, of each object that is a B , that this object is an A at least once, with nothing said about whether or not it is permanently an A .

¹¹Not to be confused with 'broad absolute syllogisms', which at *Syllogism* i.1, 4.5 means syllogisms in the broadest sense.

At *Twenty Questions* [115] 84.7 Ibn Sīnā gives an example for (*e-t*), namely

- (11.13) There is no animal that doesn't have motion denied of it at some time.

Replacing the material terms by term letters and removing the triple negation (!) gives us

- (11.14) (*e-t*)(*B, A*): For every *B* there is a time at which it is not an *A*.

For (*i-t*)(*B, A*) we replace 'every' in (11.11) by 'some'. For (*o-t*)(*B, A*) we change the quantifier and add a negation as in (11.14). These manoeuvres give the first approximations for *t*:

First approximations

(*a-t*)(*B, A*): Every *B* is at least once an *A*.

- (11.15) (*e-t*)(*B, A*): Every *B* is at least once not an *A*.

(*i-t*)(*B, A*): Some *B* is at least once an *A*.

(*o-t*)(*B, A*): Some *B* is at least once not an *A*.

In the text immediately following (11.10), Ibn Sīnā states a first refinement of the first approximation to (*e-t*)(*B, A*):

- (11.16) [Its contradictory negation]¹² has to say that '[*A*] is permanently false of some [*B*]' where *A* being permanently false of this individual means being false for so long as the essence of this individual (*shakhs*) exists.
(*Syllogism* [124] i.5, 47.5f.)

The reference to an individual essence takes us back to the sentences discussed in Section 10.3 above, which translate facts about an individual into

¹²The text needs some attention here. According to the Cairo edition this passage begins *fa-yajibū an yakūna l-ba'ḍu maslūban 'anhu dā'iman*. But what is the situation in which something has [*A*] permanently false of it? Logic requires that the situation is the one described by the contradictory negation of the last sentence of (11.10), and this is confirmed by the mention of the contradictory negation at line 47.10. But the required mention has apparently dropped out of the text at line 47.5, so I have restored it in the translation.

facts about its individual essence, as in (10.32). Ibn Sīnā has complicated his exposition by describing the contradictory negation of the broad absolute sentence, rather than the broad absolute sentence itself. If we transfer the point about existence back into the broad absolute sentence, (11.11) becomes a second and closer approximation to $(a-t)(B, A)$ as follows:

- (11.17) For every individual that is a B , there is a time at which that individual exists and is an A .

To discuss this approximation we introduce a new 2-ary symbol E , where $E(x, \tau)$ means ' x exists at time τ '. Using E we can now write second-stage approximations to $(a-t)$, $(e-t)$, $(i-t)$ and $(o-t)$:

Second approximations

- $(a-t)(B, A)$: For every object x that is a B , there is a time τ such that $E(x, \tau)$ and $A(x, \tau)$.
- (11.18) $(e-t)(B, A)$: For every object x that is a B , there is a time τ such that $E(x, \tau)$ and not $A(x, \tau)$.
- $(i-t)(B, A)$: For some object x that is a B , there is a time τ such that $E(x, \tau)$ and $A(x, \tau)$.
- $(o-t)(B, A)$: For some object x that is a B , there is a time τ such that $E(x, \tau)$ and not $A(x, \tau)$.

Note that the sentence for $(a-t)$ in (11.18) can be deduced from the sentence for $(a-t)$ in (11.15) together with (10.32); and likewise with $(i-t)$. But the same doesn't hold for $(e-t)(B, A)$. In fact if x is not eternal then there is a time when x doesn't exist, and then by (10.32) x is not an A at that time. So in the presence of (10.32), the second approximation to $(e-t)(B, A)$ is not equivalent to the first approximation to $(e-t)(B, A)$. And $(o-t)(B, A)$ behaves like $(e-t)(B, A)$ and not like $(a-t)(B, A)$.

Note that (10.32) expressed a logical property of the symbol E that we have just introduced—or more precisely a logical property of E together with any 2-ary term B . Logical properties of E in this sense can be described as forming the 'theory of E '. They are available to be quoted when we do calculations with E , as in the previous paragraph.

We reach another approximation to $(a-t)(B, A)$ by turning to the various forms of Ibn Sīnā's *dictum de omni* and asking what they tell us about

the subject side of (*a-t*). We should bear in mind that some forms of the *dictum* are specifically for sentences with a particular kind of mode. For example DDO.11 is a form of the *dictum* for sentences that are Themistian or Alexandrine absolute, DDO.12 is a form for sentences that are necessary, and DDO.13 is for sentences that are possible. At present we are asking about the subject sides of sentences that are broad absolute, i.e. Themistian absolute (see Footnote 25 in Chapter 10 above). So the chief relevant forms of the *dictum* are DDO.6, DDO.11, DDO.16, DDO.17, DDO.22. None of these forms are about sentences that ascribe necessity or possibility to the predicate, and all of them suggest that the subject side describes a subject individual as being a *B* either permanently or at some time, i.e. at least once.

For example the part of DDO.11 describing the subject side reads

(11.19) every one of the things described in the intellect or reality as being a *B* permanently, or is described as being a *B* at some time after it is not a *B*.

In DDO.22 note the phrase

(11.20) without adding that it is a *B* permanently, or [is a *B* but] not permanently.

(I have no explanation for the text preceding this in DDO.22, for example ‘whether the satisfaction [of *B*] is possible or impossible’. But for the subject individuals being described as ‘ideas’, compare Footnote 14 in Section 10.3.) There are no DDOs which describe absolute statements and unambiguously say that the subject side can place any condition other than that the subject individual is a *B* at least once.

Accordingly we can write third-stage approximations to (*a-t*), (*e-t*), (*i-t*)

and (*o-t*):

Third approximations

- (*a-t*)(*B, A*): For every object *x* that is sometimes a *B*, there is a time τ such that $E(x, \tau)$ and $A(x, \tau)$.
- (11.21) (*e-t*)(*B, A*): For every object *x* that is sometimes a *B*, there is a time τ such that $E(x, \tau)$ and not $A(x, \tau)$.
- (*i-t*)(*B, A*): For some object *x* that is sometimes a *B*, there is a time τ such that $E(x, \tau)$ and $A(x, \tau)$.
- (*o-t*)(*B, A*): For some object *x* that is sometimes a *B*, there is a time τ such that $E(x, \tau)$ and not $A(x, \tau)$.

The fourth and final approximations are to take account of the existential augment on affirmative universal sentences. Just as in categorical logic, this is needed for the moods *Darapti* and *Felapton*. Given the third approximation to (*a-t*)(*B, A*) as above, the most natural way to add the existential augment is as follows:

Fourth approximations for affirmative universal

- (11.22) (*a-t*)(*B, A*): For every object *x* that is sometimes a *B*, there is a time τ such that $E(x, \tau)$ and $A(x, \tau)$; and there are an object *x* and a time τ such that $B(x, \tau)$.

