Nonproductivity proofs from Alexander to Abū al-Barakāt: 1. Aristotelian and logical background

Wilfrid Hodges
wilfrid.hodges@btinternet.com

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1 Introduction

Aristotle in his *Prior Analytics* classifies premise-pairs as those for which 'there will be a syllogism' and those for which 'there will be no syllogism'. Following the later Arabic terminology I will distinguish these cases as 'productive' (*muntij*) and 'nonproductive'. There are other names in the literature; for example Malink [15] calls them 'concludent' and 'inconcludent'.

Aristotle describes methods for proving that a given premise-pair is nonproductive. His successors in the tradition of Aristotelian logic developed these methods; key figures in the development are Alexander of Aphrodisias ([1], around AD 200), Paul the Persian ([9] 6th century) and Abū al-Barakāt ([8], 12th century). It's hard to find any discussion of these developments in the modern literature. In the case of Abū al-Barakāt the reason is that people haven't read his logic. But there is a further problem, namely that all these later authors, and others besides them, rely on some pieces of strange terminology in Aristotle's own exposition.

These nonstandard usages in Aristotle's text are generally ignored by modern commentators, or taken to be an innocent shorthand. But many of the Aristotelian logicians treated them as definitive terminology. These logicians struggled to build a consistent language incorporating these usages, or at least to neutralise the inconsistency between the standard and the nonstandard terminology. In some cases they used the nonstandard terminology as a cue for building up a new perspective on the structure of logic.

To make sense of these developments we need to study exactly who said what, from Aristotle himself onwards. We also need to understand the underlying logical content, and we need a vocabulary of our own for reporting and discussing what the ancient and medieval authors said. This is an uncomfortably long haul involving some serious immersion both into texts and into logic. The present paper introduces the relevant items that Aristotle's successors found in his text, and proposes some terminology for the logical content. The papers [12] and [13] describe the terminologies developed by later authors, and the theories built up around these terminologies.

Except where otherwise stated, references to Aristotle's text are to the Ross edition of *Prior Analytics* [5]. Translations are my own except as stated; quoted translations are sometimes adjusted, chiefly for reasons of uniformity.

I thank Stephen Read for helpful comments, which included raising some of the main questions answered in the paper.

2 Aristotle's strange statements

The first of Aristotle's notably strange statements is at *Prior Analytics* i.14, 33b3–6:

> *éti dè kaì ek tō̂n hórōn phanerón; hoútō gàr ekhousō̂n tō̂n protáseōn tò prō̂ton tō̂*$_i$ *eskhátō*$_i$ *kaì oudenì endékhetai kaì pantì hupárkhein anagkaîon.*
>
> (1) Furthermore, this is also evident from terms, for when the premisses are related in this way, then it is not possible for the first term to belong to any of the last, and it is also necessary for the first to belong to all of the last. (i.14, 33b3–6, tr. [6] p. 21)

This could be read as saying that a certain premise-pair entails two incompatible conclusions; which is odd, since Aristotle has just told us (33a8) that the premise-pair in question doesn't entail any conclusion at all. Striker comments:

> (2) Aristotle avoids the awkwardness of repeated modal expressions by saying simply that it is both not possible for the first term to belong to any of the last and also necessary for it to belong to all of the last, but it is clear that he means that *it is possible* for the first term either to hold or not to hold of all of the last of necessity. ([6] p. 140)

A very similar strange statement occurs at 39b2–4; Patzig [16] p. 20 describes it as 'highly misleading, or rather, downright false'.

Another strange statement comes a couple of pages after (1):

> *éti éstō tò mèn prō̂ton zō̂ion, tò dè méson kinoúmenon, tò d'éskhaton ánthrōpos. hai mèn oûn protáseis homoíōs ékhousi, tò dè sumpérasma anagkaîon, ouk endekhómenon; ex anágkēs gàr ho ánthrōpos zō̂ion.*
>
> (3) Further, let the first term be animal, the middle term moving, the last man: then the premisses will be related as before, but the conclusion will be necessary, not possible, for man is an animal of necessity. (i.15, 34b14–17, trans. [6] p. 23)

The word translated as 'conclusion' is *sumpérasma*, Aristotle's usual word for the conclusion of syllogisms. Striker comments:

> (4) In fact there will be no conclusion, as the counterexamples show; but Aristotle somewhat misleadingly applies the word to the proposition that expresses the actual relation between the major and minor terms in his example. ([6] p. 148)

Albrecht Becker [11] p. 57 thought that this passage of Aristotle's text is a later interpolation. But this question is irrelevant for us, because the sentence is well-attested in the manuscript tradition, and Aristotle's readers from Alexander to Abū al-Barakāt believed it was genuine.

After another few pages, another strangeness appears. Aristotle has said that if a premise-pair in second figure has two contingent premises, then any conclusion from it would have to be (or imply) either a possible affirmative sentence or a possible negative. He continues:

> *oudetérôs d' egkhōreî. kataphatikoû mèn gàr tethéntos deikhthḗsetai dià tō̂n hórōn hóti ouk endékhetai hupárkhein, sterētikoû dé, hóti tò sumpérasma ouk endekhómenon all' anagkaîon estin.*
>
> (5) But it cannot be either of these. For (considering the affirmative) it will be demonstrated through terms that it is not possibly the case, and (considering the privative) that the conclusion is not possible but necessary. (i.17, 37a40–3)

Here 'conclusion' is again *sumpérasma*. Again Aristotle seems to be telling us that a premise-pair which he claims doesn't entail any conclusion actually demonstrates a conclusion that is a necessary truth.

Theodorus in his Arabic translation of *Prior Analytics* i.1–22 has twenty-two occurrences of what became the standard Arabic word to express 'entails', namely *yuntiju* (or its feminine form *tuntiju* or perfect tense *antaja*).

Eighteen of these occurrences are in line with these strange passages of Aristotle, not with how one usually talks about logical entailment.

We will see in [12] and [13] that Theodorus is by no means atypical. For example Paul the Persian systematically speaks of conclusions in the sense of (3) and (5) above as 'conclusions', even in an elementary textbook. He distinguishes them from the conclusions of logical entailments by calling the logical conclusions 'necessary conclusion' and the strange conclusions 'non-necessary conclusion'.

Another example is Abū al-Barakāt. He is discussing the premise-pair for *Ferio*:

(6) Some A is a B; and no B is a C.

