Basing logic on semantics — some historical themes

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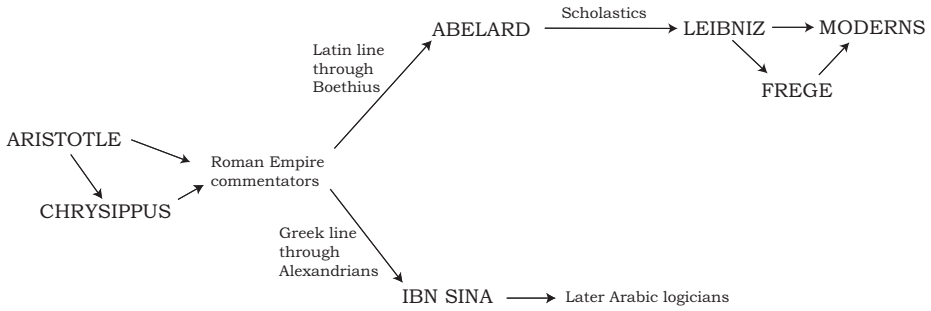
WESTERN LOGIC — THE BIG NAMES



The history of logic is never a smooth progression towards present-day logic.

Sometimes we find great depth and sophistication alongside extraordinary gaps in understanding.
This is particularly true of Ibn Sīnā.
It creates huge problems for the historical expositor.

I don't have many answers.
In these talks I'm mainly thinking aloud about some issues that particularly interest me.

1. Basing logic on language

From Aristotle onwards, the primitive notion of logic was
'ϕ *follows from* ψ and χ' where ϕ, ψ and χ are
(the meanings of) meaningful sentences.

Some authors added a second: '*A is definable in terms of B, C, . . .*' where *A*, *B*, *C*, . . . are concepts.
Thus Ibn Sīnā, Pascal, 1930s Tarski. We'll ignore.

Aristotle (followed faithfully by Ibn Sīnā):

> No science can define its own primitive notions.

So logicians can't define *follows from*. In fact they take it as raw datum from experience of reasoning.

The experience is of the act of inferring ϕ from ψ and χ.

Various hopeless 'definitions' of this act, e.g. we think ψ and χ, and find we can't help thinking ϕ too.
One should ignore. These definitions are never used and their purpose is only to remind us what inferring is.

The key point, emphasised by Ibn Sīnā and Frege, is that inferring has a purpose, and there is an objective criterion for whether the purpose has been achieved.
Viz. inferring gives us decisive and checkable evidence that 'If ψ and χ then ϕ' is true.

Now observing Aristotle's syllogistic moods, for example

> *Every A is a B. Every B is a C.*
> *Therefore every A is a C.*

(or Chrysippus' propositional inferences), people noticed that often the *syntactic form* of the sentences ϕ, ψ, χ is what guarantees the inference.

These inference-guaranteeing syntactic forms are the ancestors of today's *inference rules*.
(And note that before the mid 19th century, logic was basically about single inference steps, not about arguments.)

Basing inference rules on syntax is not at all Aristotle's approach.
In fact Aristotle shows almost no interest in language.
(Contrast his abiding interest in geometry.)

The evidence is incomplete, but it seems that the move to syntactic patterns took place in the early Roman empire and had three main causes, as follows.

Cause One. The writings of Aristotle first became public in a single edition which lies behind all modern editions. It is reported to have been the work of Andronicus of Rhodes in the 1st century BC.

Andronicus started the edition with *Categories*, which could be read as being about single words.

Then he put *On interpretation*, which could at a stretch be read as being about combining words into sentences.

Then came the *Prior analytics*, which is certainly about combining sentences into inferences.

This helped to create an impression that Aristotle saw syntax as a halfway step to constructing inferences.

Cause Two. In mid to late 2nd century AD, Apollonius the Foul-Tempered (Dyscolus) introduced the study of syntax with his book *Perì Syntáxeōs*.
Henceforth some knowledge of syntax was part of basic culture.

Several leading logicians have been actively interested in syntax.

They include Ibn Sīnā, who was bilingual Arabic-Persian.
He published a text on phonetics,
and his logical writings contain many syntactic remarks.

Leibniz was deeply interested in language
(though perhaps mostly etymology).
He claimed that many steps in reasoning are in fact purely grammatical.

Frege, though an avowed enemy of natural language, invented his logical language *Begriffsschrift* on the basis of the syntactic dependency diagrams current in German language teaching.
(More on this below.)