11.3.2 Narrow absolute, *w*

The name 'narrow absolute' appears first in *Syllogism* at iii.1, 130.4, after the Jurjān period. But the concept is the same as that of the Alexandrine interpretation and of the paradigmatic absolute (see Section 11.4), both of which were named during the Jurjān period.

In spite of Ibn Sīnā's temporary attempt to fasten Alexander's name on the Alexandrine interpretation, it seems likely that Ibn Sīnā himself designed broad absolute and narrow absolute as a parallel to broad possible and narrow possible. As he says in *Guidance* [121] 3f, the broad possible is what is not impossible, while the narrow possible is 'where neither its

truth nor its falsehood is necessary'. Correspondingly the broad absolute is what is not permanently true, and the narrow absolute is what is neither permanently true nor permanently false.

The various refinements that we listed for *t* apply equally well for *w*, giving the following final approximations:

Final approximations

- (11.23) $(a-w)(B, A)$: For every object *x* that is sometimes a *B*, there are a time τ such that $E(x, \tau)$ and $A(x, \tau)$, and a time σ such that $E(x, \sigma)$ and not $A(x, \sigma)$; and there are an object *x* and a time τ such that $B(x, \tau)$.
- $(e-w)(B, A)$: For every object *x* that is sometimes a *B*, there are a time τ such that $E(x, \tau)$ and not $A(x, \tau)$, and a time σ such that $E(x, \sigma)$ and $A(x, \sigma)$.
- $(i-w)(B, A)$: For some object *x* that is sometimes a *B*, there are a time τ such that $E(x, \tau)$ and $A(x, \tau)$, and a time σ such that $E(x, \sigma)$ and not $A(x, \sigma)$.
- $(o-w)(B, A)$: For some object *x* that is sometimes a *B*, there are a time τ such that $E(x, \tau)$ and not $A(x, \tau)$, and a time σ such that $E(x, \sigma)$ and $A(x, \sigma)$.

Note that $(o-w)$ is synonymous with $(i-w)$, because of the Janus structure of *w*. Also $(e-w)$ is almost synonymous with $(a-w)$; the difference is the existential augment of $(a-w)$. But Ibn Sīnā seems to brush away this difference when he implies that *w* sentences are equivalent to their own negations, as at:

- (11.24) As for the verifying discourse, the conclusion is necessary and permanent, even if it is from two affirmatives . . .
(*Middle Summary in Logic* [114] 144.11)

This is about second-figure syllogisms, which must have a negative premise. Ibn Sīnā quietly observes that if one of the premises is negative and narrow absolute then it can be converted to affirmative by switching around the two parts of the Janus predicate, in the same way as with a contingent sentence. It would have been more foresighted if he had said that Janus sentences are not affirmative and not negative, but part affirmative and part negative.

By the time of *Syllogism* Ibn Sīnā has begun to refer to narrow absolute sentences as *wujūdī* ('non-permanent'), which we abbreviate to *w*. Before this, Ibn Sīnā used the word *wujūdī* in other senses.¹³ So I delay further discussion of the name *wujūdī* to Section 14.4.

11.3.3 Necessary or permanent, *d*

The *d* form is derived from NP2, which in turn is the first of Ibn Sīnā's explications of the sentence '*B* is an *A* permanently' in Theophrastus's *tathlīth* (cf. Section 10.2 above). NP2 already contains the restriction to times when the subject individual exists, and DDO.24 (which is specifically for sentences classified as necessary) qualifies the subject term *B* by 'an actual *B* at some time'. This together with other refinements that we considered for *t* yields the following final approximations for *d*:

Final approximations for *d*

- (*a-d*)(*B, A*): Everything that is at some time a *B* is an *A* at all times when it exists; and there is at some time a *B*.
- (11.25) (*e-d*)(*B, A*): Everything that is at some time a *B* is always not an *A*.
- (*i-d*)(*B, A*): Something that is at some time a *B* is an *A* at all times when it exists.
- (*o-d*)(*B, A*): Something that is at some time a *B* is never an *A*.

Note that the use of (10.32) in the theory of *E* allows us to remove any mention of times of existence in the sentences for (*e-d*) and (*o-d*).

11.3.4 'For as long as it is a *B*', conventional or *ℓ*

In the interpretation which we have labelled as (*a-ℓ*)(*B, A*), the predicate *A* is understood to be true of each subject individual at each time in which *B* is true of this individual. Here *ℓ* is short for *lāzīm* ('entailed'), the name which Ibn Sīnā introduces in the late work *Easterners* [116] 65.6 for these sentences. He gives no reason for this name in *Easterners*, but we find

¹³Two occurrences at *Deliverance* [131] 65 could refer to narrow absolute sentences, but what Ibn Sīnā says about the sentences being referred to is not clear.

a fairly clear explanation of the name in *Demonstration* [125] ii.1 REUSE THIS; see Section 14.3 below. Since we want a name that has the authority of Ibn Sīnā himself, I use the name *lāzim* henceforth, translated as ‘entailed’ and abbreviated to ℓ . Ibn Sīnā is not known to have used this name before *Easterners*, and strictly the name may not be appropriate outside demonstrative logic.¹⁴

Just as the form d grew from NP2, so the form ℓ is derived from NP3. In Ibn Sīnā’s available writings NP3 appears first in *Twenty Questions* [115] Question x, where Ibn Sīnā gives the example

- (11.26) Everything black has a colour which comes together for the eye.¹⁵
(*Twenty Questions* [115] Question x, 89. 21f.)

We are to understand (11.26) as ascribing ‘a colour which comes together for the eye’ to anything black *at every time when it is black*, with nothing said about what happens when the thing changes colour or becomes transparent. Al-Barakāt discusses some sentences of the form ℓ , for example ‘when it is said that no black thing is white, it just means for as long as it is black, or the sleeper is not awake for as long as he is sleeping’.¹⁶

Expanding to full approximations for the form ℓ gives:

Final approximations for ℓ

- ($a\text{-}\ell$)(B, A): Everything that is at some time a B is an A at all times when it is a B ; and there is at some time a B .
- (11.27) ($e\text{-}\ell$)(B, A): Everything that is at some time a B is never an A while it is a B .
- ($i\text{-}\ell$)(B, A): There is something that is at some time a B and is an A at all times while it is a B
- ($o\text{-}\ell$)(B, A): There is something that is at some time a B and is never an A while it is a B .

Again the use of (10.32) allowed us to remove any mention of existence in (11.27).

¹⁴Calling the form *lāzim* is better than calling it ‘descriptive’ (*waṣfī*), a name that Ibn Sīnā is not known to have ever used for these sentences.

¹⁵This phrase comes from a theory of colour vision. I have no further details.

¹⁶Al-Barakāt *Kitāb al-muṭabar* i, 88.24–89.1.