He gives four sets of interpretations for the three term letters A, B, C, and all four sets make both premises in (6) true. His name for a set of interpretations for the terms (or more precisely a diagram giving these interpretations) is 'picture' (*ṣūra*); so he has given four 'pictures'. He comments:

(7) Given the affirmative particular minor premise and the negative universal major premise, C is denied of some A as a constant set-theoretic fact, regardless of this difference between the four pictures, which is that it entails (*antaja*) a negative universal [proposition] in some of them and a negative particular [proposition] in others. ([8] p. 131)

Here he tells us that *Ferio* has different conclusions in different interpretations. But we know that this productive formal premise-pair has one conclusion that applies in all interpretations, and that this conclusion is not a universal sentence. So his statement is confusing. Ironically this is an example of Abū al-Barakāt using the strange idiom on a productive premise-pair. That fact will make better sense when we see in the companion paper [13] how Abū al-Barakāt recast the distinction between productive and nonproductive premise-pairs.

In the papers [12] and [13] we will review how the commentators and translators after Aristotle built up idioms based on Aristotle's strange passages. Since these idioms had to be reconciled with what Aristotle says elsewhere in *Prior Analytics* i.1–22, we will also need to review his usual terminology for talking about entailments.

3 Logical definitions 1

1. By a 'material sentence' we mean a meaningful sentence of a natural language. The language can be fixed by the context of our discussion; for example when discussing Aristotle or Alexander of Aphrodisias we understand the language to be classical Greek, and when discussing Abū al-Barakāt we take it to be Arabic.

2. By a 'formal sentence' we mean a string of natural language words together with one or more letters, such that the string can be turned into a material sentence by replacing the letters by natural language phrases, where a letter that occurs twice or more is replaced by the same natural language phrase at all occurrences. The letters that play this role in a formal sentence are called the 'term letters' of the formal sentence.

> For example the four main formal sentences of Aristotle's categorical logic, as they appeared to the Arabic logicians, are in English translation

(8)	$(a)(B, A)$	Every B is an A.
	$(e)(B, A)$	No B is an A.
	$(i)(B, A)$	Some B is an A.
	$(o)(B, A)$	Not every B is an A. (Or: Some B is not an A.)

> The letters A and B are respectively the 'predicate' and 'subject' of these formal sentences. The letters A, B can be replaced by any other two distinct letters. Aristotle himself usually gives a technical description of the forms rather than quoting them; thus for $(a)(B, A)$ he says 'A is predicated of all B'. (The notation on the left is a shorthand for our convenience below, using the Scholastic shorthands a, e, i, o.)

3. By an 'interpretation' we mean an assignment of natural language phrases to letters. An 'interpretation for' a formal sentence ϕ is an interpretation I that assigns a natural language phrase to each term letter of ϕ, making ϕ into a material sentence when each term letter of ϕ is replaced by the phrase assigned to it by I. This sentence is called the 'I-instance of ϕ' and written $\phi[I]$. A 'material instance' of ϕ, or more briefly an 'instance' of ϕ, is an I-instance of ϕ where I is some interpretation for ϕ. (We sweep under the carpet that the replacement of letters by phrases may incur some tidying up, for example 'a animal' is corrected to 'an animal'.)

> When Aristotle says 'Let A be animal, B human and C white' (30b33f) he is naming an interpretation. Often he leaves it to the reader to supply the letters, as when he writes 'Terms in common to all cases: animal, white, horse; animal, white, stone' (26b24f), giving two interpretations. At 48a25 he describes the giving of an interpretation as *tȭn hórōn ékthesis*; this may be the source of the name *paráthesis* which Alexander often uses for the act of giving an interpretation.

4. An interpretation I of a formal sentence ϕ is a 'model' of ϕ if $\phi[I]$ is a true sentence. We say that I is a 'model' of a set Φ of formal sentences if I is a model of every sentence in Φ.

5. An 'entailment relation' is a relation $\vdash$ between sets Φ of formal sentences and formal sentences θ. When $\vdash$ holds between Φ and θ, we write $\Phi \vdash \theta$ and we say that 'Φ $\vdash$-entails θ'. In this case we say also that a '$\vdash$-entailment' holds between Φ and θ, and that the 'premises' of the $\vdash$-entailment are the formal sentences in Φ, and the 'conclusion' of the $\vdash$-entailment is the formal sentence θ. (Such a relation wouldn't normally be called an entailment relation unless it had some features that link it to standard logical entailment. But here it will be convenient to leave these features unspecified until we need them.)

> The primary notion of entailment in all of Aristotle's logic is the syllogism, defined as 'an argument in which, certain things being posited, something other than what was laid down results by necessity because these things are so' (24b18–20, tr. [6] p. 2).

6. A *logic* $\mathbb{L}$ consists of a collection $\mathbb{S}(\mathbb{L})$ of formal sentences, a collection $\mathbb{I}(\mathbb{L})$ of interpretations and an entailment relation $\vdash(\mathbb{L})$ that relates formal sentences of the logic. We will assume that if ϕ is a formal sentence of $\mathbb{L}$ and I is an interpretation in $\mathbb{I}(\mathbb{L})$ that assigns phrases to all the term letters of ϕ, then I is an interpretation for ϕ, i.e. that substituting the phrases for the letters in ϕ does produce a material sentence.

> Aristotle's categorical logic, described in *Prior Analytics* i.4–7, has the formal sentences described in (8) above. He sometimes considers other sentence forms, for example unquantified forms; but for our purposes these are marginal to the logic. In *Prior Analytics* i.8–12 he can be read as working with a logic got by adding formal sentences as in (8) but with 'Necessarily' adjoined; this is sometimes called his apodeictic logic. In *Prior Analytics*

> i.13–22 he adds further formal sentences, adjoining 'Possibly' or 'Contingently'. We will speak of the logic with all these sentence forms as Aristotle's 'modal logic'.

In the definitions that follow, we assume we are working with a logic $\mathbb{L}$. Then all formal sentences under discussion are assumed to be formal sentences of $\mathbb{L}$, all interpretations are assumed to be interpretations of $\mathbb{L}$, and the $\vdash$-entailment relation is assumed to be that of $\mathbb{L}$. In such a context we can normally simplify '$\vdash$-entails' to 'entails'; but this can sometimes be unsafe when 'strange' usages are under discussion.

7. (In $\mathbb{L}$) Suppose Φ is a set of formal sentences and θ is a formal sentence, and I is an interpretation for the sentences in Φ and for θ. We say that I is a 'counterexample to the entailment $\Phi \vdash \theta$' if I is a model of Φ but not a model of θ.