Cause Three. All historical lines in western logic seem to pass through the Palestinian platonist philosopher Porphyry of Tyre, late 3rd century AD.
Little of his work survives, except as borrowed by Boethius or developed by the Alexandrians (Ammonius, Simplicius, Philoponus, c. AD 600).

A mediocre logician but a brilliant academic politician.
He established a way of presenting Aristotle's logic within a basically platonist curriculum, by basing it on philosophically neutral facts of natural language.

Kant was apparently unaware of the Scholastic advances on Porphyry, or of the difference between Porphyry's logic and Aristotle's. He declared

> We have no one who has exceeded Aristotle or enlarged his logic
> (which is in itself fundamentally impossible)
> just as no mathematician has exceeded Euclid.
> (*Dohna-Wundlacken Logic*)

A testament to Porphyry's powers of spin.

2. Syntax versus semantics

When logicians express an interest in language,
there is a risk of demarcation disputes.
We will track this mainly with the Arabs.

Arabic linguistics began in the 8th century.
By Ibn Sīnā's time (early 11th century) it was
a strong discipline with a sophisticated metatheory.

E.g. Sibawayh 9th century, genius of the Basra school;
Ibn al-Sarrāj 10th century, friend of the logician Al-Fārābī;
Ibn Jinnī c. 1000, possible influence on Ibn Sīnā.

Arabic linguistics was a religious discipline,
aimed at protecting the text of the Qur'ān
(cf. Homer for the early Greek linguists,
the Vedas for the Indian).

In practice it was also nationalistic,
aimed at showing the innate superiority of Arabic.

By contrast the logicians claimed to study
things that are uniform across all languages,
because they depend on the meanings behind the syntax.
Several early Arabic logicians were in fact Aramaic.

A reported debate around AD 940,
between the linguist al-Sīrāfī
and the Aramaic logician Mattār bin Yūnus.

> Al-S: You can only reason in a specific language.
> There is no universal language;
> you can't create one inside a given language.

(WH: Compare the Kreisel dirty dishwater problem.)

> So logic in Arabic has to start with linguistic knowledge of Arabic. For example what does *wa-* mean in Arabic?
> MbY: Um, er, er, ...

Footnote: Strangely it went the other way round in medieval Europe.

The linguists, particularly the Modists (e.g. Martin of Dacia, Radolfus Brito) were so keen to find universals across all languages that they made little progress on any one language.

By contrast the logicians assumed all languages are Latin. E.g. Walter Burley (early 14th c.) reports Aristotle's views on some Latin words. He also states that negating words go before what they negate, apparently not realising that in his own language of Middle English you negate a verb by putting 'not' *after* it. ('Waste not want not.')

Ibn Sīnā rides above the dispute.
Languages have some things in common and differ in others.

If languages *L* and *M* use different devices for expressing the same meaning,
this shows that the devices belong in syntax and are suitably studied in linguistics.
For example *there is no natural linear order*
for the parts of a sentence.

But some things,
e.g. the distinction between verb and pronoun,
seem to be universal across languages and can be defined in terms of meaning.
These things are suitably studied by logicians.

Interesting example

Ibn Sīnā remarks that there could be languages which don't have words to mark the relations between subclauses ('if', 'then', 'because' etc.).

In fact the North-West Caucasian languages
(Abkhaz and others) have this property,
because each sentence has just one verb,
so there are no subordinate clauses.
Did Ibn Sīnā know this? He lived quite close to them.

3. The compositional assumption

There seem to have been a standard assumption among aristotelian logicians who thought about semantics (Al-Fārābī, Ibn Sīnā, Abelard, Frege for example).
Probably it goes back to Porphyry.
Frege (1923) articulates it as follows:

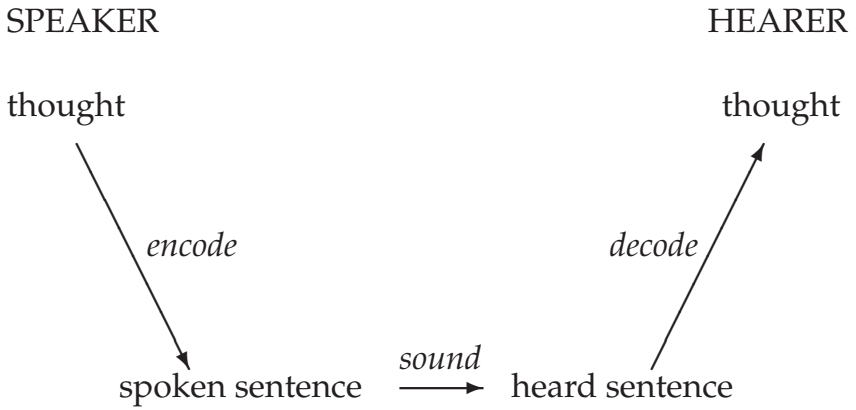
It's astonishing what language does for us,
expressing unimaginably many thoughts with a few syllables, so that even when some thought occurs for the first time to some earthling, it provides a clothing (*Einkleidung*) in which he can communicate it to someone else for whom it is new. This wouldn't be possible if we couldn't separate parts in the thought which correspond to parts of the sentence, so that the construction of the sentence counts as a representation of the construction of the thought. (*Gedankengefüge*)

It's ironic that this is called 'Fregean compositionality' when it's perhaps the least original thing Frege ever said.

In any case he fudges the central point of modern 'Fregean compositionality',
which is that the correspondence holds at the level of the separate constructions of parts of the sentence.

Following Jackendoff we will write for example [HORSE] for the thought that corresponds to the word 'horse'.

The standard assumption involves an encoding (thought $\longrightarrow$ sentence) and a corresponding decoding (sentence $\longrightarrow$ thought):



We will compare how the picture above appears in Ibn Sīnā, Frege and Tarski.

There is almost certainly a historical development from Frege to Tarski,
but somebody should trace the details
(maybe through Leśniewski).

Ibn Sīnā and Frege are parallel takes on the aristotelian tradition.

(i) Ibn Sīnā

For Ibn Sīnā the encoding function is a natural phenomenon, largely open to inspection.
It differs from language to language, and from user to user, in three main ways.

(a) Languages can chunk together several thoughts in a single expression.
Individuals can do the same by introducing new words.

(b) Thoughts are not linearly ordered but sentences are.
Languages differ in how they impose an order.
For example in Arabic the verb comes at the beginning, in Persian it comes at the end.

(c) We nearly always leave parts of a thought unspoken .
For example we express a thought about an individual person, using only the pronoun 'he' for the person.

(c) is very strongly characteristic of Ibn Sīnā's view.
But it is also very much in line with Arabic linguistics.
For example Arabic *muḍmar*, 'pronoun', literally means 'hidden'.

Note that (c) handles pragmatics and indexicality under the head of 'meaning'.
This seems to have been universal practice before the mid 20th century.

(ii) Frege

Frege also regards the translation from meanings to natural language sentences as a given.
But for him it is too unreliable for scientific purposes.
So he introduces a new language *Begriffsschrift* to represent the construction of thoughts faithfully.

Begriffsschrift sentences are not linearly ordered.
Also Begriffsschrift makes provision for chunking, via definitions.

Like Ibn Sīnā, Frege has no separate discussion of the decoding function.

(iii) Tarski

Model-theoretic versions of the Tarski truth definition often incorporate a decoding function that assigns to each expression a denotation, by recursion on complexity.

Tarski in his big 1933 paper on truth didn't do this.
Instead he gave a formula, depending on the language, which is true of all and only the true sentences of the language.
But a paper of his in 1930, clearly a preparatory paper for the truth paper, does explicitly discuss a function assigning denotations (of a sort) to expressions by induction on complexity.

Nevertheless Tarski's truth definition, in any version,
never has anything like an encoding function.

The breakthrough that allows Tarski to drop the lefthand
side of the picture is the *autonomy of syntax*.
More precisely, Tarski uses a language whose syntax is
completely defined without any reference to meaning.

The development of formal languages in the Hilbert
school will have helped Tarski here.
But he was encouraged in this direction by his teacher
Leśniewski's fanatical devotion to syntactic definitions.

4. The wrapper fallacy

We need to take note of a very unfortunate metaphor.
Both Ibn Sīnā and Frege criticise it, in similar terms,
but both use it nevertheless.

The aristotelian tradition tends to emphasise that
properties of an object come and go in time.
At 7 in the morning I'm in bed; at 9 I'm not in bed.
So a statement expressing 'X has property Y' has an
implied time reference.

Ibn Sīnā was apparently the first logician to take the time
reference as seriously as the other parts of the sentence.
For example he describes quantification over it;
this will be important for us later.