In *Middle Summary in Logic* Ibn Sīnā remarks that universal negative sentences of the form e are normally understood as being of the form $(e-\ell)$, and that

(11.28) the negative universal propositions that are useful in the sciences and are conventionally used (*muta^cāraf*) [in the sciences] are those of this kind.

(*Middle Summary in Logic* [114] 115.17f, 116.2.)

Later logicians picked up this use of *muta^cāraf*, and in the shorter form *urfi* made it an alternative name for ℓ sentences.

11.4 Ontological naming of logical concepts

Ibn Sīnā certainly belongs among the logicians who like to introduce new names for logical concepts. In the case of the sentences that he called *lāzim* ('entailed'), for example, he was entitled to believe that he was the first logician to have identified these sentences with adequate precision, so it's not surprising that he offered a name for them. But he also chose new names for types of sentence that were already in wide circulation; thus he chose the name *mumkin ḥaqīqī* for contingent sentences.¹⁷

Several of Ibn Sīnā's new names for sentence types appeared in *Twenty Questions* and works of his Jurjān period but are not found in *al-Barakāt's Kitāb al-mu^ctabar*. In the light of Section 8.2 above, this suggests that Ibn Sīnā introduced the names in question sometime between the years 1005 and 1012. Some of these names are as follows.

- (1) Ibn Sīnā uses the phrase 'for as long as the essence (*dhāt*) [of the subject individual] exists'. The context shows that this phrase is used to mean 'for as long as the subject individual exists'. (Cf. Subsection 11.3.2 above.)
- (2) Ibn Sīnā uses the phrase 'for as long as the essence [of the subject individual] is a C' ' (where C is the subject term in the relevant sentence). The context shows that this phrase is used to mean 'for as long as the subject individual is a C' '. (Cf. Subsection 11.3.3 above.)
- (3) Ibn Sīnā uses the phrase *mumkin ḥaqīqī*. Ahmed [5] p. 32 l. 3 translates this phrase as 'real contingent'. For sure this translation catches the

¹⁷After some searching I believe this name was Ibn Sīnā's invention. I would be glad to be corrected if this was not so.

right sentences, though it hides the force of *ḥaqīqī*. Inati [110] agrees in translating *ḥaqīqī* in this phrase as ‘real’. Later in this section I will defend translating *ḥaqīqī* as ‘paradigmatic’; but before this defence is given I follow the translation ‘real’.

- (4) Beginning in *Twenty Questions*, Ibn Sīnā refers to some sentences as *muṭlaq ḥaqīqī* (‘real absolute’). The clues that Ibn Sīnā gives to the identity of these sentences are less explicit, but I will argue in this section that they are the Alexandrine absolute sentences.
- (5) *Twenty Questions* also introduces some sentences as *ḍarūrī ḥaqīqī*. These come close to some fundamental questions about Ibn Sīnā’s modal logic on which there is not yet any consensus. I will argue in this section that they are the temporal permanent sentences, i.e. the *d* sentences (see Subsection 11.3.2 above) as opposed to the alethic necessary sentences.

The names in both (1) and (2) are openly ontological; they refer to essences. The following quotation from *Madkhal* might warn us against reading too much into Ibn Sīnā’s dips into ontology for his names.

- (11.29) ... although it is Customary to prolong [the discussion on] the first principles of logic with material that does not belong to Logic but only to the philosophical discipline—I mean the First Philosophy—I have avoided mentioning any of that [in Logic] and wasting thereby time, and deferred it to its [proper] place.
(*Madkhal* [118] i.1, 10.5–7, trans. Gutas [74] p. 43.)

On this I make two comments.

The first comment is that we should take the reference to essence in (1) and (2) seriously. Ibn Sīnā continues to use these references to essence right through to his late *Pointers and Reminders* (as at [135] i.4.2, 88.12). Moreover in cases (3), (4) and (5) the word *ḥaqīqī* invites us to understand it in terms of essences too, as follows. Ibn Sīnā sometimes varies *mumkin ḥaqīqī* (‘real possible’) to *mumkin bi l-ḥaqīqa* (‘possible in reality’) as at *Deliverance* 49.10 and *Syllogism* iii.5, 177.15, suggesting that he uses *ḥaqīqī* as equivalent to *bi l-ḥaqīqa*. For Ibn Sīnā the *ḥaqīqa* of a thing is in many contexts close or equivalent to the essence of the thing.¹⁸

The second comment is that by using ontological names Ibn Sīnā is not necessarily claiming that we have to understand the logical concepts in

¹⁸Goichon [72] p. 82 s.v. 171 ḤAQIQA. I slightly regret that Goichon didn’t also have an entry on ḤAQIQI.

terms of essences. We will see that in all cases the named logical concepts can be defined without any reference to essences or any other ontological notions. It's quite possible that his references to essence are to support some private intuition that he has of the relationship between logic and ontology. The fact that he offers no explanation of these references suggests that he is not concerned about making them clear to the reader. But we will see below that it may be possible to read between his lines and recover something of these intuitions. Before we do this, notice two other examples of Ibn Sīnā's ontological names. (6) below is earlier than any of (1)–(5); in fact it is already present in *Short Epitome in Logic*. (7) is later; the class named is already present in al-Barakāt, but Ibn Sīnā adopts the name much later than the Jurjān period, in *Easterners*. I will argue in Section 14.3 that his reason for the name appears first in *Burhān* and has to do with the ontological use of logic in demonstrative argument.

- (6) Ibn Sīnā uses the phrase *ḥaqīqī munfaṣil* for some *munfaṣil* hypothetical sentences in exceptive logic. The context shows that this phrase is used to pick out those *munfaṣil* sentences such that in any situation exactly one of the clauses is true.
- (7) At least in the context of demonstrative logic, Ibn Sīnā uses the name 'entailed' (*lāzim*) for sentences of the form described in (2) above. The name suggests that the subject term of the sentence names a species of the genus named by the predicate term.

We will describe all of (1)–(7) as examples of 'ontological naming'.

Now we examine the evidence for the logical meanings of the ontological namings, beginning with those involving *ḥaqīqī* (in other words, (3), (4), (5) and (6)). In each case mentioned, there is a larger class of sentences, and *ḥaqīqī* is used to pick out a smaller class within the larger. In (3) the larger class is the class of *mumkin* sentences, and *ḥaqīqī* picks out the contingent. In (6) the larger class is the class of *munfaṣil* hypothetical sentences in exceptive logic, and *ḥaqīqī* picks out those such that in any situation exactly one clause is true. We should expect a similar situation in (4) and (5).

As to (5), there are two kinds of predicative sentence that Ibn Sīnā calls 'necessary'. The first of these consists of the alethic necessary sentences, and the second consists of the permanent (*d* or *dā'im*) sentences which express (in the affirmative universal case) that the predicate holds of each subject individual for as long as that individual exists. Ibn Sīnā gives adequate evidence that when he describes a sentence as 'real necessary' (*darūri*

ḥaqīqī) he normally means that it is permanent. Thus he tells us:

- (11.30) An example of real necessary is the sentence ‘The human with necessity is an animal’. Its meaning is that for as long as the essence which is the human exists, he is an animal.
(*Middle Summary in Logic* [114] 105.9f.)