8. We say that the logic $\mathbb{L}$, or its entailment relation, has the 'Tarski property' if the following condition holds:

(9) For all sets Φ of formal sentences and all formal sentences θ, if Φ entails θ then every model of Φ is a model of θ.

This is in view of a general resemblance to Tarski's paper [17]; but we are not assuming Tarski's set-theoretic definition of 'model'.

9. (In $\mathbb{L}$) Suppose ϕ and ψ are formal sentences with the same term letters. Then we say that ϕ and ψ are 'incompatible' if there is no interpretation for ϕ and ψ which is a model of both ϕ and ψ. Generalising this, we say that ψ is 'compatible' with a set Φ of formal sentences if there is a model of Φ which is also a model of ψ.

10. In some logics $\mathbb{L}$ the entailment relation is restricted so that '$\Phi \vdash \theta$' is never true unless Φ and θ stand in a certain simple syntactic relationship. When the logic makes a requirement of this kind, and the requirement holds for Φ and θ, we will say that θ is a 'candidate' (in relation to Φ). A set Φ of formal sentences will be called 'admissible' if at least one formal sentence is a candidate in relation to it. In some cases an admissible set of formal sentences has to be considered as linearly ordered.

> Aristotle describes his candidates by means of his notion of figures and the associated notions of major and minor premises and major (or first) and minor (or last) terms. I will assume that a pair of premises (a 'premise-pair') is written with minor premise first; the minor term is then the term letter occurring

> only in the minor premise, and similarly with major in place of minor. The candidates are the formal sentences whose subject is the minor term and whose predicate is the major term.

11. (In $\mathbb{L}$) Let Φ be a set of formal sentences. We say that Φ is 'productive' if there is at least one candidate θ such that Φ entails θ. If Φ is not productive we say that it is 'nonproductive'.

12. (In $\mathbb{L}$) A 'nonproductivity proof' for Φ is an argument showing that Φ is nonproductive. A 'nonentailment proof' for Φ and θ is an argument showing that Φ doesn't $\vdash$-entail θ.

> A nonproductivity proof for Φ could proceed by giving, for each candidate θ in relation to Φ, a nonentailment proof for Φ and θ. Aristotle's nonproductivity proofs can be read this way. For reasons not entirely clear, he gives no nonentailment proofs in *Prior Analytics* i.4–7, no nonproductivity proofs in *Prior Analytics* i.8–12, and then again no nonentailment proofs in *Prior Analytics* i.13–22 (with a very few exceptions).

The definitions above are enough for us to describe the logical content of Aristotle's methods for proving nonentailment and nonproductivity. But they are not enough to cover all the notions that he himself deploys when he uses those methods. So at this point we turn to his text; then we will give some more logical definitions in the light of what we find there.

4 Aristotle proves nonproductivity of categoricals

After some preliminary definitions, Aristotle begins his discussion of entailments in *Prior Analytics* i.2f with a treatment of conversions. The topic is a particular kind of entailment: there is a single premise, and the candidates are the sentences with the same term letters as the premise but in the opposite order. In *Prior Analytics* i.2 the logic is categorical logic; in *Prior Analytics* i.3 it is modal logic.

Aristotle states some entailments, for example that 'Some A is a B' entails 'Some B is an A'. He also gives examples of nonproductivity. For example 'Not every A is a B' doesn't entail 'Not every B is an A'. For this he simply gives the counterexample that not every animal is a human, though every human is an animal (i.2 25a22–26). He mentions the nonentailment that 'Every A is a B' doesn't entail 'Every B is an A', but he gives no proof. All of these logical claims are too straightforward to need any supporting theory. As Lee comments ([14] p. 79), 'In den an. pr. ist die Konversion kein selbständiger Lehrgegenstand, sondern nur ein Hilfsmittel zur Vervollkommnung der meisten unvollkommenen Syllogismen. Demgemäss wird sie knapp in dem Mass behandelt, wie es für den Aufbau der Syllogistik erforderlich ist.'

When he moves into categorical logic in *Prior Analytics* i.4–7, Aristotle gives some proofs of nonproductivity. (But curiously no proofs of nonentailment—that will come when he turns to apodeictic logic in *Prior Analytics* i.8–12.) He uses three methods, which we can call the 'counterexample' method, the 'reduction' method and the 'set-theoretic' method. The counterexample method is by far his preferred method, and it is the main one that we will consider below. In the 'reduction' method he shows that if a premise-pair was productive, then another premise-pair which he has already shown to be nonproductive would be productive; there is an example at *Prior Analytics* i.4 26b14–21.

The 'set-theoretic' method deserves some remarks, not least because of its importance in the later history of logic. For some cases of nonproductivity, Aristotle sets out the formal premises and then attempts a hands-on argument why nothing can follow from them, using just the set-theoretic notions of one class being disjoint from another, or being included in another, or extending beyond another. For example at i.5 27b11–15 he is considering a premise-pair in second figure with both premises negative. Thus (using his letters) none or not all of N is an M and none or not all of X is an M. Then, he says,

(10) *endékhetai dḕ kaì pantì kaì mēdenì tȭ$_i$* Ξ *tò* N *hupárkhein.*
N may belong to all as well as to no X. (i.5 27b15f, tr. [6] p. 8)

One can almost see him drawing circles to illustrate the argument! (But in fact he drops off at once into terms to cover one particular configuration.) Another example where set-theoretic language is visible is at i.14,33a38f, where he is considering a modal premise-pair in second figure:

oudèn gàr kōlúei tò B *huperteínein toû* A *kaì mḕ katēgoreîsthai ep' ísōn; hȭ$_i$ d' huperteínei tò* B *toû* A*, eilḗphthō tò* Γ*.*
(11) For nothing prevents B from extending beyond A and not being predicated of an equal number of things. Let C be taken as the part by which B extends beyond A. (i.14, 33a38–40, tr. [6] p. 21)

In the paper [13] we will find Abū al-Barakāt picking up the language used in the Arabic translation of this passage, particularly the phrase 'extend beyond' (*tafaḍḍala* in Arabic), and turning it literally into pictures. Unfortunately there are problems about applying set theory to modalities, and again in this passage Aristotle falls quickly back into an argument by counterexamples. Abū al-Barakāt's applications are in categorical logic.