Now in syntax, and more generally in mathematics,
an element of a structure can have properties in terms of
where it occurs in the structure.

For example in the sentence 'Fish don't fish',
the first occurrence of 'fish' is a noun
and the second isn't.

In a common metaphor, we say that 'fish' is a verb *when* it
follows 'don't'.
The metaphor suggests misleadingly that 'fish' follows
'don't' at 7 am and not at 9 am (say).
But so far, nobody is going to be misled.

The next step in metaphor does mislead.

Given an occurrence of a in context c,
we speak of the context c as
something that has happened to a.

For example in the context '6 + 1', 6 has had 1 added to it,
so it has become 7.
Ibn Sīnā points out the absurdity of this conclusion.

Here is Frege, talking about adding negation at the beginning of a sentence. Negation, he says, needs to be completed by having a sentence attached.

> *I compare a thing that needs to be completed to a wrapping that — like a dress — can't stand up on its own but needs to be wrapped around something. . . . Of course we mustn't forget here that wrapping up and putting together are things that happen in time, but what corresponds to them in the realm of thoughts is timeless. (Verneinung)*

Time is not the issue. The issue is that a sentence with a negation at the beginning is not like a person with a dress on. Adding negation gives a *different sentence*.

Armed with these thoughts, we tackle
a strange doctrine of Ibn Sīnā.
He claims that when a compound thought is built up, later additions to the compound can alter component thoughts that were already there.

He calls this *taḥrīf*, which means twisting, particularly of meanings.
(*taḥrīf* is a grave sin if it involves twisting the meanings of phrases in the Bible or the Qur'ān.
Try googling 'tahrif' for some examples.)

The mind boggles.
Is this some theory about noncompositionality?

Some of Ibn Sīnā's examples show that his point is much more mundane.

For example Ibn Sīnā claims that by adding the word 'If' at the beginning of 'The sun is up', I change the meaning of 'The sun is up' to one that is neither true nor false.

Of course this is wrong. What is neither true nor false is 'If the sun is up', not 'The sun is up'.
No meaning has changed.

Recalling Frege's remarks, let's say that Ibn Sīnā has committed the *wrapper fallacy*.

5. Dependency grammars

Both Ibn Sīnā and Frege make some assumptions about the way in which meanings are built up in a sentence. These assumptions put them within the broad framework known today as *Dependency grammar*.

We take a moment to explain this framework.

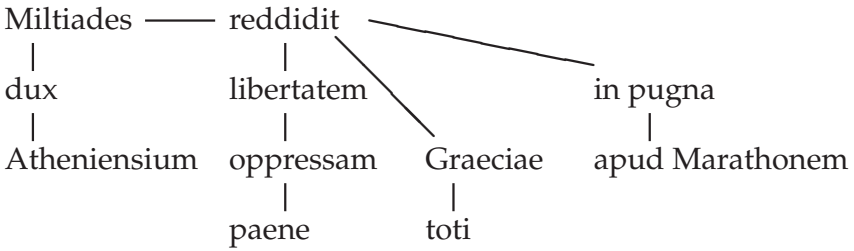
Dependency grammar is a formalism for describing one way in which compound structures are built from atoms.
The basic notion is that of a *dependency*,
which is an ordered pair of distinct atoms, perhaps together with a label.
The first atom is called the *head* of the dependency and the second is called the *dependent*.

Ibn Sīnā speaks of the dependent being 'attached' or 'added' to the head,
and the Modists call the head 'terminans'.
So we will represent a dependency by an arrow from the dependent to the head, e.g. $(a \leftarrow b)$.

By a *tree* we will mean a finite connected acyclic directed graph with a distinguished element called the *root*,
where all directed edges point towards the root.
(Because the graph is connected without cycles,
this is unambiguous.)

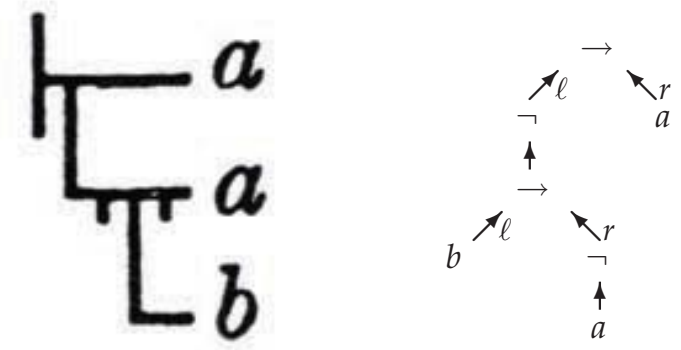
Given a set A of atoms and a set D of dependencies between these atoms,
by an (A, D)-*tree* we mean a tree where each node is labelled by an atom from A,
and for each edge which goes to a node labelled a from a node labelled b (and maybe is labelled i),
the dependency $(a \leftarrow b)_i$ is in D.
We refer to labelled trees of this kind as *dependency trees*.