At *Middle Summary in Logic* 105.5f Ibn Sīnā has given the same explanation for ‘necessary’ without the *ḥaqīqī*. This suggests to me that at least within the context of sentences with temporal conditions on their predicates, Ibn Sīnā thinks of *ḍarūrī* as short for *ḍarūrī ḥaqīqī*.

We turn to the *ḥaqīqī* absolute sentence. The most revealing mention of it is in *Deliverance*:

- (11.31) ... the absolute [sentence] which is *ḥaqīqī* is pure so that it says the same thing whether it is regarded as negative or affirmative.
(*Deliverance* [131] 68.7f.)

This unambiguously points to the Alexandrine absolute, which is a Janus sentence with the same temporal mode on both its affirmative side and its negative. We are calling this sentence form *w*. References at *Twenty Questions* Question ix, 88.22 and *Middle Summary in Logic* 119.1f confirm this reading of *ḥaqīqī* absolute.¹⁹

Now that we have considered several of Ibn Sīnā’s ontological namings, it’s natural to ask whether there is a common pattern running through all or most of the examples, that might help to explain Ibn Sīnā’s thinking. I suggest that there is such a pattern, at least for the namings (3), (4) and (5). Ibn Sīnā seems to have a picture of how a class of sentences is formed: it is formed by generalising from paradigm examples of sentences in the class. Also I suggest that he took as paradigms those sentences which are richest in the number of logical inferences that they take part in. For example in the case of *munfaṣil* hypothetical sentences in exceptive logic, the sentences that Ibn Sīnā in *Short Epitome in Logic* counts as *ḥaqīqī* are those sentences

¹⁹The *Twenty Questions* reference confirms that the sentence is a Janus sentence (but goes on to make an appalling inference from this). The *Middle Summary in Logic* reference claims that if a [formal predicative absolute] sentence ϕ is consistent with a *d* sentence with the same subject and predicate terms as ϕ , and also with a *ḥaqīqī* absolute sentence with the same subject and predicate terms as ϕ , then ϕ must be Themistian absolute. This claim is correct if *ḥaqīqī* absolute sentences are Alexandrine absolute but not if they are Themistian absolute. There is also an informative reference at *Syllogism* ii.2, 89.

'Either ϕ or ψ ' that obey all of the four laws²⁰

- (11.32) Either ϕ or ψ . ϕ . Therefore not ψ .
 Either ϕ or ψ . ψ . Therefore not ϕ .
 Either ϕ or ψ . Not ϕ . Therefore ψ .
 Either ϕ or ψ . Not ψ . Therefore ϕ .

The same is true of the sentences that al-Fārābī in his *Syllogism* describes as expressing a 'complete' (*tāmm*) conflict.²¹ The sentences that Ibn Sīnā counts as not *ḥaqīqī* are a little different from those that al-Fārābī counts as not expressing a complete conflict, but they have it in common that they obey only half of the four listed laws. It could be that Ibn Sīnā thinks of the four laws as giving the essence of a *munfaṣil* sentence, and it may be that al-Fārābī expressed the same idea through his word 'complete'. A similar argument could be made for case (3).

In case (5) it is not very plausible that the alethic necessary sentences are got by generalising the permanent sentences. But I will argue BELOW that Ibn Sīnā sees the permanent sentences as giving the meaning of the alethic necessary sentences, in the sense that the alethic necessary sentences obey the same logical laws as the permanent sentences, and are our best source of information about what these laws are. We will return to this point in more detail when we have set up the rules of (*dt*) logic in Section ?? below.

Likewise we will return to the case (7) of *lāzim* when we have taken note in Section 14.2 of how Ibn Sīnā explains these sentences. We will see there that these sentences have a distinctive role to play in justifying the steps in demonstrative reasoning.

There remain cases (1) and (2) which talk about essences. I don't know what one can usefully say about them, beyond the two comments made earlier in this section.

To conclude: leaving aside the cases (1) and (2) where Ibn Sīnā's intuitions are hidden from us, the other cases of ontological naming that we have examined all revolve around some way in which it can be said that the named sentences play a primary or fundamental role within their part of logic. It could be said that they are paradigms for certain areas of logic. I suggest that this is what caused Ibn Sīnā to find ontological names for them. With this in mind I propose to translate *ḥaqīqī* as 'paradigm'.

²⁰'When an exceptive syllogism is generated from paradigmatic hypotheticals ... those which have two parts have an affirmative and a negative clause that are the contradictory negations of each other' (*Syllogism* [124] viii.2, 400.4f).

²¹Al-Fārābī *Syllogism* [59] 33.10.

Chapter 12

Aristotelian and non-Aristotelian proofs

We saw in Section 7.1 above that Alexander of Aphrodisias described two ways in which a mode can be attached to a predicate: it can either be written alongside the predicate, or it can be ‘incorporated’ in the predicate so as to make a new predicate. In this chapter we will see how Ibn Sīnā develops the idea of incorporation. First, he lets us incorporate a mode in the subject term just as in the predicate term. And second, he points out that incorporation, generalised in this way, allows us to create new syllogisms that use his new temporal *d* and *t* sentences.¹

12.1 (*dt*) logic

In section ii.4 and books iii and iv of his *Syllogism* [124], Ibn Sīnā reviews the syllogisms of predicative modal logic. Section ii.4 considers the syllogisms of categorical logic, together with some close variants including the syllogisms where both premises are classified as Necessary. Then books iii and iv consider in turn other modalisations of categorical syllogisms, in a schedule based on their underlying categorical syllogisms and following Aristotle’s *Prior Analytics* i.8–22. In this schedule the syllogisms with

¹Independent of Ibn Sīnā, the device of incorporation and its use to justify entailments were given in a more general form suitable for first-order logic by Thoralf Skolem in [201] (1920), and since then have been widely used in model theory under the name of Morleyisation (cf. [83] pp. 56f). Ibn Sīnā’s description of the device in *Syllogism* iii.1, 130.11–131.2 (quoted below in (12.1)) is clearly inadequate as an explanation, but it says enough to allow us to reconstruct and apply the method—as we will do in Section 12.1 below.

at least one premise classified as Possible or Contingent occupy *Syllogism* iii.4–iv.6, 160.1–227.4, so that *Syllogism* iii.1–3, 125.1–159.16 takes up the syllogisms that have one premise Absolute and the other premise Necessary.