Though it is not immediately obvious from Aristotle's text, his argument at (10) covers several moods simultaneously. Philoponus [10] 70.1–21 collected together a group of rules, taken from this and other places in Aristotle, that determine whether or not any given formal premise-pair is productive. From al-Fārābī (10th century) onwards there was a growing tendency in Arabic logic to give lists of 'conditions of productivity' based on the Philoponus rules, rather than using case-by-case arguments as Aristotle did. If the rules were justified at all, it was by informal set-theoretic arguments. These arguments tended not to be rigorous, and for modal logic they were totally inadequate. In the 12th century Abū al-Barakāt may have been alone in fighting for case-by-case methods.

We turn to Aristotle's normal method of proving nonproductivity or nonentailment, namely by counterexamples. His first proof of nonproductivity is at *Prior Analytics* i.4, 26a2–9. The passage is oddly repetitive; it contains five separate clauses that all mean 'the premises are nonproductive'. After removing these clauses the following skeleton remains:

(12)

ei dè tò mèn prô̂ton pantì tô̂$_i$ mésō$_i$ akoloutheî, tò dè méson mēdenì tô̂$_i$ eskhátō$_i$ hupárkhei,
(i) But if the first [term] follows all of the middle [term], and the middle [term] is true of none of the last [term],
...
kaì gàr pantì kaì mēdenì endékhetai tò prô̂ton tô̂$_i$ eskhátō$_i$ hupárkhein,
(ii) for it is possible for the first [term] to be true of all of the last [term], and of none of it,
...
hóroi toû pantì hupárkhein zô̂$_i$on ánthrōpos híppos, toû mēdenì zô̂$_i$on ánthrōpos lìthos.
(iii) Terms of being true of all are animal, human, horse, and [of being true] of none are animal, human, stone.

At (i) Aristotle describes the premise-pair that he will show to be nonproductive. With letters added for our convenience, it is:

(13) Every B is an A. No C is a B.

The clause at (ii) allows two readings. The first takes 'possibilities' to be models of the premises, so that Aristotle is saying

(14) There is a model I of the premises which is also a model of 'Every C is an A', and there is a model J of the premises which is also a model of 'No C is an A'.

Then clause (iii) spells out the required models. Namely I is: A animal, B human, C horse, and J is: A animal, B human, C stone. Aristotle expects us to check for ourselves that I and J have the required properties. Namely for I: Every human is an animal, and no horse is a human, so I is a model of the premises. Also every horse is an animal, so I is a model of 'Every C is an A'. Likewise for J.

Aristotle doesn't say why (ii) implies that the premise-pair is unproductive. We can fill in the details as follows. By (ii), I is not a model of 'Some C is not an A'. If syllogistic entailment has the Tarski property (9), then it follows at once that the premises don't entail 'Some C is not an A'. Hence they don't entail 'No C is an A' either, since that sentence implies 'Some C is not an A'. The corresponding argument with J shows that the premise-pair doesn't entail either 'Some C is an A' or 'Every C is an A'.

The Tarski property says exactly what is needed to complete the argument. As far as I know, there is no passage of Aristotle that could be taken as a discussion of whether or not syllogistic entailment has the Tarski property. The question is raised by Alexander [1] 238.35f, who may be responding to an earlier discussion that hasn't survived.

Patzig ([16] p. 27 middle) implies that 'Aristotle's method of disproof' needs both the Tarski property and its converse, namely that if every model of Φ is a model of θ then Φ entails θ. But Aristotle's method doesn't need the converse, and Aristotle himself doesn't mention the converse. In [13] we will see that the question of the converse to (9) does arise in the work of some of the commentators; it would be best not to beg any questions by reading an answer into the text of Aristotle.

As remarked above, there is a second reading of (ii). Namely, we can take 'it is possible that ϕ holds' to mean 'it is not necessary that ϕ is false', where the necessity in question is the kind that Aristotle attributes to the conclusion of a syllogism. Since 'Every B is an A' being false is the same thing as 'Some B is not an A' being true, and likewise with the other clause mentioned in (ii), this allows us to understand (ii) as saying

(15) There is no entailment from these premises to the conclusion 'Some B is not an A', and there is no entailment from them to the conclusion 'Some B is an A'.

On this reading, (ii) represents a later part of the same argument as in the first reading. It says what has been shown immediately after the Tarski property is applied to what the terms give us, namely that neither of the particular sentences is a conclusion from the premises.

Apart from the redundant clauses that we cut out earlier, Aristotle's statement of his method in (12) above is very sparse. He uses the same style throughout his proofs of nonproductivity in categorical logic. There is a complication at *Prior Analytics* 26a39–26b21 and some later passages; it results from his trying to treat o sentences as having the form 'Some B is an A and some is not'. This complication may throw an interesting light on how he came to invent categorical logic, but it raises no issues of principle about his methods.

5 Nonentailment and nonproductivity in modals

When he comes to apodeictic logic in *Prior Analytics* i.8–12, Aristotle switches from nonproductivity proofs to nonentailment proofs, and his style changes. Instead of simply stating an interpretation, he spells out the material instances of the premises under the interpretation, and after this he states the material instance of the relevant candidate. The first example reads thus:

(16) For example, if A were motion, B animal, the term designated by C, human. For human is an animal with necessity, but an animal does not move with necessity, nor does human. (i.9, 30a29–32, tr. [6] p.14, adjusted)

The interpretation is clearly stated. But there are two ways of reading the second sentence; they reflect a fundamental ambiguity in the whole of Aristotle's modal logic.

On one reading, which I will call the 'formal' reading, Aristotle is now considering formal sentences which may contain the phrase 'with necessity'. In the present case he is showing that the premise-pair

(17) Every B is an A. With necessity no C is a B.

doesn't entail the formal sentence

(18) With necessity every C is an A.

His method is to find a model of (17) which is not a model of 'Necessarily every C is an A'. After stating his interpretation I, he gives the I-instances of the two premises, so as to show that I is a model of them. The instance of the first premise should be 'Every animal moves'; he says 'An animal does not move of necessity', but we have to read him as meaning 'An animal moves, though not of necessity'. The truth about humans and moving is

(19) It is not the case that with necessity every human moves.

This is an instance of

(20) It is not the case that with necessity every C is an A.

which is not a formal sentence of Aristotle's modal logic, though later he will introduce 'With possibility some C is not an A' to serve the same purpose.