The inspiration for dependency grammar comes from sentence analyses, for example (from 1830s Germany)



In sentences the order of the words carries information.
The indices on the dependencies partly cover this,
e.g. (reddidit←libertatem)$_{obj}$, (reddidit←Graeciae)$_{indobj}$.

Example. Frege's Begriffsschrift sentence on the left is only notationally different from the dependency tree on the right.



Besides Frege,

- The West European Modist grammarians (late 13th to early 14th century) used a dependency framework.
- Jonathan Owens (1988) argued that the syntactic theory of the classical period of Arabic linguistics was a dependency theory. (Not everybody is convinced, but I'll assume he is right.)
- As noted above, Ibn Sīnā used a dependency grammar.

In all cases there are some violations of dependency grammar. For example the Modist analysis of 'Socrates and Plato are running' contains a cycle.

Dependency grammars are not obviously the best grammars.
In fact hardly anybody uses them today.
Their appearance in both the Modists and Ibn Sīnā suggests the idea came from Roman Empire linguists or philosophers.

But recent research shows that Arabic linguistics is largely home-grown and not based on Western models (which doesn't exclude some informal influence from the West).

So we don't know where Ibn Sīnā got his trees from.
Some of his vocabulary (*taᶜalluq*, *ziyāda*) comes more easily from Arabic linguists of his own time than from the logical tradition.

6. What are dependency grammars about?

The fact that both Ibn Sīnā and the classical Arabic linguists use dependency grammars is no guarantee that they recognise the same dependencies, let alone that they both understand dependency in the same way.
Thus in a phrase consisting of preposition plus noun,

to Zayd

the linguists make 'to' the head and 'Zayd' the dependent. Ibn Sīnā seems to take it the other way round.

Covington p. 49: the dependencies in Modist theory 'correspond to no grammatical relation recognized by modern theory'.

So we need some informed guesswork. Basically there were two interpretations.

(i) Control of inflection. In the Arabic tradition, *b* counts as dependent on *a* if *a* determines the case marking (in a generalised sense) of *b*.
In the Latin tradition, a related notion of dependency (called *regimen*) appears in the Modist Simon of Dacia and has been traced back to the 8th century.

This concept of dependency makes sense only for certain languages.
So by Ibn Sīnā's criterion it is syntactic and irrelevant to logic.

(ii) Valency. In Frege's Begriffsschrift, *b* is dependent on *a* if and only if the meaning (*Bedeutung*) of *a* is a function and *b* fills one of its argument places.

This notion seems to trace back to the Stoics through Porphyry. For example, as Porphyry explains, the Stoics call an expression a 'predicate' if when a name is added to it, the result is an assertion.

There is no direct evidence that either the Stoics or Porphyry developed this notion to produce dependency trees. But they might have done it in texts now lost.

In subject-predicate sentences, Ibn Sīnā takes the predicate as attached to the subject.
He seems to be taking the predicate as dependent, contrary to Porphyry and Frege.
So his thinking seems to be neither (i) nor (ii).

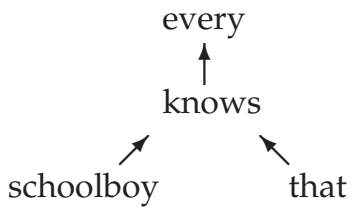
Some 20th century theories help to make sense of all this.

First, *Theta theory* discusses how certain words in a sentence ascribe roles (agent, location etc.) to other words in the sentence.
In some languages the role is marked by a case inflection.
This theory helps to reconcile (i) and (ii).

Second, users of a language generally agree fairly well about how to break down a sentence into smaller units ('constituents', Bloomfield 1933).
In a syntactic dependency tree, a segment of the sentence consisting of a node and all the nodes below it should be a constituent.
This is a constraint on possible dependency grammars.