At *Syllogism* iii.1, 130.4 Ibn Sīnā begins to consider the modal predicative premise-pairs in the second figure. At 130.4–10 he considers the premise-pairs where both premises are universal; these are *Cesare* and *Camestres*. At 130.10f he turns to the two premise-pairs where one of the premises is existential; these are *Festino* and *Baroco*. Then before making any attempt to establish productivity or find conclusions of modalisations of these four premise-pairs, he jumps to the proposal:

If you count the permanence or non-permanence as a part of the predicate, it forms a premise-pair You could say

(12.1) 'Everything which satisfies the phrase *C* is a thing of which it is permanently (*dā'imān*) denied that it is a *B*.
And nothing which satisfies the phrase *A* is a thing of which it is permanently (*dā'imān*) denied that it is a *B*.
It follows that no [*C*] is an [*A*], and that is with necessity.'

(*Syllogism* [124] 130.11–131.2.)

The natural reading of this passage, in its context after mention of the modalised second figure syllogisms, is that incorporating the temporal modes into the terms will lead to a proof of productivity and will find the required conclusion. Testing (*imtilhān*) as below will confirm that this is true.²

The premise-pair exhibited in (12.1) is *Camestres*(*d, t*). Let us test it. The repeated 'permanently' shows that the premises are meant to be read temporally; so 'necessity' in the conclusion can be read as paradigmatic necessity. Thus *B* is a 2-ary term with an object parameter and a time parameter. In the first premise the predicate together with the time quantifier form the quantified predicate

(12.2) thing that is not a *B* at any time when it exists

²Except that if incorporation leads to a conclusion of the form *t*, a further argument is needed to show that no stronger conclusion is available; and if incorporation leads to a dead end, it needs to be shown that the premise-pair is sterile. These can both be shown by a suitable modification of the method of pseudoconclusions, as in REF. But we don't know how far Ibn Sīnā himself was aware of this.

which is equivalent to

(12.3) not (a thing that is a B at some time when it exists).

We will write³ B^+ for the new term which is formed by incorporating the time quantifier into B in (12.3). Similarly we will write B^- for the new term which is formed by incorporating the time quantifier into B in the quantified predicate

(12.4) thing that is a B at every time when it exists .

Note that the phrase ‘when it exists’ is redundant in (12.2) and required in (12.4), by (10.32) in the theory of E .

Also the term ‘thing which satisfies the phrase C ’ in the first premise of (12.1) needs a temporal quantifier, and again by the theory of E the implied full phrase is ‘thing that is a C at some time when it exists’. This quantifier can be incorporated together with C . Thus the first premise of (12.1), which is a d sentence of the form $(e-d)$, can be written

(12.5) No C^+ is a B^+ .

The second premise can be paraphrased as

(12.6) Everything which is an A at some time when it exists is a B at some time when it exists.

which has the form $(a-t)$ and goes into symbols as

(12.7) Every A^+ is a B^+ .

So the two premises of (12.1) can be paraphrased as

(12.8) $(e)(C^+, B^+), (a)(A^+, B^+)$.

The two symbols B^+ and B^- are called the two ‘poles’ of the term B .⁴

Most people find these paraphrases with poles quite unpleasant to have to check whenever we use a (dt) syllogism. But we can write them out in a

³Some people write ‘sometimes- B ’ for B^+ and ‘always- B ’ for B^- .

⁴Note the curious fact that the sentences in (12.8) are neither material nor formal by our definitions in Section 4.1. Just as the term letter B in categorical logic can be replaced by a noun or noun phrase, so B^+ in (dt) logic can be replaced by a noun with the qualification ‘at some time during its existence’.

table that we can refer to as needed:

	ϕ^{dt}	$\phi^{\pm}$
	$(a-t)(B, A)$	$(a)(B^+, A^+)$
	$(a-d)(B, A)$	$(a)(B^+, A^-)$
	$(e-d)(B, A)$	$(e)(B^+, A^+)$
(12.9)	$(e-t)(B, A)$	$(e)(B^+, A^-)$
	$(i-t)(B, A)$	$(i)(B^+, A^+)$
	$(i-d)(B, A)$	$(i)(B^+, A^-)$
	$(o-d)(B, A)$	$(o)(B^+, A^+)$
	$(o-t)(B, A)$	$(o)(B^+, A^-)$

In this table, ϕ^{dt} is a sentence of one of the forms d and t , and $\phi^{\pm}$ is the same sentence written with the temporal quantifiers incorporated into the terms. Reading from the right column to the left gives the rule for de-incorporating.

Finding the conclusion of (12.1) is now easy when we remember that categorical *Camestres* has the form

$$(12.10) \quad (e)(C', B'), \quad (a)(A', B'). \text{ Therefore } (e)(C', A').$$

In (12.10) take A' to be A^+ , B' to be B^+ and C' to be C^+ . This tells us at once that the conclusion of (12.8) is $(a)(C^+, A^+)$, and according to the table (12.9) this de-incorporates as $(e-d)(C, A)$, in English

$$(12.11) \quad \text{Everything that is a } C \text{ at some time of its existence is permanently not an } A.$$

This is the same conclusion as in (12.1) with 'necessity' read as paradigmatic necessity. So incorporation leads us to a proof of *Camestres*(d, t, d). Hiding the table (12.9), the proof by incorporation can be written in the three-column format illustrated in Section 4.5 above, as follows:

	<i>Camestres</i> (d, t, d)		
	1	$(e-d)(C, B)$	premise
	2	$(a-t)(A, B)$	premise
(12.12)	3	$(e)(C^+, B^+)$	1 incorporation
	4	$(a)(A^+, B^+)$	2 incorporation
	5	$(e)(C^+, A^+)$	3,4 categorical <i>Camestres</i>
	6	$(e-d)(C, A)$	5 de-incorporation

While we are in a proving mood, here are a few more (dt) proofs by incorporation, to illustrate some features of this proof system.

Example 2 in Section 11.2 gave a sketch proof of $Camestres(t,d,d)$. That proof read the moods involved as temporal, since it referred to the Alexandrine interpretation and the Themistius interpretation, both of which are temporal. Here is the proof by incorporation:

$Camestres(t,d,d)$

- | | | | |
|---------|---|-----------------|----------------------------------|
| | 1 | $(e-t)(C, B)$ | premise |
| | 2 | $(a-d)(A, B)$ | premise |
| (12.13) | 3 | $(e)(C^+, B^-)$ | 1 incorporation |
| | 4 | $(a)(A^+, B^-)$ | 2 incorporation |
| | 5 | $(e)(C^+, A^+)$ | 3,4 categorical <i>Camestres</i> |
| | 6 | $(e-d)(C, A)$ | 5 de-incorporation |

A curious fact about (dt) syllogisms is that for each of them, the mode (d or t) of the conclusion depends on just three things: the mode of the minor premise, the mode of the major premise and the figure of the premise-pair.⁵ So for example we get a complete picture of the (dt) syllogisms in second figure by adding the conclusions of $Camestres(d,d)$ and $Camestres(t,t)$, as follows.

