# Including history in a mathematics module — practice and theory

Wilfrid Hodges

Herons Brook, Sticklepath, Okehampton
May 2010
http://wilfridhodges.co.uk/history16.pdf

2

**The speaker identifies himself**

- Recently retired university teacher of pure mathematics.
- Author of five textbooks in areas of logic (one jointly authored);
  two further jointly authored textbooks are planned, one on graph theory and one on mathematical writing.
- First degree in ancient history; knowledge of Latin, classical Greek and classical Arabic.

So I am not a historian of science, but it was always natural for me to include history in both classes and textbooks.

The best case *for* including history in teaching mathematics is a many-sided one:

it can add motivation, human interest,
rival perspectives, challenges to understanding,
appreciation of intellectual property, etc. etc.

This case is widely made, for example *Vita Mathematica: Historical Research and Integration with Teaching*, ed. Ronald Calinger, Mathematical Association of America 1996.



Also the inclusion of some *specifically historical* (and maybe optional) modules in a mathematics programme allows a wider range of learning methods:

the students can

- read original texts,
- present mini-lectures on them to other students,
- write essays

all of which add things commonly missing in mathematics teaching.

But henceforth I disregard this and concentrate on *integrating history with the regular teaching*.

There are also sound cases to be made *against* including history in mathematics classes.

One should expect this. Teaching is a practical matter, and in practical matters there are always pros and cons.

I report some of my own attempts to weave a path around the arguments *against* including history in a mathematics course.

Some of the points I will make are special to mathematics, others are more general.



**Argument 1**: The needed historical information isn't always available.

Some areas are well covered, e.g.

- Babylonian arithmetic,
- Greek geometry,
- beginnings of calculus,
- classical graph theory.

Fine if you are teaching some related mathematics. But for example I was unable to find good teaching material for someone who recently asked me about teaching history of 20th century logical foundations.

Some undergraduate mathematics is too recent.

For example optimization algorithms:

- linear programming (Dantzig 1946, in response to a request from the Pentagon).

- Dijkstra's algorithm on graph distances (1959).

How can the historians keep up?



A particular problem: getting the facts about early work of recently dead mathematicians.

One goes to people who knew them, e.g. their research students. *Disaster*: $X$'s research students knew $X$ when $X$ was supervising them, maybe late in $X$'s career.

I've met this problem several times.
One must always check the texts oneself.

Writing an encyclopedia article about Abraham Robinson (inventor of nonstandard analysis, died 1974), I consulted two students of his. They gave me wholly false information about Robinson's PhD thesis, based on his later interests.
Happily the thesis can be checked in London University Library.



**Argument 2**: Personalities are an irrelevant distraction.

Some algebraists believe that when teaching Galois theory one shouldn't mention that Galois died in a duel at the age of 20, because it has nothing to do with the mathematics.

P. M. Cohn put this argument to me around 1973.
But in his *Algebra* text of 1977 he relented.

In fact there are good cognitive reasons for including some personal information. It provides labels that help the student to remember the mathematics.

In 1985 I suggested that in logical games where the players represent universal ($\forall$) and existential ($\exists$) quantification they should be called Abelard and Eloise, after a famous medieval logician and his lover.
Reason: it matches pronouns 'he', 'she' to the two players.

A feminist mathematical logician at the University of Paris was upset when she heard another Paris logician use these names in a game where Eloise cheats.
She asked leave from me to present a paper to a feminist mathematics conference, on the dangers of personalising the mathematical content.

(I agreed, but sadly the conference was cancelled.)



When Ian Chiswell and I published *Mathematical Logic* (2007), we included photographs of many of the creators of the subject, with brief personal notes.
(We were allowed to use two previously unpublished family favourite photographs.)

A reader was puzzled that we described Emil Post as from 'Poland and USA' although he left Poland for the US when he was 7 years old.

We happened to know (from a footnote of Jan Woleński) that Post spoke proudly of his Polish origins.
But there are obvious dangers of lack of objectivity.

I once saw the Israeli logician Abraham Fraenkel listed as 'German'. He would have been mortified.



**Argument 3**: Are the exams going to contain historical material?

Students ask this question. It's part of a larger one:
if the syllabus is already heavy, adding history can make it heavier.

- In fairness to the students, they should be told what historical material is examinable
(but not necessarily what will be examined!).
- Some good historical material doesn't actually expand the syllabus, it colours it.

Example: Chiswell and I included some material on diophantine equations.
We needed to show that questions about solutions in the integers $\ldots, -2, -1, 0, 1, 2 \ldots$ are reducible to questions about solutions in the natural numbers $0, 1, 2, \ldots$.

The required device is exactly the procedure referred to by Al-Khwārizmī in his book *Kitāb al-jabr wal-muqābala* (8th century) as *al-jabr* ($>$ 'algebra').
So we called it that, with a brief historical note.

This added no new mathematical content,
apart from the observation that this device, which looks totally trivial today, was once a novelty and even today has special logical properties.