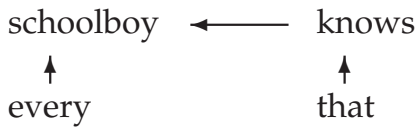
Frege's Begriffsschrift dependency trees violate this constraint,
so far as we can translate between them and natural language sentences.

For example the Begriffsschrift tree (slightly simplified to eliminate a variable) for 'Every schoolboy knows that':



Contrary to the tree, 'schoolboy knows that' is not a constituent of the sentence.

Ibn Sīnā's analysis:



Ibn Sīnā attaches 'every' to 'schoolboy',
meeting the constraint.
Below we will see why 'every' needs to be
near the top of the tree.
Together these facts may justify Ibn Sīnā's choices.

Ibn Sīnā cites the phrases

herder of sheep, thrower of stones

His general theory contains two approaches which would both yield the dependencies

herder ← sheep, thrower ← stones

But sadly he doesn't say enough to indicate which way he jumps.

The first approach is standard today:
[THROWER] has an argument place for the thing thrown,
and [STONES] fills this place.

Granted, [THROWER] can occur on its own.
Ibn Sīnā would explain this by saying that there is
a tacit existential quantification over the argument place
(a device he uses in several places).

The second approach is that [STONES] tightens up the specification of [THROWER]:

thrower who specialises in stones

Then since [STONES] ascribes a property, it should behave like predicates, and hence attach to what it ascribes a property to.
I don't think this works, but similar things are common in the aristotelian tradition.

7. Subject-predicate sentences

As we saw, Ibn Sīnā counts the predicate as dependent on the subject.
An expression representing the arrow from predicate to subject is called a *copula*.
In English it's often 'is' or 'isn't'.
In Arabic it can be a pronoun or absent altogether.

Expressions should represent thoughts, not dependencies.
It seems the reason Ibn Sīnā wants to have a word representing an arrow is that he wants to attach other items to the *arrow* rather than to nodes.
The key examples are modalities.

Suppose we want to say, about Obama, that it's possible he won't have a second term.
Without the 'possible' we have

Now where do we attach 'possibly'?
Not to 'going-to-have-a-second-term', because this will give the wrong sense.
Not to 'Obama', because it's the possibility of not having a second term, not the possibility of Obama.
There remains only the copula.

The placing of modalities is listed by Matthews *Syntax* p. 90 as one of the flaws of dependency grammars.
He cites with approval Henry Sweet's (1891) view that the modality should be attached to the sentence as a whole.

My present impression is that the predicate should be taken as the compound

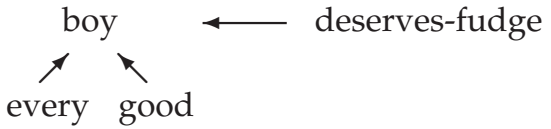
(not ⟵ going-to-have-a-second-term)

and then the modality should be attached to the head of the predicate, which is the word 'not'.

The error was Aristotle's, taking the negation as part of the copula rather than a node in its own right.
This error was corrected by Jevons and Frege.

8. Quantifiers

In a sentence with a single quantifier,
Ibn Sīnā attaches the quantifier to the subject noun.



'every' and 'good' are not attached in any order.
But their semantics are quite different.

Ibn Sīnā accepts the semantics of 'every' described by the 6th century Alexandrian Ammonius. Viz.

> 'Every X is a Y' means that for every individual a satisfying X, a also satisfies Y.
> (*Commentary on 'On interpretation'* 89.10.)

Remark: This by itself tells us nothing about the parsing of the sentence. For example if we read the condition as $\Phi(\text{every}, X, Y)$ and we take Y as the head, we can give Y argument places for 'every' and 'X', and semantics

$$\lambda ij\ \Phi(i, j, Y).$$

Likewise Ibn Sīnā is entitled to apply it as he does, with 'X' as the head.

One of Ibn Sīnā's more radical innovations

Since properties come and go, a statement assigning a property to an individual has an implicit time marker which may be quantified, universally as in

> *All horses are mammals*

(where the quantification is over the lifetime of each horse), or existentially as in

> *All horses sleep*

(i.e. each horse sleeps sometimes).

Ibn Sīnā may (I think he does) believe that these time quantifiers generalise to any implied variables.

His analysis (though not his example):

Question: The same fudge for each boy, or not?
How is this distinction reflected in the semantic analysis?

Note that for Ibn Sīnā *the order of the components is irrelevant to meaning*.
So the distinction has to be represented some other way.