The other reading I will call the 'evaluative' reading. On this reading Aristotle is still working with categorical formal sentences, but he has introduced a new way of evaluating them—not just whether they are true, but also whether they are true with necessity. Then he is showing that the truth of 'Every B is an A' and the necessary truth of 'No C is a B' don't entail the necessary truth of 'Every C is an A'. The notion of an interpretation is as before, but he revises the notion of model. He seeks an interpretation I such that the I-instance of 'Every B is an A' is true and the I-instance of 'No C is a B' is a necessary truth. The categorical statement 'Every human moves' is true, but it is not a necessary truth.

This paper is concerned with what the commentators and translators found in Aristotle, not with what Aristotle himself meant. But my own understanding is that Aristotle throughout his modal logic is systematically ambiguous between the formal reading and the evaluative reading, and that his main surviving Greek commentators, Alexander and Philoponus, copy this ambiguity. The earliest surviving modal syllogistic in Arabic logic is that of Avicenna; although there are traces of both readings in his writings, he makes a definite switch towards the formal reading, and emphasises the point by inventing names for the types of formal sentence.

In any case it is very much easier to analyse what is happening under the formal reading, and so I will adopt the formal reading henceforth. (Most recent studies of Aristotle's modal logic do the same, as one can see from the profusion of labels for sentence forms.)

When he comes to study the full modal logic in *Prior Analytics* i.13–22, Aristotle switches back in general from proving nonentailment to proving nonproductivity. His proofs consist of two proofs of nonentailment, one for 'With necessity every C is an A' and one for 'With necessity no C is an A'.

For the two subproofs he ranges between the sparse style of *Prior Analytics* i.4–7 and the fuller presentation that he adopted in *Prior Analytics* i.8–12. One of the fuller examples is:

> *éstō gàr tò mèn* A *kórax, tò d' eph' hō̂i* B *dianooúmenon, eph' hō̂i dè* Γ *ánthōpos. oudenì dē tō̂i* B *tò A hupárkhei; oudèn gàr dianooúmenon kórax. tò dé* B *pantì endékhetai tō̂i* Γ*; pantì gàr anthrṑpōi tò dianoeîsthai. allà tò A ex anágkēs oudenì tō̂i* Γ*; ouk ára tò sumpérasma endekhómenon.*

(21) For let A be raven, what is designated by B, thinking, and what is designated by C, human. Then A is true of no B (for nothing that is thinking is a raven); but with possibility B is true of every C, for every human may be thinking. However, with necessity A is true of no C; therefore, the conclusion is not possible. (34b32–37, tr. [6] p. 24 adjusted)

Write I for this interpretation. The formal sentences 'A is true of no B' and 'With possibility B is true of every C' are in our notation 'No B is an A' and 'With possibility every C is a B'; this is the premise-pair being proved non-productive. The sentences 'Nothing that is thinking is a raven' and 'Every human may be thinking' are the I-instances of the premises, showing that I is a model of the premises. The formal sentence 'With necessity A is true of no C', i.e.

(22) With necessity no C is an A,

is found as follows. We look for a sentence with subject 'human' and predicate 'raven' that expresses what Striker in (4) calls the 'actual relation' between these two terms. This is

(23) With necessity, no human is a raven.

which is the I-interpretation of (22). Since I is a model of (22), it is not a model of 'With possibility some C is an A' (and this proves that the premises don't entail any of 'With possibility some C is an A', 'With necessity some C is an A', 'With possibility every C is an A'.)

Another example is (3), quoted earlier. The material sentence

(24) With necessity, [every] human is an animal.

expresses the 'actual relation' between the terms 'human' and 'animal'. Using the interpretation that Aristotle defines in (3), this sentence is the material instance of the formal sentence

(25) With necessity, C is an A.

This proves that the premises don't entail 'With possibility, some human is not an animal'. Since 'the premisses' in (3) are presumably the formal premises, I would guess that Aristotle means by 'the conclusion' here the sentence (22), though Striker (4) takes it to be 'the proposition that expresses the actual relation between the major and minor terms', which is (23).

Aristotle's third 'strange' remark (5) has a similar analysis, and again the sentence that he calls 'the conclusion' might be either the material sentence expressing the actual relation between major and minor terms, or it might be the formal sentence which has the material sentence as an instance.

The sentences (23), (22), (24) and (25) play a distinguished role in Aristotle's own explanations, and they are what he has in mind when he makes his 'strange' references to conclusions. I will refer to them as 'pseudoconclusions' of the premises; (23) and (24) are 'material pseudoconclusions', and (22) and (25) are 'formal pseudoconclusions'. (The name 'pseudoconclusion' may not be an entirely new coinage. The paper [13] will give grounds for thinking that its Arabic translation was used by Ibn al-Muqaffa^{c} for the same sentences.)

6 Logical definitions 2

We continue the definitions of Section 3, with the same numbering sequence.

13. Let Φ be a formal premise-pair in one of Aristotle's figures; write A and C for its minor and major terms respectively. By a 'material pseudoconclusion' of Φ we mean a true material sentence of the form $\theta[I]$ where θ is a candidate in relation to Φ and I is a model of Φ; the formal sentence θ is then called a 'formal pseudoconclusion' of Φ.

> Note some provable facts about formal pseudoconclusions. In the papers [12] and [13] we will see them in use.
>
> **Fact 1** *θ is a formal pseudoconclusion of Φ if and only if θ is a candidate of Φ which is compatible with Φ.*
>
> **Fact 2** *(Assuming the Tarski property) If Φ has a formal pseudoconclusion θ then Φ has no syllogistic conclusion which is incompatible with θ.*

Fact 3 *(Assuming the Tarski property) If Φ has two formal pseudoconclusions θ_1 and θ_2 with the property that every candidate is incompatible with at least one of θ_1 and θ_2, then Φ is nonproductive.*

14. Let Φ be a formal premise-pair, let ϕ and θ be candidates in relation to Φ and I a model of Φ. When we say that $\theta[I]$ 'rules out' ϕ, we mean: $\theta[I]$ is true (i.e. a pseudoconclusion of Φ), and θ is incompatible with ϕ.

Fact 4 *(Assuming the Tarski property) If the formal premise-pair Φ has a pseudoconclusion that rules out the formal sentence θ, then θ is not a syllogistic consequence of Φ. (This follows from Fact 3 above.)*

Aristotle uses *anaireî* at 33b12 to express 'rules out'; he is invoking a pseudoconclusion in order to prove a nonentailment. This is the only occurrence in *Prior Analytics*, but Alexander frequently uses the idiom.