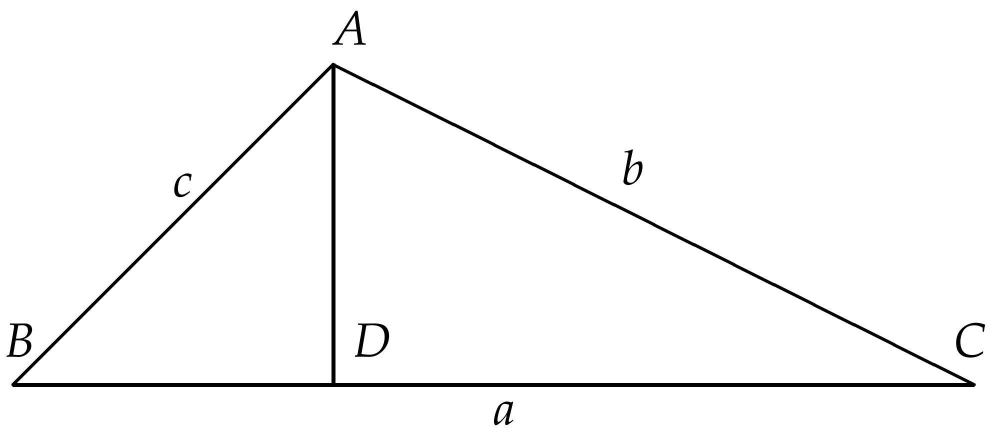
**Argument 4**: Mathematicians sometimes misunderstand the history.

This happens very easily when mathematicians read their own concerns back into earlier authors who didn't have the same concerns or the same concepts.

I hope I can't illustrate this from my own work. But there is a very nice example given by Glen van Brummelen in the current Bulletin (vol. 25 no. 1) of the *Bulletin of the British Society for the History of Mathematics*.

A mathematics teacher asked Van Brummelen who first gave the law of cosines, $b^2 = a^2 + c^2 - 2ac \cos B$.

17




Euclid *Elements* ii.13 proves that the square on $BA$ plus the square on $BC$ equals the square on $AC$ plus twice the rectangle $BC.BD$, which is equivalent to
$a^2 + c^2 = b^2 + 2a(c \cos B)$.

But Euclid couldn't even have stated this equivalence, because he didn't know the cos function.

18

From this and many similar examples, we see that exact credits for mathematical results are often impossible.

A clash between two principles:

(1) People should be given credit for what they discovered.

(2) One shouldn't ascribe to earlier scientists ideas that they couldn't have had.

**Argument 5**: Historians sometimes misunderstand the mathematics.

In my area of logic, historians are often part-philosophers, and philosophical prejudices can distort the history.

Example:

- Gottlob Frege made great advances through formalising a conceptual analysis of 'number'.
- Alan Turing in 1936 invented computers through a conceptual analysis of 'calculating'.

These examples have encouraged philosophically minded historians of mathematics to interpret other mathematical advances as conceptual analyses.



In fact very little worthwhile mathematics has come from conceptual analysis.
For a good reason: conceptual analysis is thinking about ideas we *already have*,
and most mathematical advances result from *new ideas*.

For example cohomology was a completely new way of looking at old questions. It could never have been discovered by conceptual analysis.

In logic the same goes for cut elimination, ultraproducts, stability, indiscernibles etc. etc.
We can say what these ideas are and when they appeared, but they might as well have been gifts of Rāmānujan's goddess.

Another example I met recently. Roman Empire logicians wanted to analyse Euclid's geometry using Aristotelian sentence forms 'Every $A$ is a $B$', 'Some $A$ is a $B$' etc.

One device they used was ordered pairs, as in

> *Every pair of lines $(x, y)$, where some line $z$ is parallel to both $x$ and $y$, is a pair of parallel lines.*

A highly respected historian describes this move as 'ridiculous' and says that certain classical arguments using it are logically invalid.
In fact the standard set-theoretic semantics for first-order logic uses *exactly* this device.
The historian just didn't know the relevant logic.



This example illustrates a gap in Ivor Grattan-Guinness's dichotomy 'History or Heritage? An important distinction in mathematics and for mathematics education', *American Math. Monthly* 111 (2004) 1–12.

According to G-G, *history* studies the development of an idea during a particular period, while *heritage* studies the effect of the idea upon later work.

23

For me, history in a maths class is nearly always used to support the mathematics.
Often it consists of earlier alternative approaches (like the Roman Empire approach to Euclid's reasoning).

It's irrelevant whether these earlier approaches had any influence (so not heritage).

But also it's essential to understand the *mathematical content* of the earlier approaches, and this may involve translation into modern terms (so not history).

24

Mention of Ivor Grattan-Guinness brings me to a student of both of us in London, Maria Panteki.
At the University of Thessaloniki she made many important contributions to the history of mathematics.
I learned much more from her than she did from me.
Two years ago she died of a brain tumour, long before her time.