Historical aside

Today we would describe the distinction in terms of whether the *scope* of 'every' includes 'a', or vice versa.
The notion 'scope of a quantifier' is nowhere in Ibn Sīnā.

When did it first appear? It was introduced by Russell in 1908 in 'Mathematical Logic as based on the theory of types', in the form 'scope of a real variable'.
The switch to 'scope of a quantifier' seems to be due to Quine in his *Mathematical Logic* 1940.

Walter Burley (early 14th c.) had an inchoate notion of scope ('dominium'), viz. everything to the right in the sentence. But this would clearly be useless for Ibn Sīnā.

The question of characterising the scope of a node in a tree structure was tackled by Hodges in *Logic* 1976, and by Chomsky's student Tanya Reinhart in her PhD thesis of the same year.

Both gave the same answer: the scope of node n is everything including or below the node m immediately above n. (Reinhart said 'command' rather than 'scope'. Also she excluded n and m from n's command, but this is less significant than it might seem.)

We can apply this notion to Ibn Sīnā's dependency tree above, and it gives immediately that the scope of 'every' includes 'a' and not vice versa.
But Ibn Sīnā had read neither Hodges nor Reinhart.

In fact Ibn Sīnā approaches the question by skolemising the existential quantifier.

By Ammonius (commentary on On Interpretation 89.13ff), 'Some X is an Y' means that for some individual a satisfying X, a satisfies Y.

Ibn Sīnā generalises this: 'For every X and some Z, Y holds' means that there is a function F taking Xs to Zs, such that for every individual a satisfying X, $F(a)$ satisfies Z and Y is true of the pair $(a, F(a))$.

He mentions two special cases that simplify: (i) $F(a)$ is the same for all a. (ii) F is definable so it can be eliminated.

Remark: In the skolemised form, the existential quantifier becomes an existential function quantifier.
The relative order of a universal object quantifier and an existential function quantifier is irrelevant to the truth conditions.
Cf. Henkin on partially ordered quantifiers.

So the Skolem interpretation is highly suitable if we have no relative order of the quantifiers.
One up to Ibn Sīnā.

9. Quantifiers and negation

But when he tries to negate the sentence,
Ibn Sīnā runs into problems.
The passage is 39.15ff in the first chapter of his *Qiyās* (his major commentary on Aristotle's *Prior Analytics*).
It begins

> We land ourselves in the following difficulties.

Indeed the passage is very difficult. Here I give my present view of it.

Chris Martin argues in detail that
Boethius had no notion of propositional logic,
and hence no notion of negating a proposition.

It follows almost certainly that the notion of negating
a proposition is not in Porphyry,
and hence that it never reached Ibn Sīnā through the
tradition coming from Porphyry.

However, Arabic has a verb *laysa* which, added at the
beginning of a sentence, negates the sentence.
That is (at least in Ibn Sīnā's explicit usage), unless the
sentence begins with an existential quantifier.

If I apply remarks of Jamal Ouhalla correctly,
the exception is because a noun phrase at the beginning of
an Arabic sentence is in topic position,
and hence can't be indefinite. So

laysa + 'some X . . . '

can't be read as the negation of 'some X . . . ',
and has to be read as

For some X, not . . .

Although Ibn Sīnā can negate most propositions by
adding *laysa* at the front, he never leaves it there.
Instead he always tries to move the negation inwards by
De-Morgan-type rules.
He seems to regard the negation-normal form as the 'real'
negation.

Because of the quirk with *laysa*,

laysa Every good boy receives a fudge.

goes by De Morgan to

laysa Some good boy receives a fudge.
I.e. Some good boy doesn't receive a fudge.

Not clear how he parses this; but anyway he now has to
reconcile the two quantifiers.

Here Ibn Sīnā hits his problem.
The 'a' quantifier is existential, so it needs skolemising.
But since we no longer have 'every boy',
what is the point of introducing a function over all boys?

For us today this is a non-problem.
An existential quantifier within the scope of a negation
has the force of a universal one, so we can't skolemise it.
But as with quantifiers, *Ibn Sīnā has no notion of scope of
negations*.

In fact Ibn Sīnā abandons general theory at this point
and falls back on intuition.

References

The key texts are by Ibn Sīnā, but they are available only in Arabic apart from a few translations on my website at `http://wilfridhodges.co.uk`. Besides these the following are relevant.

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