7 Aristotle's descriptions of entailment

Some logicians thought that the 'strange' passages (3) and (5) gave them licence to use 'conclusion' as Aristotle uses it in these passages. They could justify the results by explaining that they were using 'conclusion' to mean 'pseudoconclusion'. But what did these logicians reckon they were entitled to do in view of (1) and Aristotle's use of 'demonstrate' in (5)?

Papers [12] and [13] will address this question. As background we need some facts about how Aristotle normally expressed entailments.

When Aristotle has sentences that he can write out as premises and conclusion, he commonly writes an entailment as a conditional statement: 'If [the premises] then it is necessary that [the conclusion]'. Alternatively he puts the conclusion into the future tense to imply necessity (cf. Patzig [16] p. 18). Typical examples are:

(26) *ei gàr tò* P *pantì tō̂i* Σ, *tò dè* Π *tinì mē hupárkhei, anágkē tò* Π *tinì tō̂i* P *mḕ hupárkhein.*
For if R belongs to all S but P does not belong to some S, it is necessary for P not to belong to some R. (28b17–19, tr. [6] p. 10)

(27) *ei gàr pantì tò* Π $t\hat{\bar{o}}_i$ Σ *hupárkhei, tò dè* P *tinì* $t\hat{\bar{o}}_i$ Σ*, kaì tò* Π *tinì* $t\hat{\bar{o}}_i$ P *hupárxei.*
... for if P belongs to all S and R to some, then P will also belong to some R. (28b26f, tr. [6] p. 10)

In this idiom the word that he normally uses for 'if' is *ei*.

This conditional idiom no longer works if Aristotle has a name X for the premises (for example 'the premises') or a name Y for the conclusion. One possible solution would be to adopt a transitive verb meaning 'entails', and say 'X entails Y'; but I know no Aristotelian verb that will fit in this role. Another solution would be to adopt a transitive verb meaning 'is entailed by', and say 'Y is-entailed-by X'. There are verbs that could in principle play this role, for example *hépesthai* and *akoloutheîn*, but in practice Aristotle uses these verbs in other senses.

Aristotle's normal solution for this problem is to adopt a convention that the conclusion is something that happens, and the premises are either a context in which it happens, or a situation that causes it to happen. So the main clause will say that a certain conclusion happens, or that some conclusion happens, or that no conclusion happens. The premises will appear indirectly, either in a subordinate clause or in some qualifying phrase. We will analyse the two parts of this construction: first the main clause and then the qualifying clause or phrase.

The main clause normally contains a verb expressing 'happens' or 'occurs' or 'results'. Aristotle's favourite verb for this purpose is *sumbaínei*, for example in Aristotle's definition of 'syllogism':

(28) *... tethéntōn tinôn héterón ti tôn keiménōn ex anákgēs sumbaínei* $t\hat{\bar{o}}_i$ *taûta eînai.*
... certain things being posited, something other than what was laid down results by necessity because these things are so. (i.1, 24b19f, tr. [6] p. 2)

Other verbs that he uses are *gínetai* 'becomes' and *sumpíptei* 'comes to be'. (Alexander's preferred verb for this use is *sunágetai* 'is brought about'. Aristotle doesn't use this verb in this construction, but Alexander could have quoted Aristotle's *Rhetoric* [7] 1357a8 and 1395b25 for related uses of the verb.)

He may also add that what happens (or fails to happen) is 'necessary'.

(29) *tethéntos gàr toû* B ... *mē̂ hupárkhein oudèn sumbaínei pseûdos.*
For if it is assumed that B is not true of ..., nothing false results. (*Prior Analytics* i.17, 37a35f)

oudèn gàr anagkaîon sumbaínei tỗᵢ taûta eînai.

(30) For nothing necessary results from those things being so. (*Prior Analytics* i.4, 26a4f)

Sometimes, as in the conditional construction, Aristotle expresses necessity by putting the main clause verb into the future, for example as *éstai* 'will be' in the phrase 'there will be a syllogism'.

One unfortunate feature of this idiom is that it can leave open several possibilities for what exactly it is that 'happens'. If the conclusion happens, then there are other things that may happen at the same time—for example that the conclusion is drawn, or that it is established what the conclusion is. Of course one can take care to discriminate between these possibilities; but Aristotle himself doesn't always do so. For example when he says 'there will be a syllogism', should we read him as saying that there will be a conclusion, or that there will be a drawing of a conclusion? A statement like (30) leaves the question open, since either the conclusion itself or the deducing of it could be described as 'necessary'.

In fact the commentators are clearly much more interested in the conclusion itself than they are in the process of drawing it. There are also texts to support them, for example (29) which describes the thing that happens as 'false'; prima facie, statements can be false but processes can't be. One could also cite 26a3f, where the syllogism that happens is said to be 'of the two extreme terms'; or (32) below, where the syllogism that happens is spelled out as a formal sentence. When Paul the Persian talks of 'necessary conclusions', he is presumably relying on passages like (30) and assuming that the conclusion is what results or fails to result.

Aristotle takes a risk when he uses words like 'necessary' in this construction. From *Prior Analytics* i.8 onwards he will be considering sentences that are necessary in the sense of being necessary truths. Although he himself is very clear about the difference between saying that a conclusion follows necessarily from the premises and saying that the conclusion is a necessary truth (*Prior Analytics* i.10, 30b31–40), there are still places where there is room for disagreement about which he had in mind. But for Paul the Persian this problem doesn't arise, because he considers only categorical logic.

We turn to the qualifying phrase or clause that mentions the premises. This phrase or clause can take several forms. Very often it is a genitive absolute. Sometimes it is a prepositional phrase. It can also be a subordinate clause beginning *eàn* or *hótan* ('if' or 'whenever').

An example to illustrate the genitive absolute is the definition of syllogism at (28). The genitive absolute is 'certain things being posited'. There is also a dative participial clause 'because these things are so'. As I read it, both of these phrases refer to the premises, but the genitive absolute says that the premises are the context of the conclusion while the dative participial clause says that they are the cause of the conclusion.

Alexander uses prepositional phrases in this construction more than Aristotle does. But Aristotle does have examples:

(31) *... ex autôn mèn tôn eilēmménōn protáseōn oudeìs éstai sullogismós.*
... there will be no syllogism from the premisses themselves as they are taken. (i.15 35a4f, tr. [6] p. 24)

An example with a subordinate clause is:

(32) *hótan oûn tò* A *pantì tôi* B *endékhetai kaì tò* B *pantì tôi* Γ*, sullogismòs éstai téleios hóti tò* A *pantì tôi* Γ *endékhetai hupárkhein.*
Now whenever it is possible for A to belong to every B and for B to belong to every C, there will be a perfect syllogism to the effect that it is possible for A to belong to every C. (i.14, 32b8–10, tr. [6] p. 20)

In this example the content of the subordinate clause is the premises themselves; but sometimes the subordinate clause states that the premises are posited, as at i.15, 35a3–5.