$Camestres(d,d,d)$

- | | | | |
|---------|---|-----------------|----------------------------------|
| | 1 | $(e-d)(C, B)$ | premise |
| | 2 | $(a-d)(A, B)$ | premise |
| | 3 | $(e)(C^+, B^+)$ | 1 incorporation |
| (12.14) | 4 | $(a)(A^+, B^-)$ | 2 incorporation |
| | 5 | $(a)(A^+, B^+)$ | 4 $(B^- \subseteq B^+)$ |
| | 6 | $(e)(C^+, A^+)$ | 3,5 categorical <i>Camestres</i> |
| | 7 | $(e-d)(C, A)$ | 5 de-incorporation |

The proof (12.14) uses the non-Aristotelian rule $(B^- \subseteq B^+)$, which applies also for any term letter; for convenience it can be thought of as a rule in the theory of E . If we try to prove the productivity of $Camestres(t,t)$ by a similar incorporation argument, we find we need the converse implication $(B^+ \subseteq B^-)$, which is false. In fact there is a proof of the sterility of $Camestres(t,t)$ by pseudoconclusions; see [98] REFERENCE.

⁵This is not true if one also allows the fourth figure. Thus in fourth figure we have $Dimatis(t,t,t)$, but $Calemes(t,t)$ is sterile. Nor is it true for Aristotle's modal syllogisms in the third figure; we have $Disamis(ABS, NEC, ABS)$ but $Datisi(ABS, NEC, NEC)$.

A combination of proofs by incorporation and pseudoconclusions allows us to catalogue the (dt) syllogisms as follows:

First and Third Figures:

	premise-pair	productive	conclusion
(12.15)	(d, d)	Yes	d
	(d, t)	Yes	t
	(t, d)	Yes	d
	(t, t)	Yes	t

Second Figure:

	premise-pair	productive	conclusion
(12.16)	(d, d)	Yes	d
	(d, t)	Yes	d
	(t, d)	Yes	d
	(t, t)	No	

Inspection of these tables, and comparison with the modal syllogisms listed in *Short Epitome in Logic* in Chapter 3 above, confirms the following striking fact.

(12.17) **(dt) Syllogism Theorem.** The alethic syllogisms that use just the modes 'necessary' and 'absolute' and are valid according to *Short Epitome in Logic* are exactly the valid (dt) syllogisms, but with t replaced by 'absolute' and d replaced by 'necessary', and after removal of the second figure syllogisms in *Short Epitome in Logic* where both premises have the mood 'absolute'.

12.2 What does the (dt) Syllogism Theorem tell us?

The (dt) Syllogism Theorem relates two groups of predicative syllogisms: the alethic syllogisms listed as valid in *Short Epitome in Logic*, and the provably valid syllogisms of (dt) logic. Ibn Sīnā gives no listing of the dt syllogisms, but we have seen that in *Syllogism* iii.1 he gives what is verifiably an efficient way of proving all the valid syllogisms of this logic, using incorporation. From the logical context in *Syllogism* iii.1 it seems highly likely that he knew he had given such a method, or at least that he was convinced of

this without a complete proof of it. In any case he also gave several hands-on proofs of *(dt)* syllogisms.

According our dating of the relevant writings, Ibn Sīnā had the list of alethic syllogisms in *Short Epitome in Logic* by 994 at latest, whereas his earliest known hands-on proof of a *(dt)* syllogism is in *Twenty Questions*, in (11.7) above, dated around 1012. The *(dt)* Syllogism Theorem sets up a translation between the *(dt)* syllogisms and a subset of the alethic syllogisms, namely those alethic syllogisms whose premise-pairs are not pairs of absolute sentences in second figure. As far as I can see, Ibn Sīnā never says that these alethic syllogisms should be removed from the list in *Short Epitome in Logic*, or points to any facts about these alethic syllogisms that explains why the *(dt)* Syllogism Theorem has to leave them out of the reckoning. However, there are passages from *Twenty Questions* up to *Pointers and Reminders* in which Ibn Sīnā gives arguments claimed to show that no second figure premise-pairs with paradigmatic or broad absolute premises are productive. As they stand, none of these arguments are convincing, though the later arguments are more cogent than the earlier.

Thus in *Twenty Questions* [115] Question ix, Ibn Sīnā claims to show that ‘there is no syllogism in second figure from two paradigmatic absolutes’, with the following argument:

The absolute which is of this kind is like the possible in that there is no conclusion from it, and that is because it is possible to make an affirmative universal absolute sentence and a negative universal absolute sentence together, as we showed. . . . Likewise
(12.18) [Aristotle] used “No horse is moving absolutely, and every animal is moving absolutely”, in the sense that every animal has a time at which it moves voluntarily, and every horse doesn’t move at some time.

(*Twenty Questions* Question ix, 88.21f and 89.4–6.)

We quoted this reference to Aristotle at (10.5).

The connection to second figure here seems to be that

(12.19) Every (sometime-)animal is moving at some time;
and every (sometime-)horse is at some time not moving.

which can be read as a premise-pair in mood *Cesare(t,t)*. From the opening words of (12.18) it seems that Ibn Sīnā wants to use the two sentences in the premise-pair as pseudoconclusions, one universal affirmative and the other universal negative, to prove by Aristotle’s method that the premise-pair is sterile. But this is not how the method of pseudoconclusions works,

so the argument falls flat. In any case it applies only to the moods *Cesare* and *Camestres*. Ibn Sīnā's claim that 'there is no syllogism in second figure from two paradigmatic absolutes' (which as far as we know is new in *Twenty Questions*) is presumably based on failed attempts to deduce conclusions by hands-on arguments.

Another style of argument for the same conclusion is that *Cesare*(t, t), *Camestres*(t, t) and *Festino*(t, t) can't be reduced to first figure in the standard way, because ($e-t$) fails to convert symmetrically.⁶ We meet this argument at *Middle Summary in Logic* [114] 125.17, which is a definite improvement on the argument in *Twenty Questions*, though it still doesn't cover all cases.

By the time of *Pointers and Reminders*, Ibn Sīnā knows that in second figure there is no syllogism from two broad absolutes, i.e. from two t premises. (Thus *Pointers and Reminders* [135] i.7.4, 145.14.) But he may still be relying on the lack of proofs for such syllogisms; a different proof might still be found. He adds that there can't be a proof of such a syllogism by *reductio ad absurdum*, because 'the conditions for a contradictory negation are not legitimate for these propositions' (*Pointers and Reminders* [135] i.7.4, 147.3).⁷

A point worth noticing is that in all the contexts where second figure absolute premise-pairs are ruled to be sterile, 'absolute' means t or w , i.e. broad absolute or paradigmatic absolute. Does Ibn Sīnā believe that the sterility of these (dt) premise-pairs tells us something about the alethic syllogisms too?

