Where Frege is coming from

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The aristotelian tradition was like the tradition of a craft. Much of it was passed on by practice rather than textbooks. This accounts for the frequent 'rediscoveries' of things that were there below the surface from early times.

Examples below are:

(1) reduction of hypotheticals to categoricals (used by Ibn Sīnā, 'rediscovered' by Wallis and Boole among others);

(2) semantics of particles (used by Ibn Sīnā, 'rediscovered' by Bertrand Russell among others).

I concentrate on two questions:

- How was Frege's logical calculus of *Begriffsschrift* different from its aristotelian predecessors?
- How was Frege's semantics different from its aristotelian predecessors?

One should go to the best of the aristotelians,
not second rate logicians like Kant and Lotze.

The logical calculus

A typical aristotelian inference (syllogism in Celarent):

No Bs are Cs.
Every A is a B.
Therefore no As are Cs.

An obvious historical question:

> Since Frege's predecessors must have seen that their logic was hopelessly inadequate to formalise (say) the Elements of Euclid, why didn't they do something about it?
> Why did it take over 2000 years to get a decent logic off the ground?

Ibn Sīnā is using logic to justify *each inference step separately*.

The formalisation can change from one step to the next.

From Ibn Sīnā's *Autobiography* (Gutas' translation):

> I read Logic and all the parts of philosophy once again. … I compiled a set of files for myself, and for each argument that I examined, I recorded the syllogistic premisses it contained, the way in which they were composed, and the conclusions which they might yield, and I would also take into account the conditions of its premisses [i.e. their modalities] until I had Ascertained that particular problem. … Having mastered Logic, Physics and Mathematics …

Example 1: Changing from individuals to pairs.

Ibn Sīnā, *Qiyās* 256.12–15:

> When you say 'If a line falls on two lines in such a way that the two angles which are on one side etc., then the two lines are parallel', this can be paraphrased … thus: 'Every pair of lines, on which a line falls in such-and-such a way, is a parallel pair'.

Leibniz does something similar to justify relational reasoning:

> Painting is an art, therefore he who learns painting, learns an art.
> Proof: He who learns painting learns a thing which is painting.
> But painting is an art. ...
> (*Letter to Vagetius*, Parkinson p. 89)

The point is that both 'he' and 'which' are in the nominative, giving a double subject.

Leibniz:

> There are *valid non-syllogistic inferences* which cannot be rigorously demonstrated in any syllogism unless the terms are changed a little, and this altering of the terms is the non-syllogistic inference. There are several of these, including arguments from the direct to the oblique — e.g. 'If Jesus Christ is God, then the mother of Jesus Christ is the mother of God'. (*New Essays on Human Understanding* p. 479f., his emphasis.)

Example 2: 'Reduction of hypotheticals to categoricals' works by switching from individuals to situations.

> "If the proposition X is true, the proposition Y is true." An undoubted meaning of this proposition is, that the *time* in which the proposition X is true, is *time* in which the proposition Y is true. (Boole *Laws of Thought* p. 163)

Frege, unpublished notes '*Logic*' (between 1879 and 1891):

> We cannot give too many warnings against the danger of confusing points of view and switching from one question to another, a danger to which we are particularly exposed because we are accustomed to thinking in some language or other and because grammar ... is a mixture of the logical and the psychological. (PW p. 6)

Two things become *harder* if you formalise globally:

(1) You can't paraphrase so as to switch between propositional and predicate rules.

Frege's solution: Include both propositional and predicate rules separately within the proof calculus.

In place of linguistic subject (= Leibniz's nominative case), Frege allows us to choose a deep component s in a proposition and write the proposition as $\phi(s)$.

He obscures this by speaking as if the issue was the structure of languages, rather than where the rule applies. Half-believing the traditional view that the eigenterms must be at the syntactic top level, he claims that the top level moves according to how we analyse the proposition. Here his position lies between traditional and modern.

(2) There are problems about bringing things low down in the structure of the proposition to the top level by paraphrase.

Frege's solution: Introduce new inference rules that apply arbitrarily deep down.
Example: Deduce $\phi(t)$ from $\phi(s)$ and $s = t$.
Slightly obscured by his decision to express inferences by combining logical axioms with a single inference rule of modus ponens.

But some things are *easier* if we globalise.
Under local formalisation, each step has to be complete in itself, starting with meaningful statements and finishing with a meaningful conclusion.
In particular we can't say

'Suppose $\phi(a)$'

and resolve this assumption two pages later.
It has to be resolved within the one step.