These indirect constructions are not in the interests of clarity. Classical Greek allows one to put all sorts of things into subclauses and indirect phrases, leaving it to the reader to work out from the context what their relationship to the main verb is.

In fact the looseness of the whole construction allows Aristotle to use it when what happens is a pseudoconclusion, not a syllogistic conclusion. His strange statement (1) above is a perfect example, where the main clause states that two sentences can occur as pseudoconclusions, and the premises appear in a genitive absolute. Although the formal syntax is the same as when Aristotle is describing an entailment, in (1) he can't be read as saying that an entailment occurs.

The strange statement (5) also says that certain pseudoconclusions occur, and again there are genitive absolutes. But in this case they serve to classify the forms of the pseudoconclusions; they say nothing at all about the premises. There is a prepositional phrase 'through terms', but now it doesn't refer to the premises; this change reflects the fact that a pseudoconclusion is worked out from the terms and not from the premises.

8 Appendix

THE TABLES BELOW ARE BEING REVISED AS THE OTHER TWO PAPERS ARE WRITTEN

The charts in this appendix show the passages in which Aristotle discusses proofs of nonproductivity or nonentailment, together with references to passages covering the same questions in Alexander's commentary on *Prior Analytics* i (Alex), Paul the Persian's *Logic* (Paul), Theodorus' translation of *Prior Analytics* i.1–22 (Theo), Avicenna's *Qiyās* (Avic) and Abū al-Barakāt's *Mu^{c}tabar* (Bara). The division into distinct passages is a little arbitrary; I was guided partly by the paragraph divisions in [2], [3] and [4].

Passages that show signs of Aristotle's 'strange' terminology are in **bold**. Strangeness is an imprecise notion and largely in the eye of the beholder; but I count a locution as strange if it describes a pseudoconclusion as a conclusion, or more generally if it seems to suggest that pseudoconclusions are entailed or derived in the same sense as logical conclusions are. I have not highlighted passages which speak of pseudoconclusions being deduced *from the premises*, because (contrary to some claims made about Alexander in the modern literature) I don't believe there are any such passages in these works.

Avicenna also has some thirty nonproductivity proofs in his propositional logic in *Qiyās*, and in some of them he uses strange locutions. None of these passages relate directly to places where Aristotle notes nonproductivity, though some do indirectly. It seemed best to leave the listing of these passages of Avicenna to the paper [13].

A. Proofs of categorical nonproductivity

#	Aristotle	content	Alex	Paul	Theo	Bara
		1st FIG				
1	i.4, 26a2–9	$(e),(a) \nvdash$	55.10–57.4	19.13–18	192.7–9	132(2)
2	i.4, 26a9–13	$(e),(e) \nvdash$	57.6–18	19.19–25	193.1–4	132(1)
3	i.4, 26a30–34	$(a),(i) \nvdash$	61.8–62.9	20.18–23	194.11–195.3	133
4	“ “	$(a),(o) \nvdash$	61.8–62.9	20.30–21.4	“ “	134
5	i.4, 26a36–39	$(e),(i) \nvdash$	62.13–30	20.24–29	195.4–7	135(1)
6	“ “	$(e),(o) \nvdash$	62.13–30	21.5–10	“ “	135(2)
7	i.4, 26b3–10	$(o),(a) \nvdash$	63.8–65.11	20.4–10	195.8–15	135(3)
8	i.4, 26b10–14	$(o),(e) \nvdash$	65.12–68.8	20.11–17	195.15–18	136(1)
9	i.4, 26b21–25	$(i),(i) \nvdash$	68.9–69.4	21.11–16	196.7–12	136(4)
10	“ “	$(o),(i) \nvdash$	“ “	21.17–23	“ “	136(2)
11	“ “	$(i),(o) \nvdash$	“ “	21.24–30	“ “	136(5)
12	“ “	$(o),(o) \nvdash$	“ “	22.1–6	“ “	136(3)
		2nd FIG				
13	i.5, 27a18–20	$(a),(a) \nvdash$	81.3–82.1	22.7–12	198.14–16	142(1)
14	i.5, 27a20–23	$(e),(e) \nvdash$	82.2–13	22.23–28	198.18–20	140(1)
15	i.5 , 27b4–6	$(a),(o) \nvdash$	85.16–29	23.30–24.5	199.17–19	143(3)
16	i.5, 27b6–8	$(e),(i) \nvdash$	86.2–10	23.25–29	199.19–22	143(4)
17	i.5, 27b14–16	$(o),(e) \nvdash$	87.1–6	23.13–18	200.3–6	140(2)
18	i.5, 27b23–28	$(i),(a) \nvdash$	88.15–28	22.29–23.2	201.3–6	142(2)
19	i.5, 27b29–32	$(e),(o) \nvdash$	92.2–18	24.6–10	201.8–11	140(3)
20	i.5, 27b32–34	$(a),(i) \nvdash$	92.19–24	23.19-24	201.11–13	143(1)
21	i.5, 27b36–39	$(i),(i) \nvdash$	92.25–93.12	24.11–15	201.14–202.3	143(2)
22	“ “	$(o),(i) \nvdash$	“ “	24.16–22	“ “	“ “
23	“ “	$(i),(o) \nvdash$	“ “	24.23–28	“ “	143(5,6)
24	“ “	$(o),(o) \nvdash$	“ “	24.29–34	“ “	141
		3rd FIG				
25	i.6, 28a30–33	$(e),(a) \nvdash$	101.11–23	25.10–15	204.8–11	–
26	i.6, 28a33–35	$(e),(e) \nvdash$	101.24–31	25.16–21	204.11–13	–
27	i.6, 28b22–24	$(o),(a) \nvdash$	104.13–106.3	25.31–26.5	205.19f–206.1	–
28	i.6, 28b36-38	$(e),(i) \nvdash$	106.23–31	26.20–25	206.10–12	–
29	i.6, 28b38–29a2	$(e),(o) \nvdash$	107.2–8	26.26–31	206.12–15	–
30	i.6, 28b38–29a3	$(o),(e) \nvdash$	107.10–31	26.6–10	206.15f	–
31	i.6, 29a6–10	$(i),(i) \nvdash$	108.2–15	27.1–6	207.4–9	–
32	i.6, 29a6–10	$(i),(o) \nvdash$	108.2–15	27.7–12	“ “	–
33	i.6, 29a6–10	$(o),(i) \nvdash$	108.2–15	27.13–18	“ “	–
34	i.6, 29a6–10	$(o),(o) \nvdash$	108.2–15	27.19–24	“ “	–