⁶Thus ($e-t$)(B, A) $\Leftrightarrow$ (e)(B^+, A^-) and ($e-t$)(A, B) $\Leftrightarrow$ (e)(A^+, B^-) $\Leftrightarrow$ (e)(B^-, A^+); we can construct counterexamples to (e)(B^+, A^-) $\Leftrightarrow$ (e)(B^-, A^+).

⁷For Ibn Sīnā's strange problems with contradictory negations of temporal sentences, see REF.

Chapter 13

Possibility and contingency

13.1 Making the possible absolute

13.2 The Ibn Sīnā-Johnston semantics for (*dt*) logic

13.3 Modal syllogisms with possible premises

13.4 The absence of ampliation

Part V

**Hamadhān and Isfahān
(c. 1020–1037)**

Chapter 14

The later works

14.1 Dating the later works

In Chapter 11 above we dated Ibn Sīnā's leaving the Jurjān region to no earlier than 1015, taking the death of Hilāl in May of that year as a fixed point. According to al-Jūzjānī's continuation of Ibn Sīnā's *Autobiography* pp. 48–51, Ibn Sīnā went from Jurjān to Rayy, and then from Rayy to Qazwīn and then Hamadhān. This began a period of migration during which Ibn Sīnā was under pressure from various local rulers to take employment with them as either doctor or business manager. The wandering came to an end when Ibn Sīnā made his services available to 'Alā' al-Dawla I-Muḥammad. Ibn Sīnā's support of al-Dawla led to his imprisonment in the castle of Fardajān near Hamadhān by enemies of al-Dawla. When al-Dawla came to Hamadhān and released Ibn Sīnā from Fardajān in 1023, Ibn Sīnā went back briefly to Hamadhān and then moved to al-Dawla's new headquarters at Iṣfahān. He remained in Iṣfahān to the end of his life, except when accompanying al-Dawla on expeditions. It was on one of these expeditions that Ibn Sīnā died in 1037.

Yahya Michot [119] dates Ibn Sīnā's *Letter to the Vizir* to 1015, written while Ibn Sīnā was in Hamadhān, and he accepts the case made by Madelung that the letter was addressed to Abū Sa'd Muḥammad who was vizir in Hamadhān at that date. But the date is not sensitive for us, because although the letter is mainly about competence in formal logic, it says nothing to tie itself to any particular date in Ibn Sīnā's logical development.

The next logical work that we have from Ibn Sīnā in this period is *Guidance (Hidāya)*, which according to al-Jūzjānī's continuation of Ibn Sīnā's *Autobiography* was written while Ibn Sīnā was imprisoned in the castle of Far-

dajān, hence in 1023. Al-Jūzjānī also tells us that he encouraged Ibn Sīnā to write the encyclopedic *Shifā'* at about this time:

- (14.1) Then I asked him to comment on the works of Aristotle, but he said that he was not free to do so at that time. "But if you would be satisfied with my composing a work in which I would set forth what, to me, is sound in these sciences, without debating with those who disagree or devoting myself to their refutation, I would do that." I was satisfied with it and so he began ...
(Al-Jūzjānī in *Autobiography* [117] p. 54, trans. Gohlman loc. cit. p. 55.)

We should note that Ibn Sīnā's opinion of *Cure* was not the one attributed to him by al-Jūzjānī in (14.1). Compare (2.12) in Section 2.2 above, where Ibn Sīnā admits that in works before *Easterners* (and this must include *Cure*) he included material that he didn't regard as sound, in order not to 'create schisms' with the Peripatetics.

In Section 1.2 we dated *Syllogism* to 1024. In the light of al-Jūzjānī's account in *Autobiography* and al-Jūzjānī's preface to *Cure* which is published as a preface to *Madkhal*, Gutas [74] p. 107f suggests a likely schedule for the completions of the Logic sections of *Cure*:

- (14.2) *Eisagōgē*, 1022 in Hamadhān.
Categories to *Burhān* and perhaps *Debate*, 1024 in Hamadhān after release from Fardajān.
The remaining books in Logic of *Cure*, 1024–1027 in Iṣfahān.

Gutas also gives 1027 as a date for the completion of *Cure* as a whole. In the preface to *Easterners* Ibn Sīnā explains that this work was written after *Cure* was completed; Gutas [74] p. 132 gives December 1028 as a date by which *Easterners* was completed. The book published under the title 'Logic of the Easterners' is what remains of the Logic section of *Easterners* after at least one early episode of vandalism; on this see Section 1.2 above.

Al-Jūzjānī tells us, in his continuation of Ibn Sīnā's *Autobiography* [117] pp. 66f, that Ibn Sīnā finished *Cure* during a journey, and wrote *Deliverance* on the same journey. This was in a year when al-Dawla attacked Shābūr Khwāst; probably Ibn Sīnā was accompanying al-Dawla. Since this journey took place in the year in which *Cure* was completed, and we have already dated this completion to 1027, we look for evidence of when al-Dawla attacked Shābūr Khwāst. Gohlman [117] p. 133 points to evidence that al-Dawla made such journeys in 1026 and 1030. A slippage of one year

is better than a slippage of three years, so we opt for the year 1027 as the year when *Deliverance* was completed. However, al-Jūzjānī also tells us that the first part of *Deliverance*, the part on logic, was the essay *Middle Summary in Logic* that Ibn Sīnā wrote in Jurjān ([117] pp. 74–77); in Section 11.1 we dated *Middle Summary in Logic* to 1015.

This double-dating raises a question: when Ibn Sīnā incorporated *Middle Summary in Logic* in *Deliverance*, did he make any changes to the text to fit its new context? For example at *Deliverance* [131] 133.6 and 133.14 we read:

- What is said of ‘everything’ in this case is not the same as in the Book of Syllogism (*kitāb al-qiyās*). . . . And the necessary in this (14.3) case is not the same as the necessary in the Book of Syllogism (*kitāb al-qiyās*).
(Cf. DDO.13 in Section 10.4 above.)

What is the Book of Syllogism? Is it (1) Aristotle’s book *Prior Analytics*, or is it (2) the part of *Middle Summary in Logic* that corresponded to Aristotle’s book? Or has Ibn Sīnā adjusted to the context in *Najāt* by introducing a reference to (3) the recently-written book *Syllogism*? Gutas [74] p. 116 (including his footnote 2) assembles various kinds of evidence that Ibn Sīnā with al-Jūzjānī’s help put together *Deliverance* from several earlier works in a hurry and with almost no editing, ‘practically without composing a single line anew’. So we are justified in assuming that the Book of Syllogism is either (1) or (2). The choice between these two is difficult because *Middle Summary in Logic* belongs to the period of maximum indeterminacy in the relation between Aristotle’s formal logic and that of Ibn Sīnā. See REF for further discussion.