Curiously Frege stays with the traditional view, which he defends in his late paper on *Foundations of geometry*.
He does this by requiring that at each step we state what fact is known that was not known previously.
This is almost exactly Ibn Sīnā's view on arguments from assumptions.

Semantics

Question: What are the meanings of prepositions, quantifiers, copular verbs etc.?

On the Ibn Sīnā-Frege view, parameters have to be universally quantified out at each step.

Frege does this with his 'latin letters'.

Ibn Sīnā seems to be doing it with a global modality '*dā'iman*' meaning 'in all cases';
but there are many problems of interpretation.

Ibn Sīnā (discussing semantics):

> It's a black mark against Aristotle that he mentions among the simple expressions the noun and the verb, but ignores the particles. (*Ibara* 29.15)

So Ibn Sīnā reckons the question has been answered since Aristotle, i.e. by the aristotelian commentators.
He probably means Porphyry (3rd c) and Ammonius (5th c).

Porphyry-Ammonius-Ibn Sīnā theory of meaning types

A common noun has a meaning that consists of a 'character' (*ṭabīʿa*) for distinguishing things that satisfy it from things that don't.
This makes the noun semantically 'absolute' and 'independent', in the sense that its meaning can be given without reference to other words in the sentence in question.

Also some noun meanings are very much easier to explain if you can use other nouns already defined.
Arabic linguists' example:

fusaiwa = small silent fart

(Did Allah teach Adam this word ostensively? *Surat al-Baqara* 31.)

What kind of independence?

Sometimes a noun has a meaning that can't be given without reference to meanings of other words, but not necessarily in the same sentence.
Such a noun said to be 'of second imposition'.
Example: 'verb'.

Particles are words that are used only in combination with other words in fixed patterms.
Example: a preposition like 'of' combines with a noun to form a noun phrase.
Likewise 'every' combines with a noun to form a noun phrase; or better, it joins with a noun to form the subject of a sentence.

This idea appears already in Hellenistic linguistics.

Ammonius and Ibn Sīnā:

To give the meaning of a particle,
describe how the meanings of its compounds depend on
the meanings of the words joined to it.

Ibn Sīnā's general theory of the meanings of particles etc.:

> … a particle like 'not' or 'in', whose meaning is completed by being linked to something. A person who says 'Zayd is in' or 'Zayd is not' won't have expressed a complete meaning so long as he fails to say 'in the house' or 'not a human'. (*Isharat* 44.12ff)

Example: Ammonius' explanation of the meaning of 'Every'.

> Determiners …combine with the subject terms and indicate how the predicate relates to the number of individuals under the subject; …'Every man is an animal' signifies that 'animal' holds of all individuals falling under 'man'.

Compare Russell *On Denoting*:

> *everything* [is] to be interpreted as follows:
>
> C(everything) means '$C(x)$ is always true'.

Frege, '*What is a function?*':

> The functional sign cannot occur on one side of an equation by itself, but only when completed by a sign that designates … a number. … The peculiarity of functional signs, which we here called 'unsaturatedness', naturally has something answering to it in the functions themselves. … a function is completed by a number …

Frege notes a common confusion between functions and their values.

Ibn Sīnā notes a common confusion between arguments and values. It seems to be the 'black box' picture of functions: squaring makes 2 *become* 4.

> And there is a common kind of error about things that are joined together. It occurs through not recognising that an idea taken with another idea is not the whole arising from it and the thing taken with it; just as one added to six, when we consider it together with six, is not the sum of one and six, which is seven. (*Ibara* 15.9ff)

A commoner confusion than the one mentioned by Frege.

If Ibn Sīnā had extended this from participles to all common nouns, he would have reached Frege's position in the Grundsatz of the *Grundlagen*:

> Seek the meanings of words in the interconnections of the sentence, not in the words taken independently.
> Nach der Bedeutung der Wörter muss im Satzzusammenhange, nicht in ihrer Vereinzelung gefragt werden.

Ibn Sīnā comes close to extending the 'incomplete meaning' account to participles, because they have an 'indeterminate subject'.

> For example the word "walking". It signifies the act of walking and the indeterminate subject, and that the act belongs to the subject. (*Ibara* 18.9f)

He probably believes that the participle contains a suppressed form of the verb prefix which indicates the person of the agent.