B. Proofs of modal nonentailment

#	page	type	Alex	Theo	Avic
		1st FIG			
1	i.9, 30a23–30	$(a\text{-}nec), (a) \nvdash (a\text{-}nec)$	129.23–130.24		
2	i.9, 30b3–6	$(i\text{-}nec), (a) \nvdash (i\text{-}nec)$	133.17–135.19		
		2nd FIG			
3	i.10, 30b19–35	$(e\text{-}nec), (a) \nvdash (e\text{-}nec)$	137.24–139.27		151.12
4	i.10, 31a10–15	$(o), (a\text{-}nec) \nvdash (o\text{-}nec)$	143.4–28		
5	i.10, 31a15–17	$(o\text{-}nec), (a) \nvdash (o\text{-}nec)$	143.28–145.20		
		3rd FIG			
6	i.11, 31a37–b5	$(i\text{-}nec), (e) \nvdash (o\text{-}nec)$	147.5–10		
7	i.11, 31b20–29	$(i\text{-}nec), (a) \nvdash (i\text{-}nec)$	148.27–33		
8	i.11, 31b37–32a1	$(i\text{-}nec), (e) \nvdash (o\text{-}nec)$	150.3–13		
9	i.11, 32a4f	$(a), (o\text{-}nec) \nvdash (o\text{-}nec)$	150.25–151.30		
		1st FIG			
10	i.15, 34b33	$(a\text{-}pos), (e)$	195.18–34		197.12

C. Proofs of modal nonproductivity

#	Aris	Alex	Theo	Avic
1	i.14, **33a34–b8**	171.19–172.5		188.9–189.5
2	i.15, **34b14–17**	191.1–8	**235.7–12**	189.7–17
3	i.15, 35a20–24	201.4–24	**237.8–11**	
4	i.15, 35b8–10	203.11–35	**238.9–12**	
5	i.15, 35b14–19	(204.2–23)		
6	i.16, 36a27–31	211.19–212.3		
7	i.16, 36b3–7	214.19–28	**242.17**	
8	i.16, 36b7–10	214.28–215.5	**243.3**	
9	i.16, 36b7–12	" "		
10	i.16, 36b12–15	215.4–19	**243.7**	
11	i.17, **37a32–37b10**	229.15–27	**247.5–8**	??205.7
12	i.17, 37b35–38	233.24–29	**249.1**	
13	i.19, 38a27–34	236.15–237.23	(251.1–9)	218.16
14	i.19, 38b2f	237.23–238.11	**251.13–16**	
15	i.19, 38b14–20	239.20–240.11	(252.8–12)	222.1, 222.10
16	i.20, **39b2–6**	244.34–40		
17	i.22, 40a35–38	251.11–21	**259.11–15**	
18	i.22, 40b10–12	253.27–254.9		

Primary references

[1] Alexander of Aphrodisias, *Alexandri in Aristotelis Analyticorum Priorum Librum I Commentarium*, ed. Maximilian Wallies, Reimer,, Berlin 1883.

[2] Alexander of Aphrodisias, *On Aristotle Prior Analytics 1.1–7*, trans. and commentary by Jonathan Barnes, Susanne Bobzien, Kevin Flannery SJ and Katerina Ierodiakonou, Ancient Commentators on Aristotle, Duckworth, London 1991.

[3] Alexander of Aphrodisias, *On Aristotle Prior Analytics 1.8–13 (with 1.17, 36b35–37a31)*, trans. and commentary by Ian Mueller with Josiah Gould, Ancient Commentators on Aristotle, Bloomsbury, London 1999.

[4] Alexander of Aphrodisias, *On Aristotle Prior Analytics 1.14–22*, trans. and commentary by Ian Mueller with Josiah Gould, Ancient Commentators on Aristotle, Bloomsbury, London 2013.

[5] W. D. Ross, *Aristotle's Prior and Posterior Analytics: A Revised Text with Introduction and Commentary*, Clarendon Press, Oxford 1949.

[6] Aristotle, *Prior Analytics Book I*, trans. and commentary by Gisela Striker, Clarendon Press, Oxford 2009.

[7] W. D. Ross ed., *Aristotelis Ars Rhetorica*, Clarendon Press, Oxford 1959.

[8] Abū al-Barakāt, *Kitāb al-muᶜtabar fī al-ḥikmat al-ilāhiyya*, Byblion, Jbeil Lebanon 2007.

[9] Paul the Persian, *Logica*, in Jan Pieter Nicolaas Land ed., *Anecdota Syriaca* Vol. 4, Brill, Lugdunum Batavorum (Katwijk) 1875, pp. 1–30.

[10] John Philoponus, *In Aristotelis Analytica Priora Commentaria*, ed. M. Wallies, Reimer, Berlin 1905.

Secondary references

[11] Albrecht Becker, *Die Aristotelische Theorie der Möglichkeitsschlüsse*, Junker, Berlin 1933.

[12] Wilfrid Hodges, 'Nonproductivity proofs from Alexander to Abū al-Barakāt: 2. Alexander and Paul', preprint.

[13] Wilfrid Hodges, 'Nonproductivity proofs from Alexander to Abū al-Barakāt: 3. The Arabic logicians', preprint.

[14] Tae-Soo Lee, *Die Griechische Tradition der Aristotelischen Syllogistik in der Spätantike*, Vandenhoeck and Ruprecht, Göttingen 1984.

[15] Marko Malink, *Aristotle's Modal Syllogistic*, Harvard University Press, Cambridge Mass. 2013.

[16] Günther Patzig, *Aristotle's Theory of The Syllogism: A Logico-Philological Study of Book A of the Prior Analytics*, tr. Jonathan Barnes, Reidel, Dordrecht 1968.

[17] Alfred Tarski, 'On the concept of logical consequence', in Alfred Tarski, *Logic, Semantics, Metamathematics: papers from 1923 to 1938*, ed. John Corcoran, Hackett Publishing Company, Indianapolis, Indiana (1983), pp. 409–420