There remain two surviving late works of Ibn Sīnā containing material on formal logic. These are *Philosophy for ‘Alā’ al-Dawla* and *Pointers and Reminders*. The dating of these works will not be critical for us, because neither of them contains much new work in logic. The reasons are different in the two cases. *Philosophy for ‘Alā’ al-Dawla* was written in response to a request for an elementary introduction to philosophy for ‘Alā’ al-Dawla’s courtiers, and it is written in Persian in a reasonably easy-going style. *Pointers and Reminders* by contrast is a sophisticated work, certainly not an introduction to anything, but in logic it is backward-looking. For example its treatment of hypothetical logic is a summary of material from *Middle Summary in Logic*.

There is a further point to mention about these two works. All through his career as a logician Ibn Sīnā struggled with a syllogistic mood that we

named *Camestres* (ABS, NEC, NEC) in Section 7.2 above. In *Syllogism* iii.2 he found an answer to Aristotle's second claimed refutation of this mood, and in the process he discovered a counterexample in modal logic to a common assumption about shapes of proofs; the assumption was true for categorical syllogisms but not for (*dt*) logic. It was probably soon afterwards that he discovered a second counterexample to the false assumption, and he put this into *Pointers and Reminders* [135] i.7.4, 145.5–12 (see REF in Section 14.3 below). Also it was probably after he had these counterexamples that it occurred to him to check the shapes of proofs in Euclid. In *Philosophy for 'Alā' al-Dawla* he goes way above the heads of the courtiers and analyses the shape of Proposition 1 of Euclid's *Elements*, showing that it is much more complicated than anything known in categorical logic. (See REF in Section 14.2 below.) Probably if Ibn Sīnā had not already resigned himself to abandoning further research in formal logic, he would have begun a new research programme to analyse the shapes of real-life proofs. As far as I know, these two items are the only logical innovations in either *Philosophy for 'Alā' al-Dawla* or *Pointers and Reminders*. They do at least confirm the early reports that the works were written after *Syllogism*. We have no further information on the dating of *Philosophy for 'Alā' al-Dawla*. Gutas [74] pp. 155–159 dates *Pointers and Reminders* between 1030 and 1034. David C. Reisman and Yahya Michot had argued for a date between 1027 and 1030, but sadly both died before this disagreement could be resolved.

Another undated work is *Notes* (*Ta'liqāt*), which may be a set of notes by one of Ibn Sīnā's students—Ibn Zayla has been suggested—on tutorials with Ibn Sīnā. It contains a useful account of the 'subject' (*mawḍū'*) of the science of logic, a topic that is known to have interested Ibn Sīnā in the mid to late 1020s.

Since *Pointers and Reminders* has somehow gained itself an undeserved reputation as an advanced work in logic, let me mention two features of the logical part of *Pointers* that I have seen quoted as evidence of the originality of the work. The first is the listing of syllogistic moods, which runs first through the first figure, then through the second and then through the third.¹ In fact Ibn Sīnā had two formats for listing syllogistic moods, and he chose between them for each work. In *Short Epitome in Logic, Guidance* and *Pointers and Reminders* he listed the categorical moods, more or less as in *Prior Analytics* i.4–6, and attached to the categorical moods some comments on related modal moods. In *Middle Summary in Logic, Deliverance*

¹Street [5] p. xxii regards this as an innovation in *Pointers and Reminders*.

and *Syllogism* he followed *Prior Analytics* i.4–22 and listed the modal forms after the categorical ones, classifying them by the kinds of mode used in them. In these examples I get the impression that he preferred the second (longer) listing when he felt he had more to say about the modal moods.

The second alleged original logical feature of *Pointers and Reminders* is the use of the notions of *taṣawwur* and *taṣdīq* as classificatory notions. In fact this use of these notions is already present in *Easterners*, and it is more conspicuous in *Easterners* than in *Pointers and Reminders*. Unless I miss something, the only thing that it contributes to formal logic is to emphasise the difference between expressions that describe or refer to an individual or concept, and expressions that state a truth or falsehood.² Joep Lameer in his sensitive study [152] of Mulla Ṣadra’s reflections on the distinction between *taṣawwur* and *taṣdīq* regards this as an epistemological distinction rather than a logical one.

Gutas believes that Ibn Sīnā’s logic was still developing in the late 1020s. At [74] p. 204 he claims that *Pointers and Reminders* i.3.10 [135] 83.15–84.3 exhibits a ‘refinement of Aristotle’s modal syllogistic, apparently fully developed in Avicenna’s later years’, and at [74] p. 440 he suggests that there was a ‘developing exposition of Avicenna’s logic’ around the years 1026–1027. These views seem to be based on two misunderstandings: (1) that the statements about ‘conditions’ in *Pointers and Reminders* i.3.10 are a new development in Ibn Sīnā’s thinking, and (2) that these ‘conditions’ are closely related to the modes of Ibn Sīnā’s logic. On these conditions see Section 5.5 above. The misunderstanding (2) may be partly derived from A.-M. Goichon [73] p. 52f, which Gutas cites.

14.2 Necessity on the quantifier

14.3 The *lāzim* sentences

During the Jurjān period Ibn Sīnā mentioned some sentences which have temporal modes but are not *t* or *d* sentences. He did also mention some

²The same logical material can often be arranged in different orders or under different heads for different expository purposes without implying any difference in the logical content overall. In our textbook [38] Ian Chiswell and I found it convenient to introduce quantifier-free predicate logic before quantifiers. Hilbert and Ackermann in their textbook [81] introduced sentences with one quantifier before sentences with nested quantifiers. Both these orders of exposition are unusual, but all four of us were teaching the same first-order logic.

sylogisms using these sentences, though he didn't succeed in building tidy logics around them like his (*dt*) logic. In this section we consider the sentences that in *Easterners* he came to describe as *lāzim*; in Section ABOVE we abbreviated this name to ℓ . *Easterners* contains no justification for the name *lāzim*, but we do find a justification in Section ii.1 of the volume of the *Shifā'* called *Demonstration* (*Burhān*), which we are dating to 1024..

Ibn Sīnā makes little attempt to find syllogisms that involve ℓ sentences. But here is one, which we name *Barbara*(d, ℓ, d):

- Everything that is sometimes a C is a B throughout its existence.
Everything that is sometimes a B is an A so long as it is a B .
(14.4) Therefore everything that is sometimes a C is an A throughout its existence;
(REF)

Again this has no Aristotelian proof. Ibn Sīnā OFFERS THE FOLLOWING HANDS-ON PROOF. In *Deliverance* again Ahmed misses the conclusion, though this time I don't see why. REFS for this proof (from Esots paper) are:

14.4 The *wujūdī* sentences

14.5 Adding induction to predicative logic

.

Chapter 15

Late proof theory

15.1 Incorporation

15.2 The modal challenge

15.3 Shapes of proofs

15.4 Compound proofs and proof search

Chapter 16

To be placed

- 16.1 The overlap between recombinant and exceptive hypothetical logic**
- 16.2 Negating subject terms**
- 16.3 Sterile hypothetical premise-pairs**
- 16.4 Making absurdity proofs direct**

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